Lecture 2

Recap

We had discussed in the last class that it is not enough to assess equilibrium of a deformable body solely based on the resultant of the external forces and moments being zero. We also need to ensure that any isolated subsystem of the system is also under equilibrium for a deformable body.

Analysis of Deformable Bodies

In this course, we will restrict ourselves to situations in which the acceleration is zero and where the geometry changes are only restricted to deformation.

There are THREE steps for analyzing a deformable body:

1) Study of forces and equilibrium requirements

To ensure that a material body is in equilibrium, we isolate the body from its surrounding environment and replace the environment with the forces it exerts on the body

A free-body-diagram (FBD) is a visual representation showing all external forces acting on the isolate body

Static equilibrium of an FBD implies Ⓐ and Ⓑ must be satisfied:

$$\sum \underline{F} = \underline{0} \quad \text{(Net resultant force = 0)}$$

$$\sum \underline{M}_o = \underline{0} \quad \text{(Net resultant moment = 0)}$$

2) Study of deformation and conditions of geometric fit

Deformation or geometry changes to a material body cannot be arbitrary; they will follow some rules s.t. each deformed portion of the member fits together with adjacent portions

Put another way, we want to find what are the requirements for geometric compatibility to ensure that the deformations are not arbitrary.

3) Application of force-deformation relations

We need to know a mathematical relation between the forces (the cause) and the deformation (the effect)

Typically, all three steps are required which results in three types of equations that need to be solved for deformable bodies:

1) Equilibrium conditions

2) Kinematic relations (relates strain to displacements)

3) Constitutive relations (relates stress to strains)

Extra set of eqns for deformable bodies

Statically Determinate Vs Statically Indeterminate

When the unknown support reactions and internal forces of a system can be determined using the Equilibrium conditions alone, then the system is called statically determinate

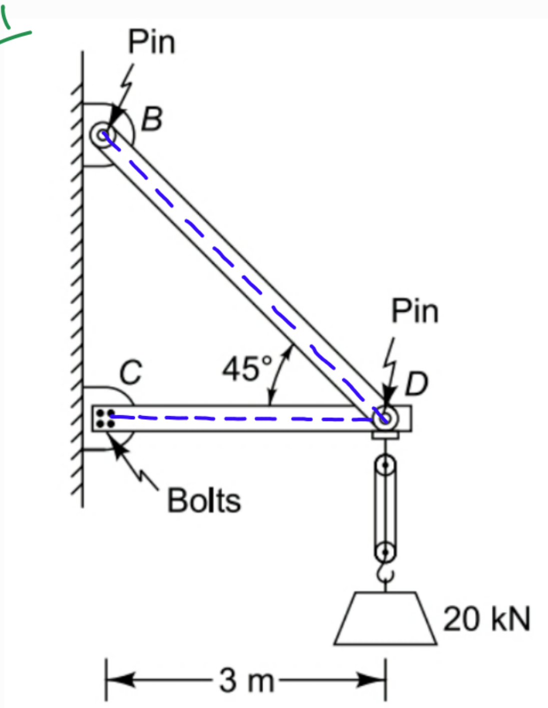
↳ for these cases, the three set of equations are uncoupled

When the unknown reactions and internal forces cannot be determined from the Equilibrium conditions alone, then such systems are statically indeterminate

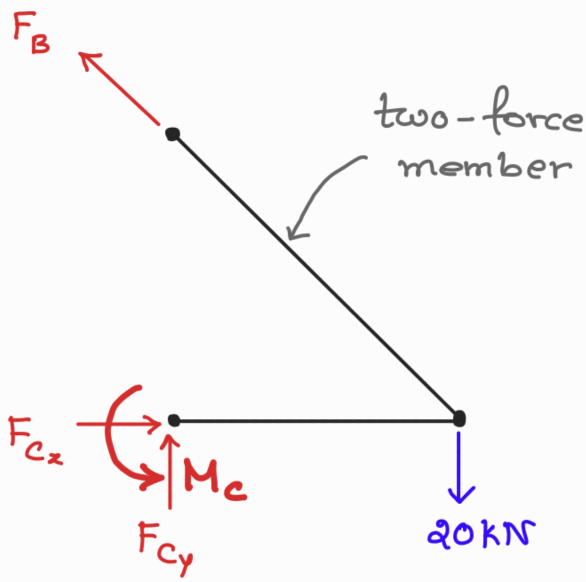
For these systems, the equilibrium eqns, kinematic relations, and the constitutive relations form a coupled system of equations, which implies that one has to solve all these three sets of equations together to find unknown quantities (e.g. internal forces, deformations)

