Response of Real Materials

The constitutive relation (i.e. the stress-strain relation) in solid mechanics was introduced in the last lecture. The means by which the constitutive relation is determined is by carrying out experimental tests on the material in question

Experimental tests at microscale (microstructure-level)

One can do experiments at microscopic or even atomistic level to understand microstructure properties of a material

- X-ray crystallography
- Scanning electron microscopy (SEM)
- Atomic force microscopy

} can be used to find microscale texture, grain boundaries, local interaction forces

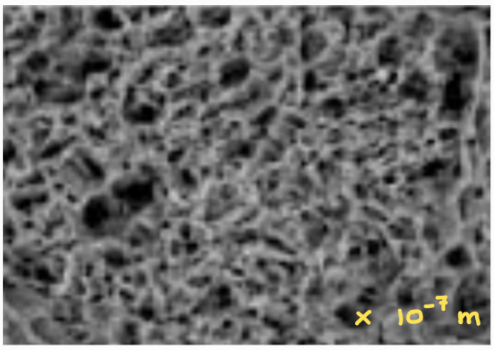
↓

Perform microscale simulations using methods such as MD (Molecular Dynamics) getting insights from micro-scale experiments

←

Use upscaling tools to relate the micro-scale properties to macroscopic properties such as Young's modulus

However, such experimentation is costly, therefore, they are not done often!



SEM image of a titanium alloy

Experimental tests at macroscale

- Tensile test
- Compression test
- Shear test

Obtain load-deformation graph

↓

Determine stress-strain curve

↓

Fit mathematical models for constitutive relations using the data

↓

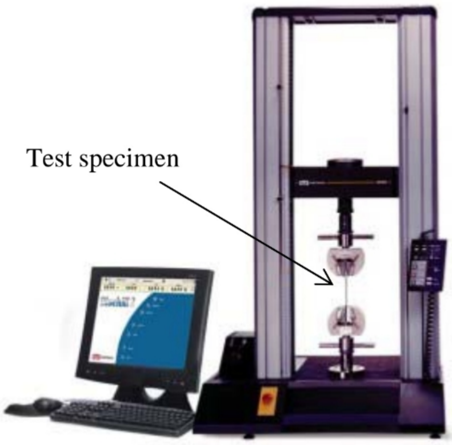
Obtain macroscale properties such as Young's modulus, etc.

- Cyclic test
- Other tests
  - Three-point bending test
  - Brinell hardness test
  - Creep test

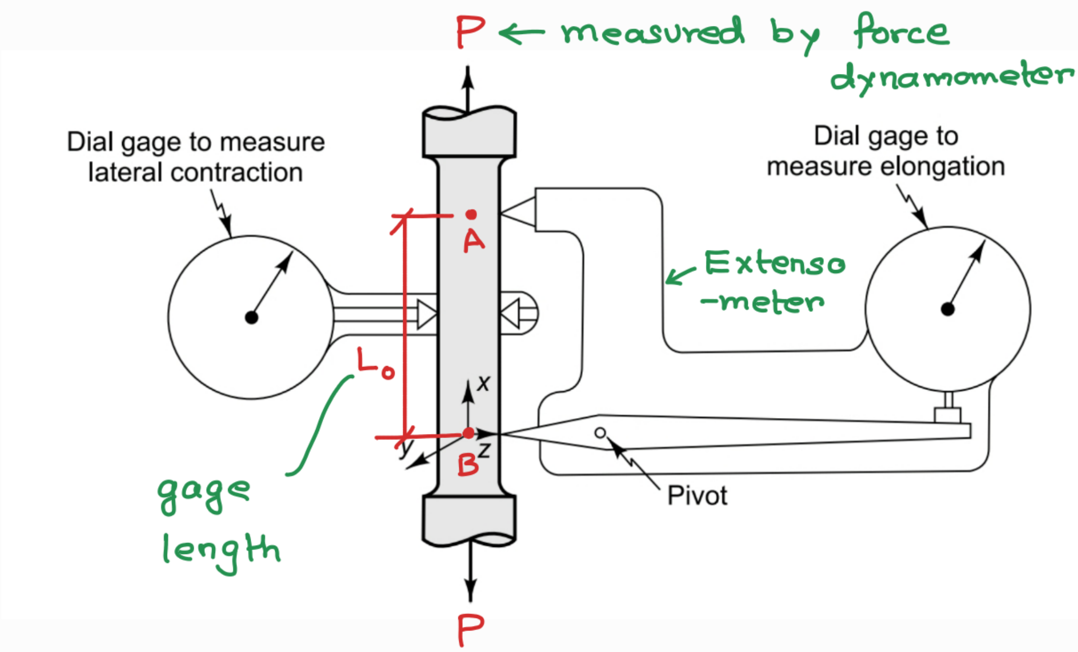
Uniaxial Tension Test

In this test, a cylindrical dog-bone-type specimen is gripped and stretched, usually at some give rate of stretching, i.e. $\frac{d\epsilon}{dt} \sim 0.001/s$ until fracture. The ends of the specimen are enlarged with a smooth fillet to ensure no fracture happens at the gripped ends and that no effect of the holding jaws falls on the stress induced within the testing length.

dog-bone type test specimen

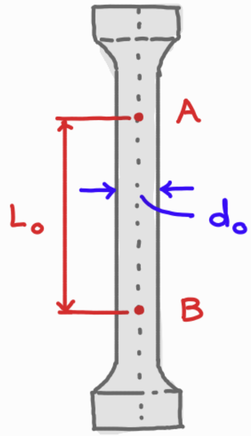


testing machine



- Pulled in the direction of cylinder axis
- Elongation & lateral contraction are measured
- Force required to hold the specimen at a given stretch is noted

