Tutorial 3: Traction and Stress Equilibrium equations

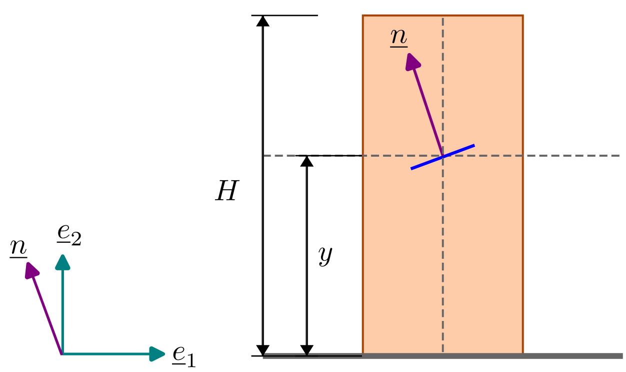
APL 108 - 2025 (Mechanics of Solids)

1. Show that $\underline{T}^n = \sum_i^3 \underline{T}^i\,(\underline{n}\cdot\underline{e}_i) = \sum_i^3 \underline{T}^{\hat{i}}\,(\underline{n}\cdot\underline{\hat{e}}_i)$, i.e, the formula is independent of what three planes are chosen to determine $\underline{T}^n$!

2. Suppose $\left[\underline{T}^1\right] = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\left[\underline{T}^2\right] = \begin{bmatrix}1\\5\\7\end{bmatrix}$, $\left[\underline{T}^3\right] = \begin{bmatrix}0\\7\\9\end{bmatrix}$ in $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ coordinate system.
   What will be the traction on a plane with normal $\underline{n} = \underline{\hat{e}}_1$ where $(\underline{\hat{e}}_1, \underline{\hat{e}}_2, \underline{\hat{e}}_3)$ is obtained from rotation of $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ about $\underline{e}_3$ by 45°? What are the normal and shear components of traction on this plane?

3. Show that the component of a traction vector on $\underline{n}$-plane in the direction $\underline{m}$ equals the component of the traction on $\underline{m}$-plane in the direction $\underline{n}$, i.e, $\underline{T}^n\cdot\underline{m} = \underline{T}^m\cdot\underline{n}$.

4. Consider a vertical bar having mass density ρ. Assume its length be to H and is subjected to uniform body force due to gravity. Find the traction vector on an infinitesimal internal section of the bar located at the center of its cross-section with outward normal
   $$\underline{n} = -\sin\theta\,\underline{e}_1 + \cos\theta\,\underline{e}_2$$
   and at a height of y from the base (see figure). Also find the normal and tangential components of the traction vector on this plane.



5. The state of stress at a point is given by $\left[\underline{\underline{\sigma}}\right] = \begin{bmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$.

   What should be σ_{11} such that there is at least one plane at that point on which the traction vanishes? Also, find the corresponding plane normal.

6. Suppose the stress matrix at a point equals $\left[\underline{\underline{\sigma}}\right] = \begin{bmatrix} a & 0 & d \\ 0 & b & e \\ d & e & c \end{bmatrix}$.

   Determine the plane having its normal perpendicular to z-axis such that the traction on that plane is tangential to the plane.

7. The sectional view of a dam is shown in Fig.1.

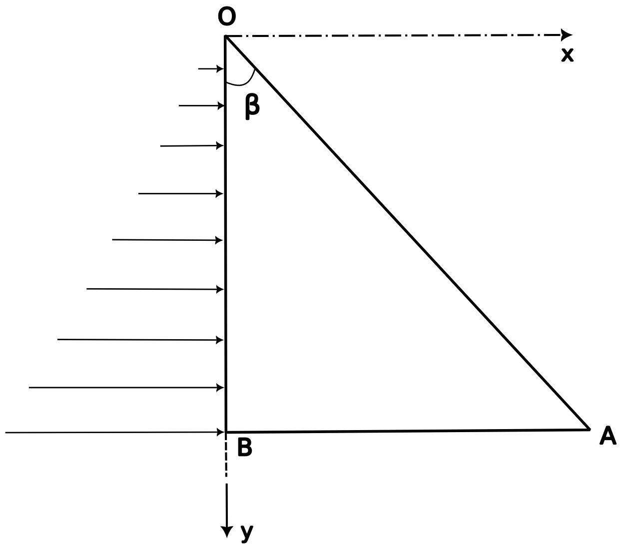


Figure 1

The pressure of water on the vertical face (denoted by line OB) is also shown. With the axes Ox and Oy, as shown in Fig.1, the stress components at any point (x, y) are given by (γ = specific weight of water and ρ = specific weight of dam material)

$$\sigma_{xx} = -\gamma\, y$$

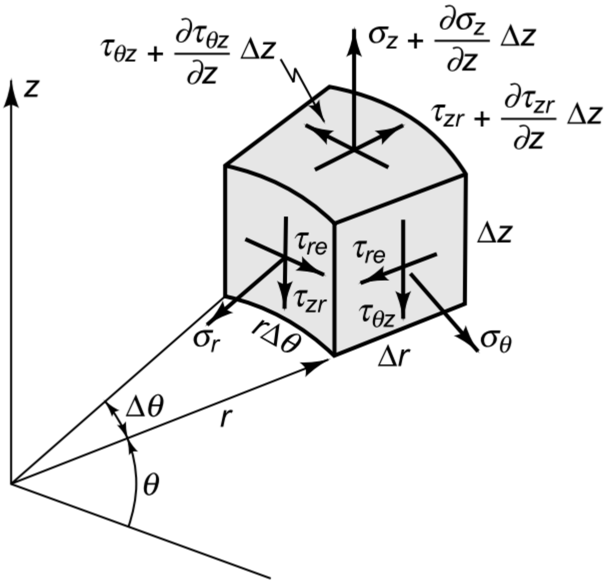
$$\sigma_{yy} = \left(\frac{\rho}{\tan\beta} - \frac{2\gamma}{\tan^3\beta}\right)x + \left(\frac{\gamma}{\tan^2\beta} - \rho\right)y$$

$$\tau_{xy} = \tau_{yx} = -\frac{\gamma}{\tan^2\beta}x$$

$$\tau_{yz} = 0,\ \tau_{zx} = 0,\ \sigma_{zz} = 0$$

Check if these stress components satisfy the differential equations of equilibrium. Also, verify if the boundary conditions are satisfied on the vertical face OB.

8. Show that if a general state of stress is to be described in cylindrical coordinates, the requirement that $\sum \underline{F} = \underline{0}$ leads to the following three equations:



$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + 2\frac{\tau_{r\theta}}{r} = 0$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = 0$$