Consider some examples of statically indeterminate systems:

Ex 1



FBD of entire structure



Unknowns

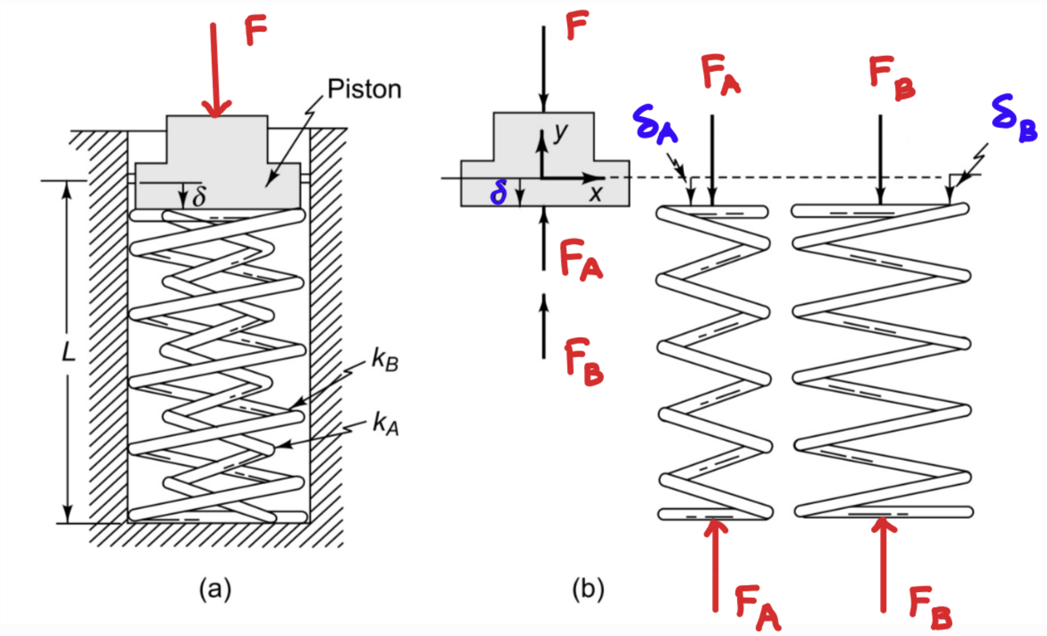
4 unknowns:
- F_B (magnitude only)
- F_C (magnitude & dir.)
- M_C (magnitude)

Knowns

Coplanar force sys.
- $\Sigma F_x = 0, \quad \Sigma F_y = 0$
- $\Sigma M_z = 0$

3 equations

Ex 2 A machine part applies a load F on a piston which fits into a cavity with two coaxial springs. Each spring deforms proportional to the force applied, and having different spring constants, k_A and k_B



What is the load carried in each spring?

Solu

* Study of forces and equilibrium requirements

Since there are no forces along x or z-directions

$$\Rightarrow \Sigma F_x = 0 \quad \text{and} \quad \Sigma F_z = 0$$

Also, since all forces are assumed to have same line of action $\Rightarrow \Sigma M = 0$ and $\Sigma F_y = 0$

$$\Rightarrow \quad F = F_A + F_B \quad \text{— (1)}$$

* Study of deformation and conditions of geometric compatibility

Since the springs are in touch with the piston, the springs and the piston must experience the same amount of deformation

$$\therefore \quad \delta_A = \delta_B = \delta \quad \text{---} \ (2)$$

* Application of force-deformation relations

$$F_A = k_A \delta_A \quad \text{---} \ (3)$$

$$F_B = k_B \delta_B \quad \text{---} \ (4)$$

Adding (3) and (4)

$$F_A + F_B = k_A \delta_A + k_B \delta_B \quad \text{---} \ (5)$$

Using (1) and (2) in (5)