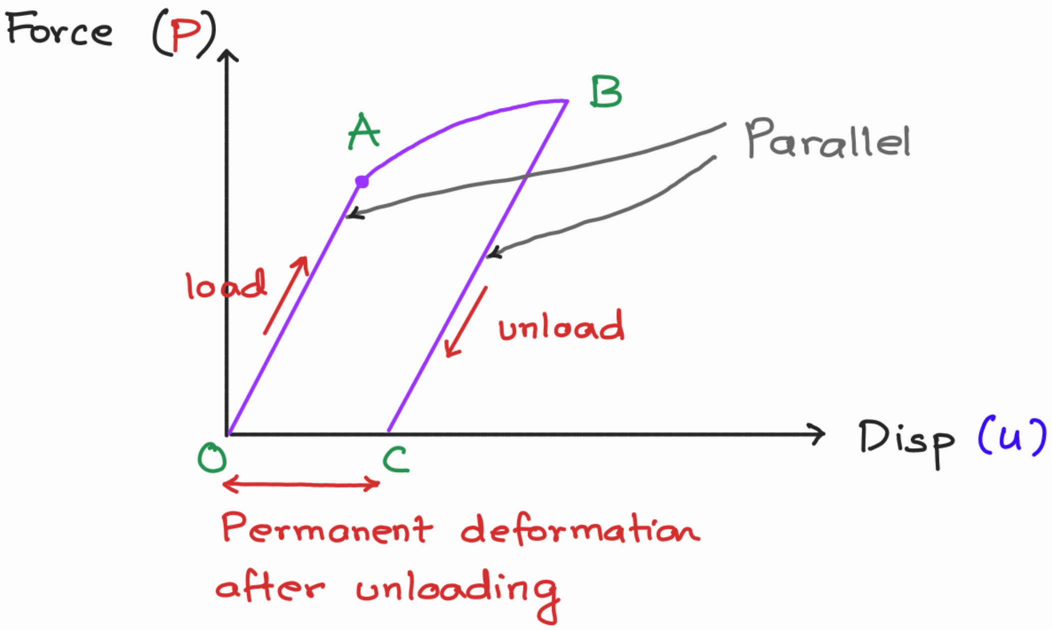
- The undeformed specimen has diameter d_o and two material points A and B are marked on it L_o (gage length) apart



- The increased distance between points A and B, L, and the deformed diameter, d, are measured by extensometers. Displacement along axis, $u = L - L_o$
- The axial load P acting on the specimen is measured by a force dynamometer
- The force/axial displacement ($P-u$) curve is plotted

Engineering Materials

For many (hard) engineering materials, the force/disp curve will look something like this:



Some common observations

1) It will be found the force is initially proportional to the displacement as with the linear portion OA

2) If the load has not reached point A, and the material is then unloaded, the force-displacement curve will trace back along the line OA down to zero force and zero displacement

3) The loading curve remains mostly linear upto a certain force level, called proportionality limit, and elastic upto (a nearby) elastic limit of the material (pt A).

Beyond this point, permanent deformations are induced that is upon unloading to zero force (from pt B to C) the specimen will have a permanent elongation

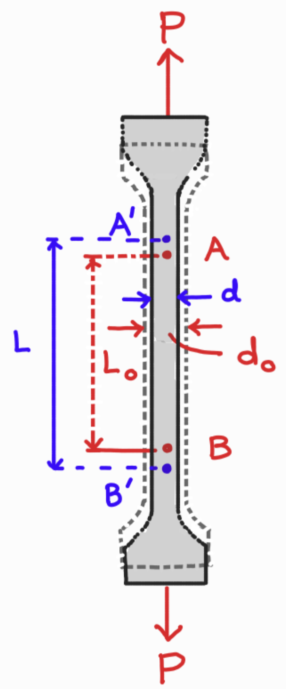
4) Above the elastic limit (pt A), the material hardens, that is the force required to maintain further stretching keeps increasing. (However, for materials like soils, they can soften)

5) The rate (speed) at which the specimen is stretched makes not much difference to the results observed (at least if the speed and/or temperature is not too high)

6) The strains upto the elastic limit are small, less than 0.002 (or 0.2%)

Stress-strain curve

There are two definitions of stress used to describe the tension test.



$$A_o = \pi d_o^2/4$$

$$A = \pi d^2/4$$

1) Nominal stress or Engineering stress

$$\sigma_{nom} = \frac{P}{A_o} = \frac{\text{Force}}{\text{original cross-sectional area of the specimen}}$$

2) True stress

$$\sigma = \frac{P}{A} = \frac{\text{Force}}{\text{(smaller) current cross-sectional area of the specimen}}$$

Bear in mind that the force P and c/s area A are changing as the experiment progresses.

For small elongations with the linear elastic range OA, the C/S area of the material undergoes negligible change and both defns of stress are more or less equivalent

Similarly, one can describe the deformation in two alternative ways

1) Engineering strain : $\epsilon = \dfrac{L - L_0}{L_0}$

or

2) True strain : $\epsilon_t = \log_e \left(\dfrac{L}{L_0} \right)$

Both have been defined earlier

$L_0 \leftarrow$ the original length between pts A and B

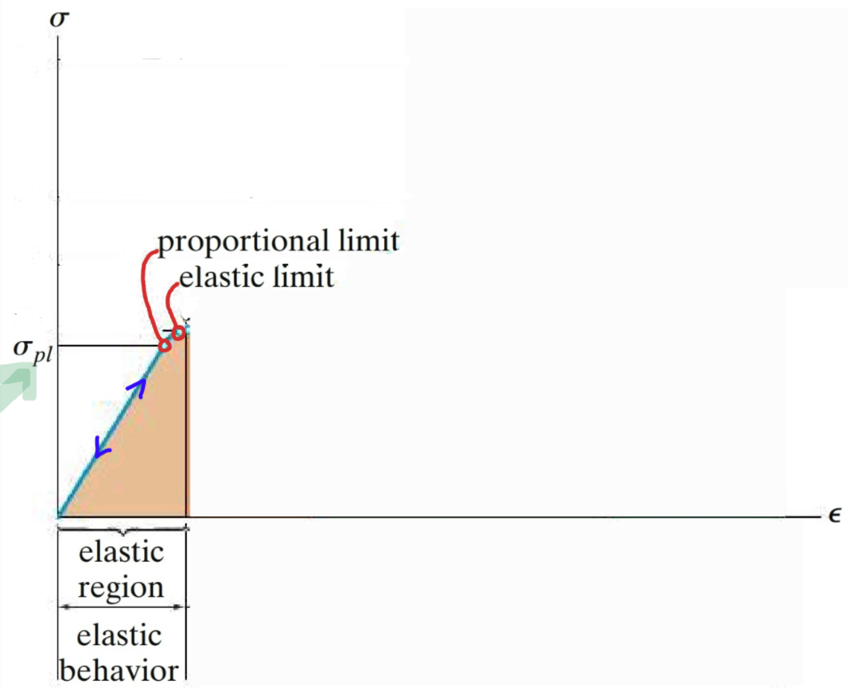
$L \leftarrow$ current length betn pts A and B

The stress-strain diagram for a uniaxial tension test can now be described using the true stress/strain or engineering stress/strain definitions.

