

Tutorial 6: Strain

APL 108 - 2025 (Mechanics of Solids)

1. Think of the following displacement field in the body:

$$\begin{aligned}u_1 &= 0.05x_1 + 0.03x_2^2, \\u_2 &= 0.07x_1x_2 + 0.08x_1^2, \\u_3 &= 0.\end{aligned}$$

- (a) Find the longitudinal strain of a line element along $\underline{e}_1$ direction at any point in the body.
 - (b) Determine the shear strain between line elements along $\underline{e}_1$ and $\underline{e}_2$
 - (c) Find volumetric strain for this displacement field. Does it vary from point to point?
 - (d) What is the shear strain between line elements along $\underline{e}_1$ and $\underline{e}_3$ at any point (x_1, x_2) ?
 - (e) Determine the average local rigid-body rotation tensor.
2. The displacement field for a body is given by

$$\underline{u} = k(x^2 + y)\hat{i} + k(y + z)\hat{j} + k(x^2 + 2z^2)\hat{k}$$

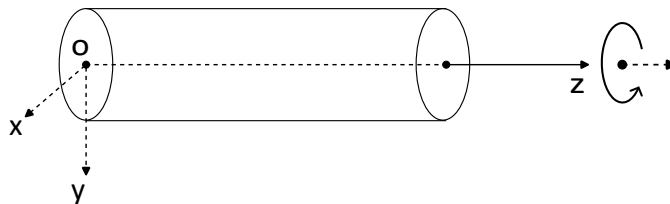
Find the volumetric strain, shear strains γ_{xy} and γ_{yz} , and the average local rotation tensor of the body at point $(2, 2, 3)$.

3. The displacement gradient matrix at a point in a body is given by

$$[\underline{\underline{H}}] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Write the condition for zero average local rotation.

4. For a circular rod subjected to a torque (shown in figure below), the displacement components obtained at any point (x, y, z) are as follows:



$$\begin{aligned}u_x &= -\tau yz + ay + bz + c, \\u_y &= \tau xz - ax + ez + f, \\u_z &= -bx - ey + k\end{aligned}$$

where a, b, c, e, f and k are constants and τ denotes twist.

(a) Select the constants a, b, c, e, f, k such that the end section $z = 0$ is fixed in the following manner:

- Point o has no displacement.
- The element Δz of the axis rotates neither in the plane xoz nor in the plane $yo z$
- The element Δy of the axis does not rotate in the plane xoy .

(b) Determine the strain components.

(c) Verify whether these strain components satisfy the compatibility conditions.

5. For the displacement field $u_x = k(x^2 + 2z)$, $u_y = k(4x + 2y^2 + z)$, $u_z = 4kz^2$ with $k = 0.001$, determine the change in angle between two lines segments PQ and PR at $P(2, 2, 3)$ having direction cosines before deformation as follows:

$$\begin{aligned}\text{PQ: } n_{x1} &= 0, \quad n_{y1} = n_{z1} = \frac{1}{\sqrt{2}} \\ \text{PR: } n_{x2} &= 1, \quad n_{y2} = n_{z2} = 0\end{aligned}$$

6. Verify whether the following strain field satisfies the equations of compatibility. Here p is a constant.

$$\begin{aligned}\epsilon_{xx} &= py, \quad \epsilon_{yy} = px, \quad \epsilon_{zz} = 2p(x + y) \\ \gamma_{xy} &= p(x + y), \quad \epsilon_{yz} = 2pz, \quad \epsilon_{zx} = 2pz\end{aligned}$$

7. Given the following formulas for strain components:

$$\begin{aligned}\epsilon_{xx} &= 5 + x^2 + y^2 + x^4 + y^4, \\ \epsilon_{yy} &= 6 + 3x^2 + 3y^2 + x^4 + y^4, \\ \gamma_{xy} &= 10 + 4xy(x^2 + y^2 + 2), \\ \epsilon_{zz} &= \gamma_{yz} = \gamma_{zx} = 0.\end{aligned}$$

(a) Determine whether the above strain field is possible. If it is possible, determine the displacement components in terms of x and y . Assume that $u_x = u_y = 0$ and $\omega_{xy} = 0$ at the origin.

(b) For the state of strain given in the previous problem, write down the spherical and deviatoric parts and also determine the volumetric strain.