$$F = (k_A + k_B)\,\delta$$

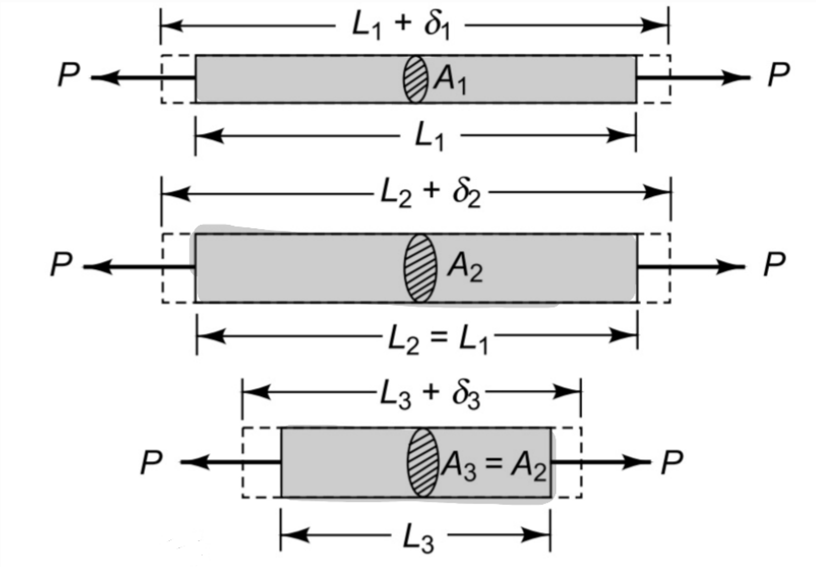
$$\Rightarrow \delta = \frac{F}{k_A + k_B} \quad \text{---} \ (6)$$

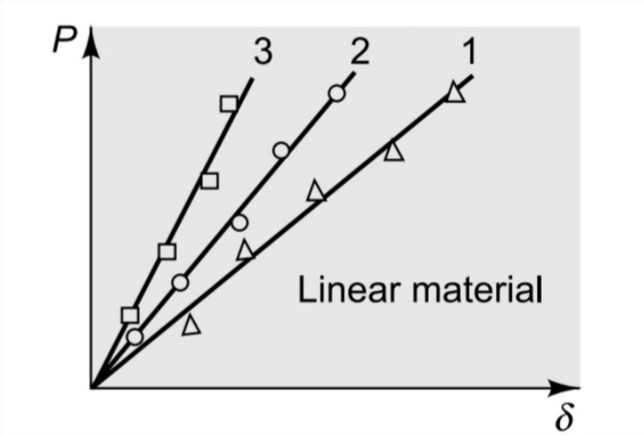
Therefore, $F_A = \dfrac{k_A}{k_A + k_B} F$, $F_B = \dfrac{k_B}{k_A + k_B} F$

Uniaxial Loading and Deformation

The basic and a common type of deformation in several structural members (such as trusses) is elongation or compression under uniaxial loading.

Say three bars are loaded by two equal and opp. forces and we look at the deformation of these bars made of SAME MATERIAL but different lengths and C/s areas





For each bar, the load is gradually increased from zero and at several values of the load a measurement is made of the elongation δ.

If the maximum elongation is VERY SMALL ($< 0.1\%$ of L) the most materials load-vs-elongation graph looks linear

If the uniaxial load-vs-elongation relation of the mater. is linear, then the relation can be fit using a straight line. The slope of the line is called Young's modulus of elasticity, and usually denoted by symbol E

$$E = \frac{\overbrace{P/A}^{\text{average stress}}}{\underbrace{\delta/L}_{\text{average strain}}}$$

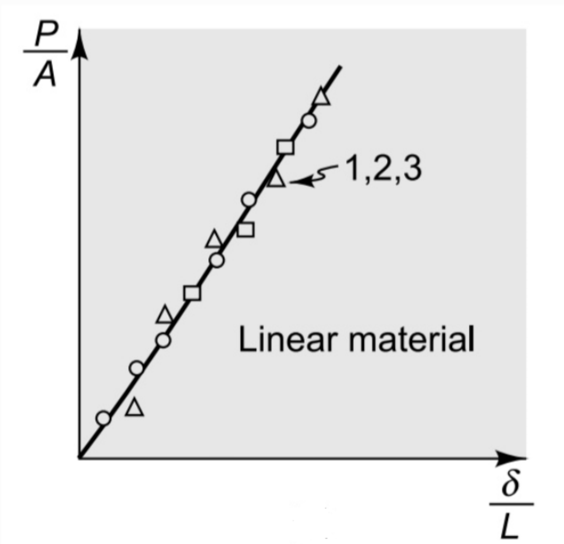
For a linear material behavior, we thus obtain an expression of elongation/compression under uniaxial loading:

$$\boxed{\delta = \frac{PL}{AE}}$$

← Most simple form of Hooke's law

only uniaxial case

Average stress

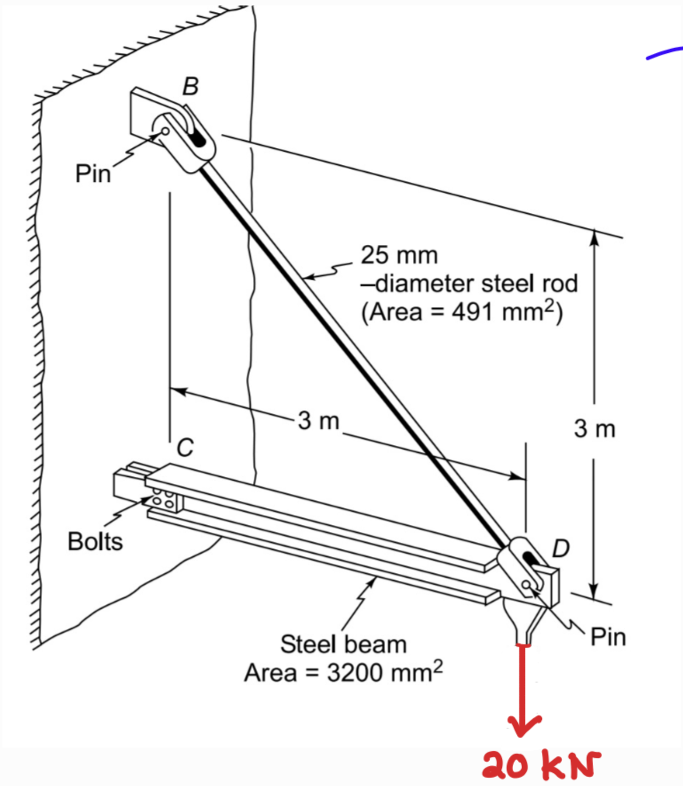


Average strain

For the same material, the average stress-vs-average strain behavior is same

Let's now analyze deformation of a truss

Ex 3 :



Modeling assumptions

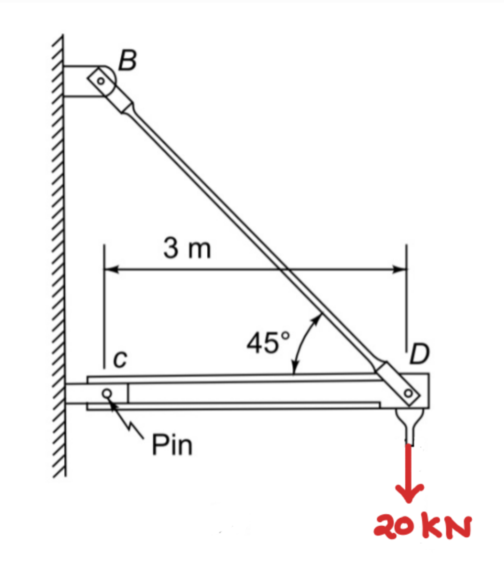
* Coplanar force system

* Pin connections are assumed frictionless

* Bolt connection idealized as frictiones pin connection

$E = 205 \text{ GPa}$ (Steel)