Elastic Regime

- Elastic behavior occurs here

  – The curve is linear upto proportionality limit, σ_{PL}



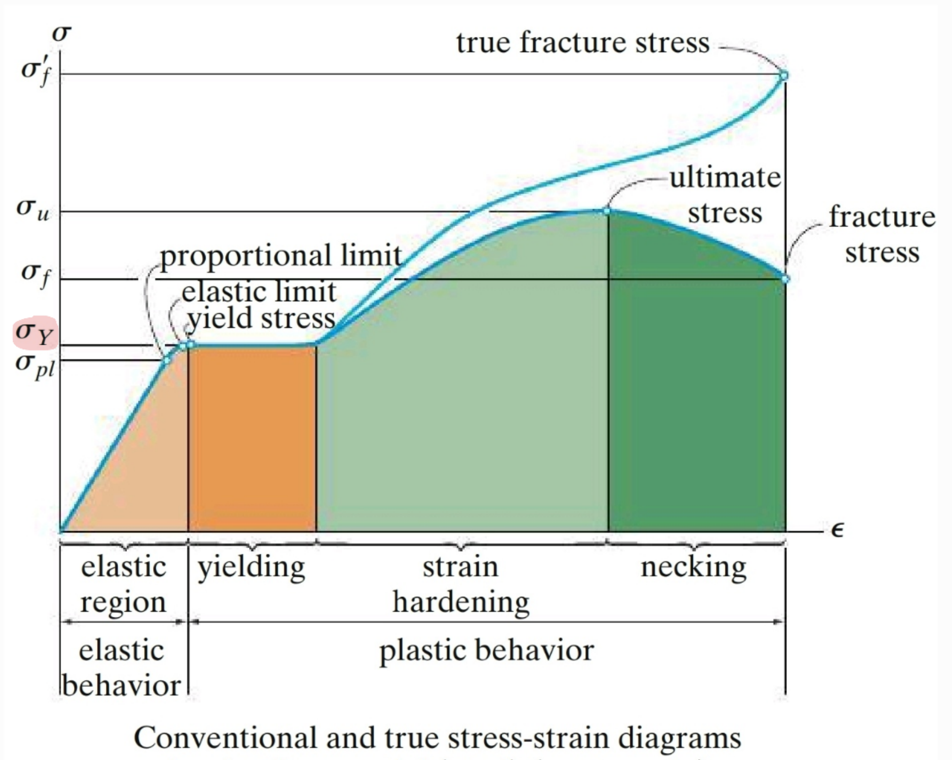
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

  – If the stress exceeds the proportionality limit, the curve tends to bend out. This continues until the stress reaches the elastic limit

this limit is difficult to determine in experiments

Up until the elastic limit, loading and unloading the specimen do not induce any permanent strain.

Inelastic Regime



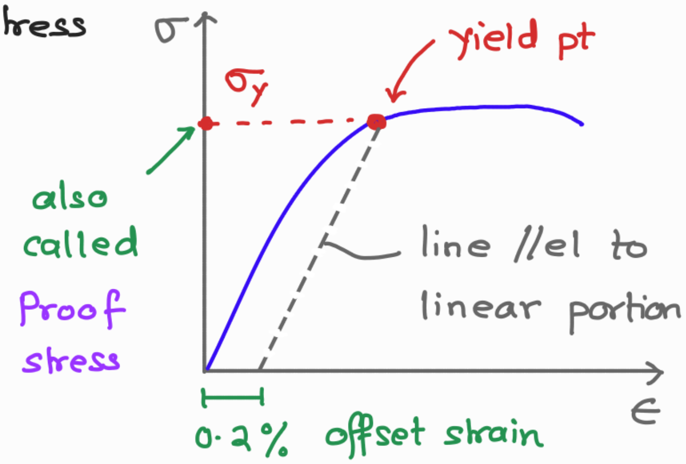
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

- **Yielding**

– A slight increase in the stress above elastic limit causes permanent deformation and the specimen will not regain its original shape upon unloading

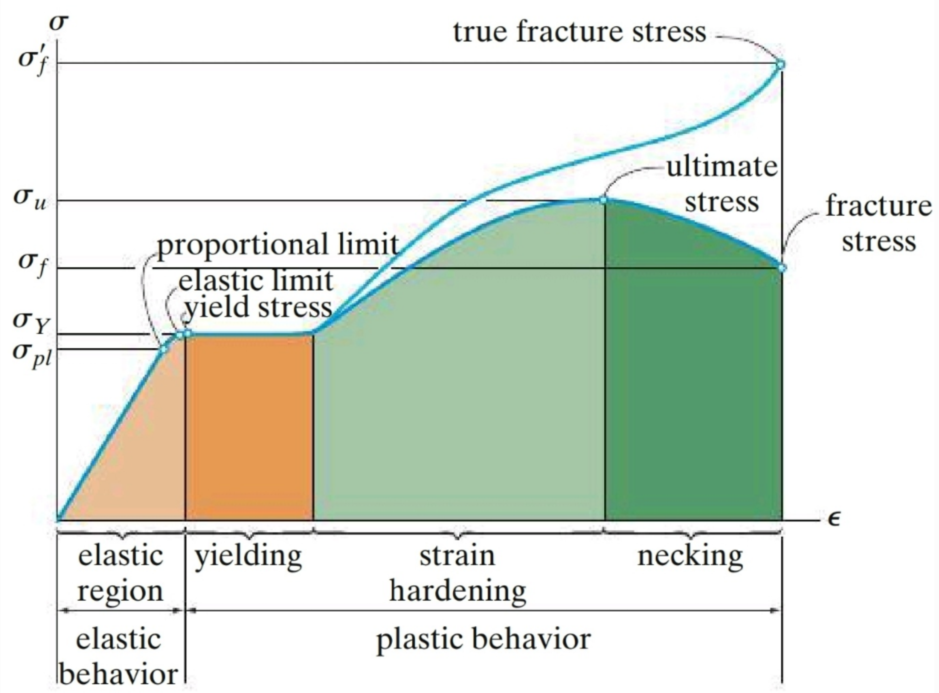
- **Yield Stress, σ_Y**, is the stress at which the material continues to deform without further increase in the stress. The associated deformation (strain) at yield stress is called plastic deformation (or plastic strain)

For most materials (such as aluminum), there is no distinct yield point. In such cases, yield stress is taken to be the stress for a stipulated permanent (or plastic) strain (0.05 – 0.3%, usually 0.2%)



- Strain hardening

When yielding has ended, an increase in load can be supported by the material resulting in a curve that rises but becomes flatter until it reaches a maximum stress called ultimate tensile strength (UTS), σ_u



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

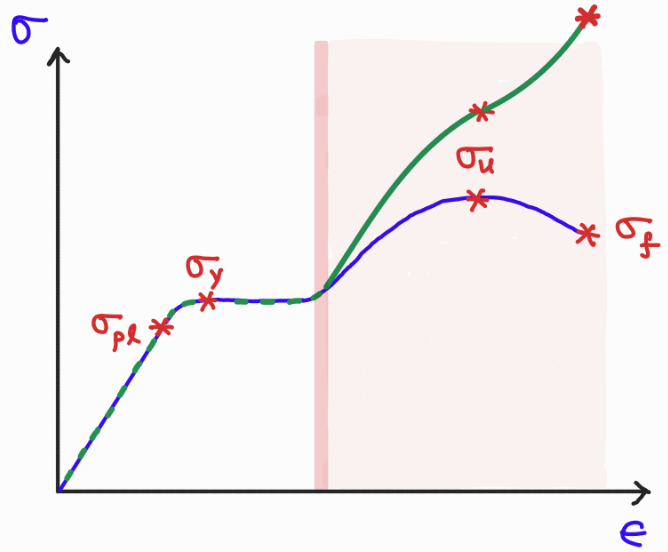
Necking

- Necking phenomenon

Upto the ultimate tensile stress, as the specimen elongates, its C/s area decreases uniformly. But just after the ultimate stress, the C/s area will begin to decrease in a localized region. As a result, a "neck" tends to form as the specimen elongates further and breaks at fracture stress, σ_f

Failure

True $\sigma-\epsilon$ vs Nominal (engineering) $\sigma-\epsilon$



— True $\sigma-\epsilon$

— Nominal $\sigma-\epsilon$

The difference is prominent in the strain hardening upto to fracture point

The main cause of the difference is due to the major reduction in the c/s area of the specimen and therefore the true stress $\sigma = \frac{P}{\frac{\pi d^2}{4}}$ becomes much larger than nominal stress $\sigma_{nom} = \frac{P}{\frac{\pi d_o^2}{4}}$ during the strain-hardening phase upto fracture.

Young's Modulus (E)

The slope of the stress-strain curve over the linear region, before the proportionality limit.

$$E = \frac{\sigma}{\epsilon} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Young's modulus has the units of stress (e.g. N/m^2) and is a measure of how "stiff" a material is.

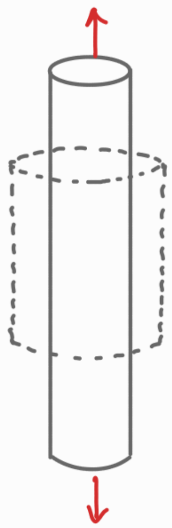
$\boxed{\sigma = E \epsilon}$ → represents one-dimensional linear elastic constitutive relation

Poisson's Ratio (ν)

→ pronounced like "new"

As the specimen in uniaxial tension test stretches, it gets thinner in the lateral (C/s) direction. Poisson's ratio is a measure of the ease with which it thins when pulled.

It is defined as negative of the ratio of lateral strain ϵ_w to the longitudinal strain ϵ:



$$\nu = -\frac{\epsilon_w}{\epsilon} = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

→ typically a positive number ~ 0.3

What is the tension test data used for ?

1) It's direct use happens in many structural applications such as bridges, buildings, or components which are relatively long and slender (e.g. truss)

These components undergo tension and/or compression, very much like the specimen in tension test. The tension test data (Young's modulus, Yield strength, UTS, Poisson's ratio) then give direct information on the amount of stress that these components can safely handle before dangerous straining or failure.

2) Tension test data (and similar other test data) can be used to predict what will happen when a component of complex 3D shape is loaded in a complex way, nothing like in the simple tension test.

Another way to think: One must be able to predict the world around us without having to resort to complex, expensive, time-consuming material testing! A simple tension test data would help us to get some properties for computer simulation of complex mechanics

Tension test data for a number of metals

	Young's Modulus E (GPa)	0.2% Yield Strength (MPa)	Ultimate Tensile Strength (MPa)
Ni	200	70	400
Mild steel	203	220	430
Steel (AISI 1144)	210	540	840
Cu	120	60	400
Al	70	40	200
Al Alloy (2014-T651)	73	415	485

Tensile test data for some metals (at room temperature)

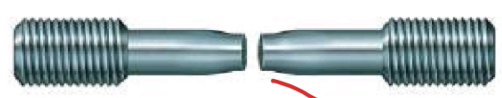
* Ductile vs Brittle materials

Engineering materials can be grouped into two broad classes → Ductile : can undergo large permanent (plastic) deformations, stretching, and necking before fracture

↓

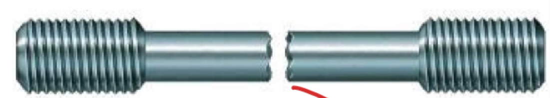
Brittle

These materials fracture without undergoing much permanent deformation – clean break. The UTS of a brittle material is same as fracture stress