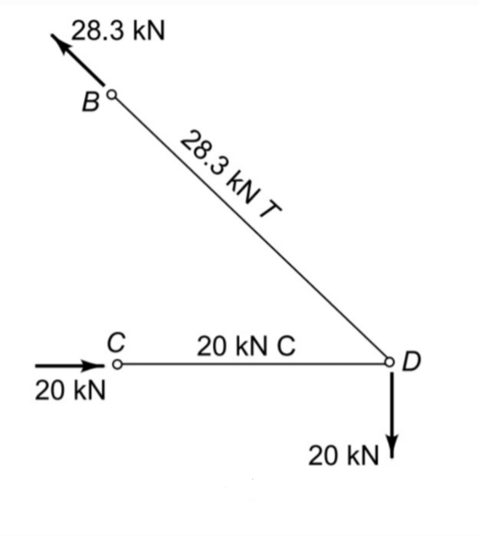
$= 205 \times 10^6 \text{ kN/m}^2$

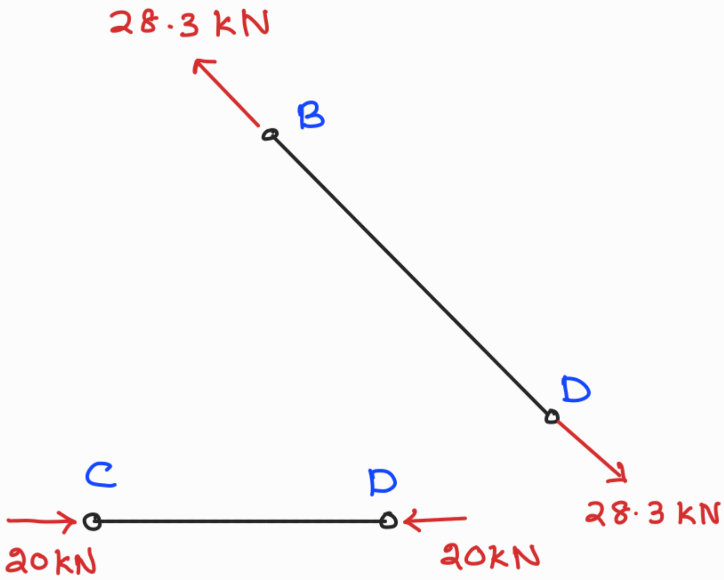


Three equilibrium eqns
Three unknown reactions
] Statically Determinate

All unknown forces can be obtained without using deformation information

* Using force equilibrium





In analyzing a deformable structure, the equilibrium should be satisfied in the deformed equilibrium configuration. However, in most engineering applications, deformations are so small that it is sufficiently accurate to apply the equb^{m} requirements to the undeformed configuration

* Force-deformation relation

Under the assumption of very small deformation, the expression for deformation under uniaxial loading can be used :

$$\delta_{BD} = \left(\frac{FL}{AE}\right)_{BD} = \frac{(28.3)(4.242)}{(0.491\times10^{-6})(205\times10^{6})} = 1.19\times10^{-3}\ m$$

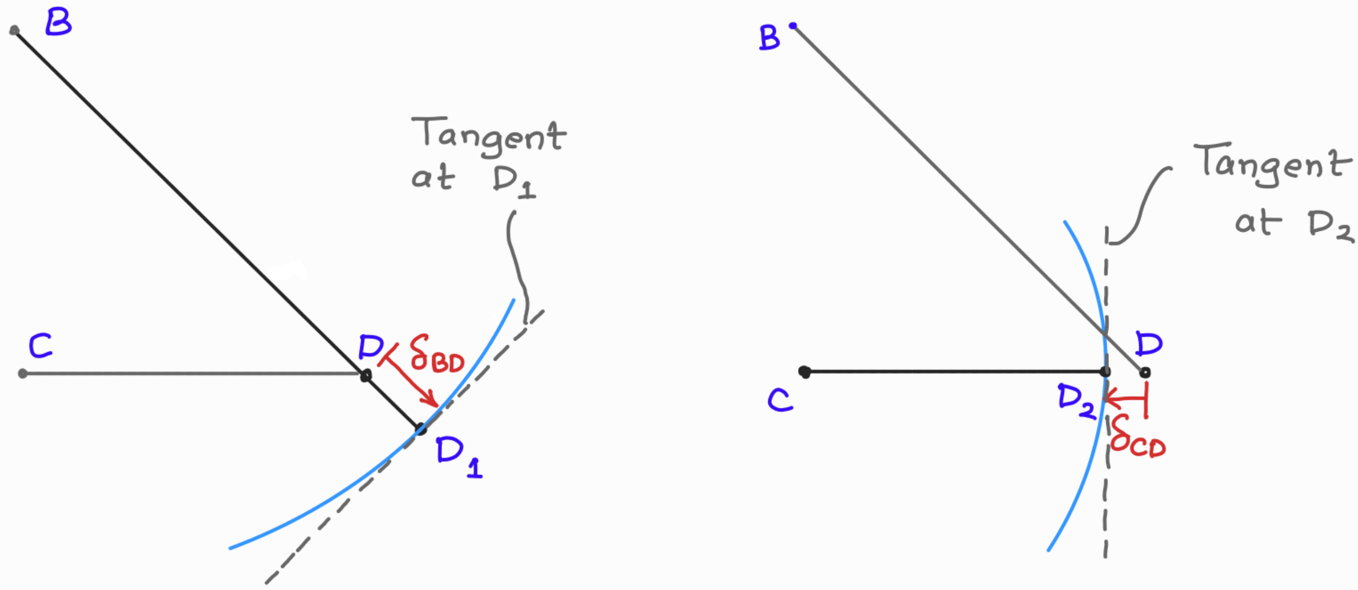
(extension)

$$\delta_{CD} = \left(\frac{FL}{AE}\right)_{CD} = \frac{(20)(3)}{(3.20\times10^{-6})(205\times10^{6})} = 0.0915\times10^{-3}\ m$$