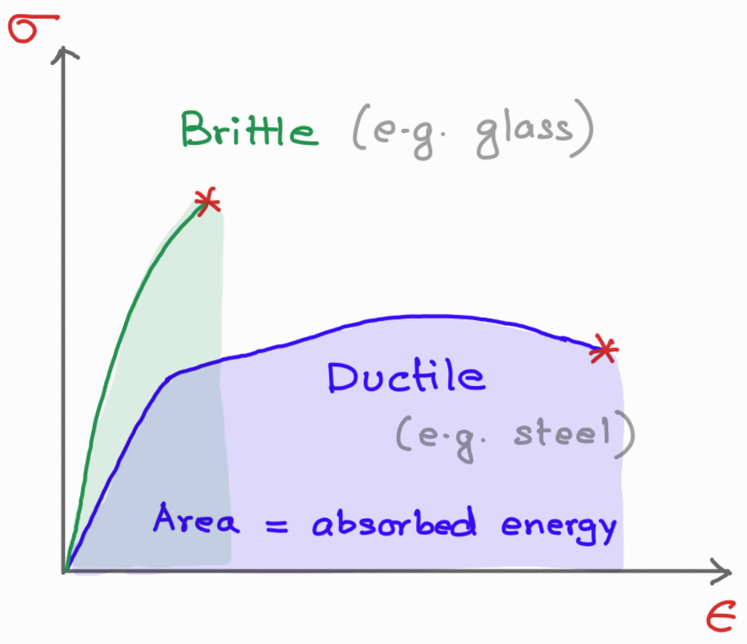
Failure of a ductile material

necking phenomenon



Tension failure of a brittle material

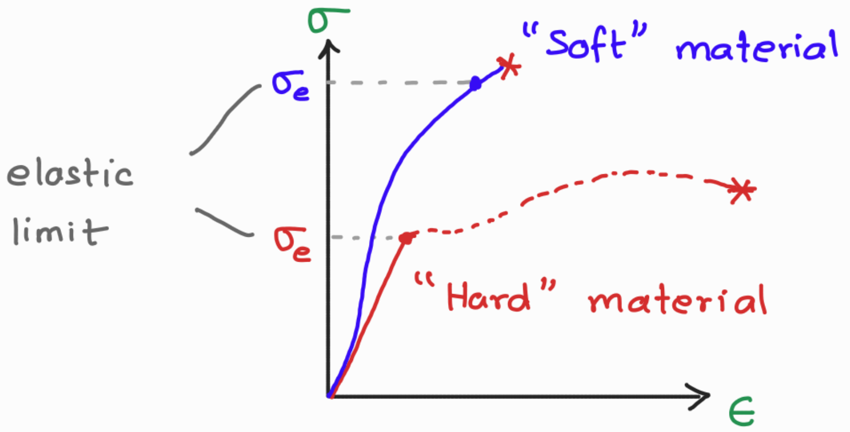
no necking



* Soft materials

Tension test data for traditionally non-engineering materials can be quite different e.g. rubber

- For many "soft" materials, the elastic limit, σ_e, (or the yield stress, σ_y) can be very high compared to hard engineering materials, and is close to failure stress



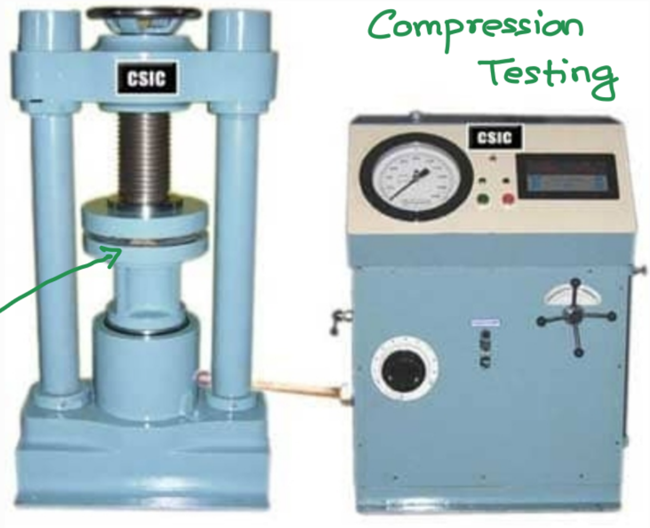
- Most of the $\sigma-\epsilon$ curve for soft materials is elastic and the material does not undergo permanent deform-ation upon unloading.

- Note the $\sigma-\epsilon$ curve is non-linear (curved), unlike prominent straight linear elastic portion for a typical metal

Uniaxial Compression Tests

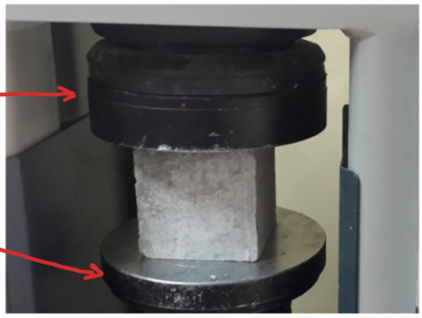
Many materials are used, or designed for use, in compression only. E.g. soils and concrete. Hence, these material are tested in compression

Concrete cubes



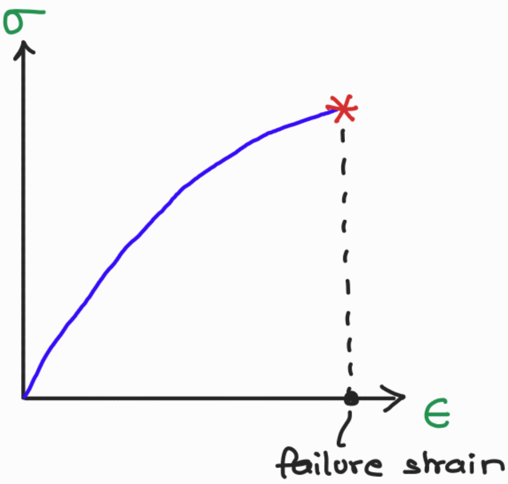
Compression Testing Machine

placed under these plates



crushed cube

The typical $\sigma - \epsilon$ response of concrete



At failure, concrete crushes catastrophically

Failure strain typically much less than 1%

For many materials, e.g. metals, a compression test will lead to similar results as the tension test.

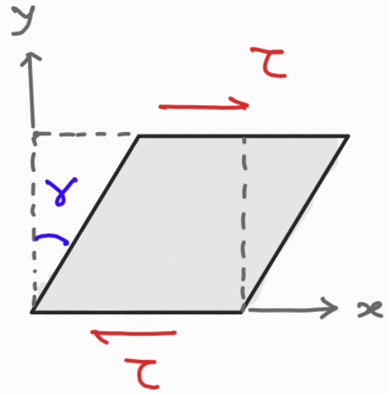
- The yield strength in compression will be approx. the same as (the negative of) the yield strength in tension

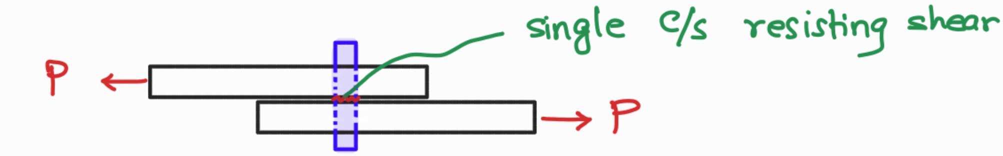
- Plot of true stress-strain curves for both tension and compression (absolute values for compression) would more or less coincide

- Plot of engineering stress-strain curves for both tension and compression would differ (Why?) bcoz of the defn of engineering $\sigma - \epsilon$ curve.

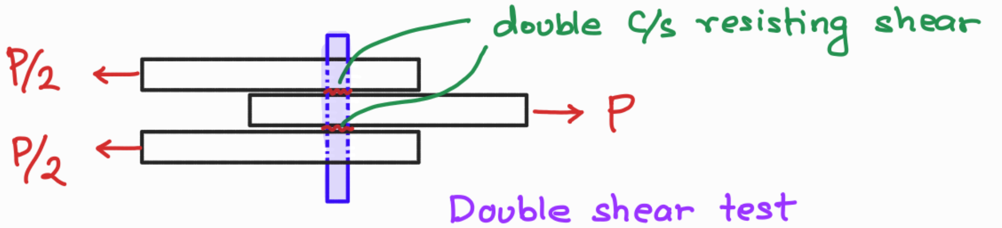
Shear Tests

Here the material is subjected to a shear strain $\gamma = 2\epsilon_{xy}$ by applying a shear stress $\tau = \tau_{xy}$

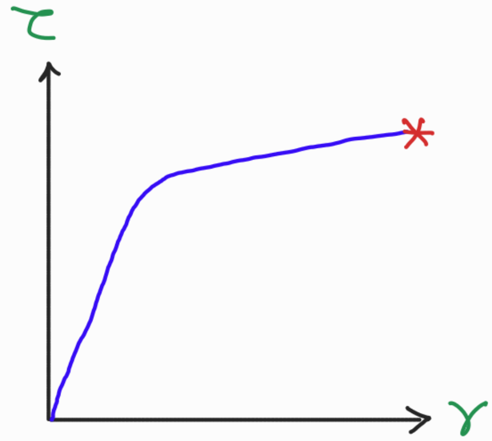




Single shear test



Double shear test



The shear stress at failure, called shear strength, can be greater or lesser than the ultimate tensile stress (UTS). The shear yield strength is however 0.5 - 0.75 times the tensile yield strength.

In the linear small-strain region, the shear stress will be proportional to the shear strain; the constant of proportionality is called shear modulus, G

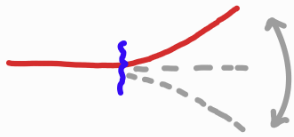
$$G = \frac{\tau}{\gamma}$$

Cyclic Tests

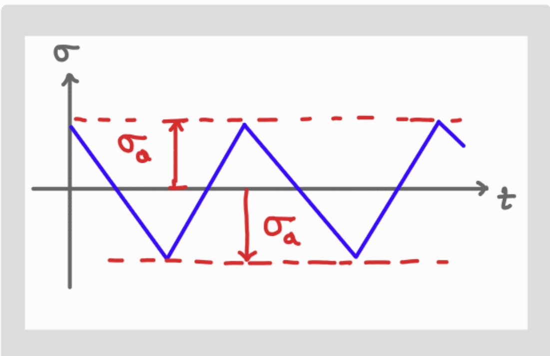
Cyclic loading occurs on rotating machines, structures subject to vibration such as wind turbines, railway/road vehicles, aerospace structures, and even tools used in the manufacturing processes.

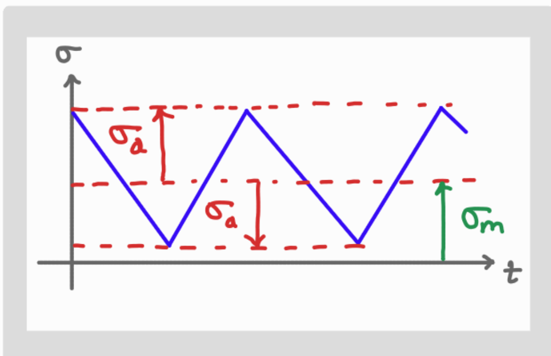
In many cases, failure (or fracture) of a material may not happen in a single application of loading, but it can happen on repeated application of loading and reloading

e.g. you will see that electrical maintenance workers often break (or fracture) a wire by repeated bending in a back and forth fashion.



Anything move back and forward is likely to be subjected to this tension/compression-type cyclic loading.





Consider the above figures where the stress at a point in the body varies with time.

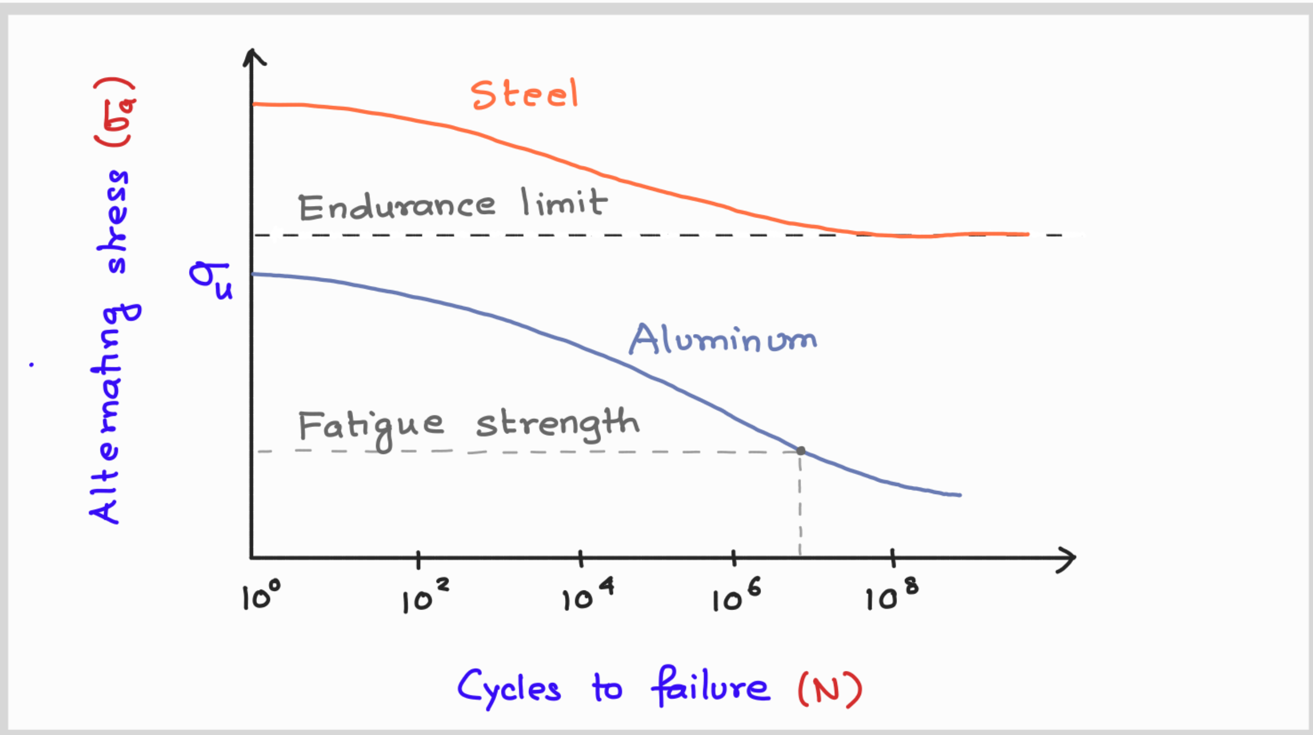
Experiments show that the alternating stress component σ_a is the most imp. factor in determining the # of cycles of load a material can withstand before fracture. σ_m is called the mean stress and is less imp. especially if $\sigma_m < 0$ (compressive).