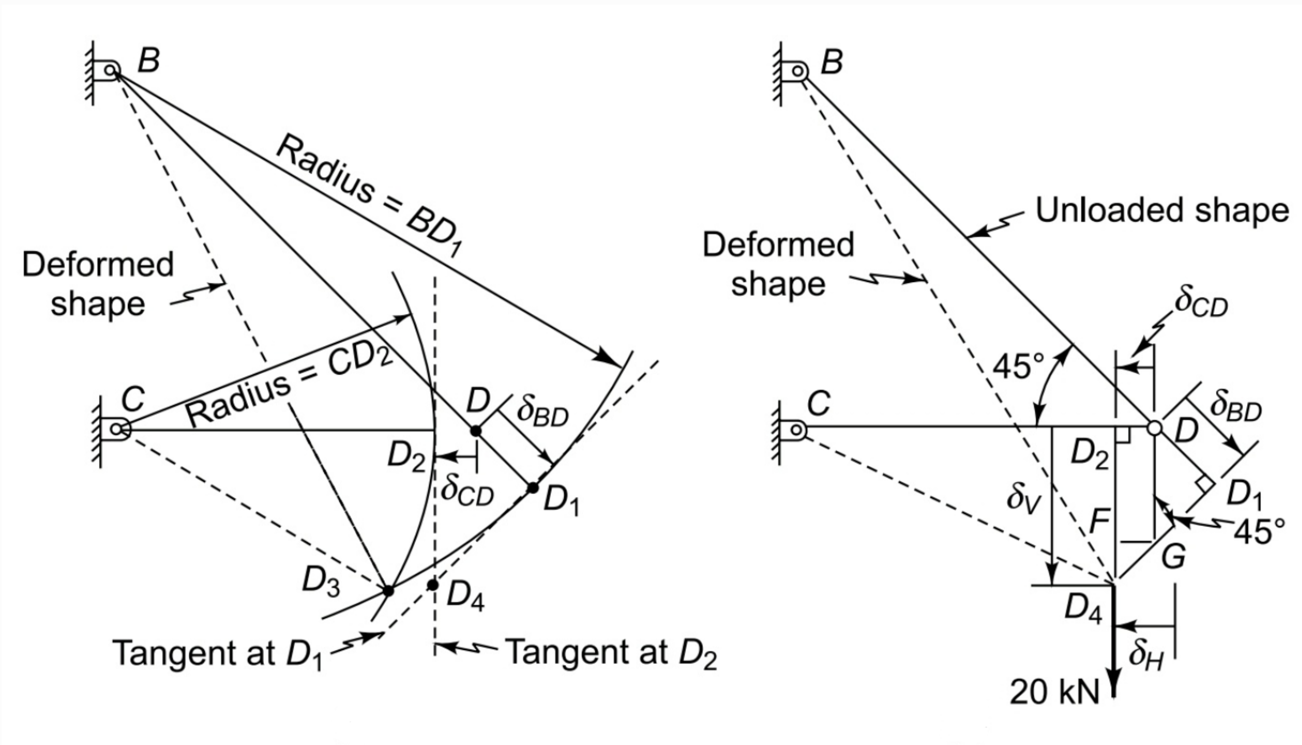
(compression)

* Geometric Compatibility

It requires that the bars BD and CD move in such a way that, while the bars change lengths, they remain straight and connected at D.



The displacement pin D position would be at D_3, the intersection of arc BD_1 and arc CD_2. However, for very small deformation, we can think of the point D to be displaced to a point D_4, which is the meeting point of the corresponding tangents



$$\delta_H = \delta_{CD} = 0.0915 \text{ mm}$$

$$\delta_v = D_2F + FD_4 = \sqrt{2}\,\delta_{BD} + \delta_{CD} = 1.77 \text{ mm}$$