FATIGUE is the process of fracture under cyclic load, well below σ_u (UTS) or even σ_y (yield).

Fracture may occur after

- a few cycles $\Rightarrow$ low-cycle fatigue, or
- after thousands/millions of cycles $\Rightarrow$ high-cycle fatigue

Plot of alternating stress σ_a (S) vs. # of cycles to failure (N)



Also called S-N curve

How is an S-N Curve Generated?

Creating an S-N curve involves fatigue testing, where a material sample is subjected to cyclic loading under controlled conditions. The process includes:

1. **Test Setup:**
    - A specimen is mounted in a fatigue testing machine, such as a servo-hydraulic system.
    - Cyclic loads are applied, typically as sinusoidal waveforms, at a set frequency.
2. **Stress Amplitude Variations:**
    - The material is tested at different stress levels, and the number of cycles to failure is recorded for each.
3. **Plotting the Data:**
    - Stress amplitudes are plotted on the vertical axis (S), and the number of cycles to failure (N) is plotted on a logarithmic horizontal axis.

The resulting curve shows how a material's fatigue life decreases as stress amplitude increases.

https://tactun.com/fatigue-testing-s-n-curves-and-their-significance/

Idealized material constitutive relations

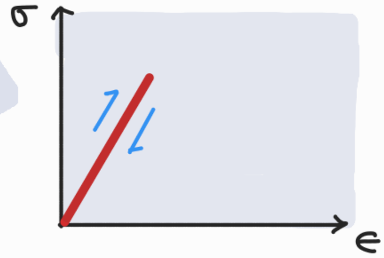
The response of real materials to various loading conditions was discussed in previous lectures. Now comes the task of creating mathematical models which can predict this response

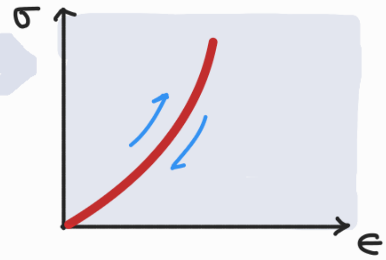
We will now characterize material responses into ideal models

- Rigid material – no strain regardless of stress

  useful for studying gross motions

- Elastic material
  - Linear elastic
  - Non-linear elastic





e.g rubber, soft tissues

  ~ loading and unloading paths are same

  ~ no permanent (or plastic deformation)

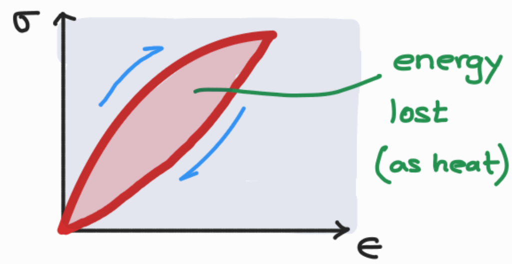
  ~ body returns to original shape upon unloading

  useful for designing for elastic deformations

- Viscoelastic material (strain-rate dependent)

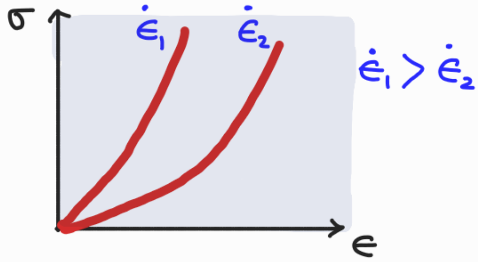
  ~ loading and unloading curves don't coincide, but form a "Hystersis" loop

  ~ stress depends upon strain-rate $\frac{d\epsilon}{dt}$



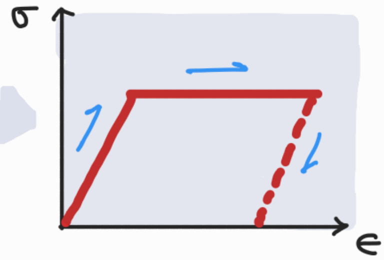
Loading & unloading in tension test

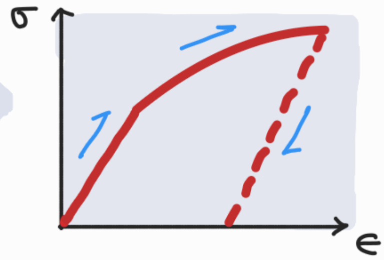
useful for designing solid materials with "fluid-like" characteristics with small deformations



Different rates of stretching

- Elasto-plastic material
  - Elastic-perfectly plastic
  - Elasto-plastic with strain hardening





useful for designing bodies with large deformations (with permanent strain)

- Viscoplastic material (strain rate dependent)

~ a combination of elasto-plastic & visco-elastic models

~ particularly, plasticity is strain-rate $\left(\frac{d\epsilon}{dt}\right)$ dependent

useful for study of materials at high temperatures

e.g. asphalt

Continuum Models and Micromechanics

The models mentioned in the previous section are **continuum models**. What this means is explained in what follows.

Stress and Scale

In the definition of the traction vector, it was assumed that the ratio of force over area would reach some definite limit as the area ΔS of the surface upon which the force ΔF acts was shrunk to zero. This issue can be explored further by considering Fig. 1.1. Assume first that the plane upon which the force acts is fairly large; it is then shrunk and the ratio F/S tracked. A schematic of this ratio is shown in Fig. 1.2. At first (to the right of Fig. 1.2) the ratio F/S undergoes change, assuming the stress to vary within the material, as it invariably will if the material is loaded in some complex way. Eventually the plane will be so small that the ratio changes very little, perhaps with some small variability ε. If the plane is allowed to get too small, however, down below some distance h^* say and down towards the atomic level, where one might encounter "intermolecular space", there will be large changes in the ratio and the whole concept of a force acing on a single surface breaks down.

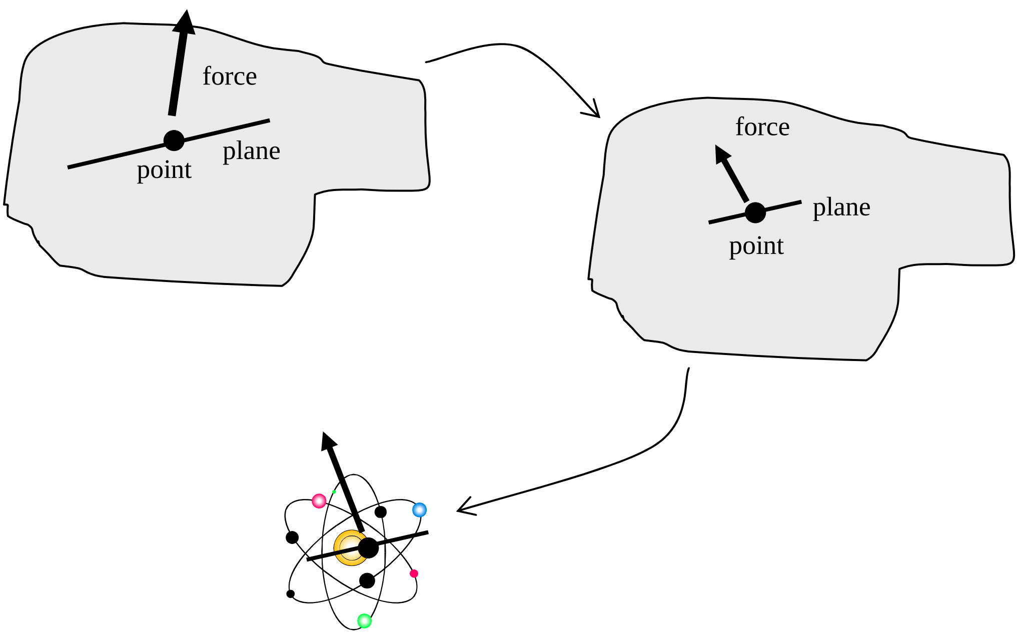


Figure 1.1: A force acting on an internal surface; allowing the plane on which the force acts to get progressively smaller

In a continuum model, it is assumed that the ratio F/S follows the dotted path shown in Fig. 1.2; a definite limit is reached as the plane shrinks to *zero size*. It should be kept in mind that the traction in a *real* material should be evaluated through

$$\mathbf{t} = \lim_{\Delta S \to (h^*)^2} \frac{\Delta F}{\Delta S} \tag{1.1}$$

where h^* is some minimum dimension below which there is no acceptable limit. On the other hand, it is necessary to take the limit to zero in the *mathematical* modelling of materials since that is the basis of calculus.

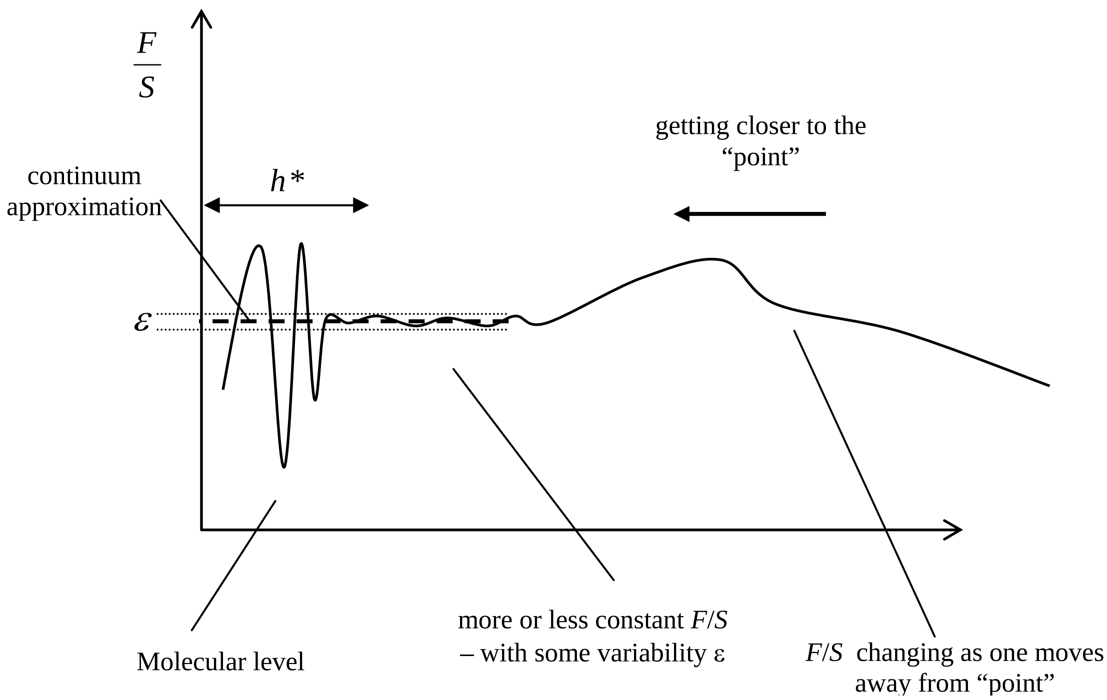


Figure 1.2: the change in traction as the plane upon which a force acts is reduced in size

In a continuum model, then, there is a minimum sized element one can consider, say of size $\Delta V = (h^*)^3$. When one talks about the stress on this element, the mass of this element, the density, velocity and acceleration of this element, one means the average of these quantities throughout or over the surface of the element – the discrete atomic structure within the element is ignored and is averaged out, or "smeared" out, into a **continuum element**.

The continuum element is also called a **representative volume element** (RVE), an element of material large enough for the heterogeneities to be replaced by homogenised mean values of their properties. The order of the dimensions of RVE's for some common engineering materials would be approximately (see the metal example which follows)

Metal:	0.1mm
Polymers/composites:	1mm
Wood:	10mm
Concrete:	100mm

One does not have any information about what is happening inside the continuum element – it is like a "black box". The scale of the element (and higher) is called the

macroscale – continuum mechanics is mechanics on the macroscale. The scale of entities within the element is termed the **microscale** – continuum models cannot give any information about what happens on the microscale.

Example: Metal

Metal, from a distance, appears fairly uniform. With the help of a microscope, however, it will be seen to consist of many individual grains of metal. For example, the metal shown in Fig. 5.4.3 has grains roughly 0.05mm across, and each one has very individual properties (the crystals in each grain are aligned in different directions).

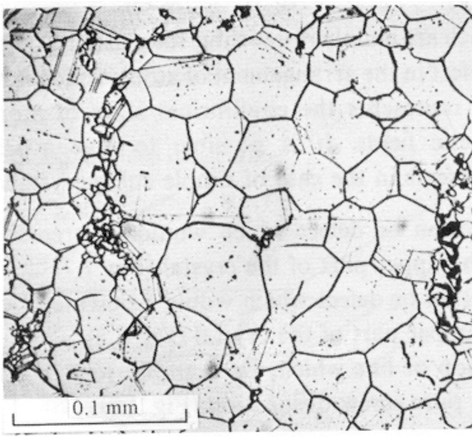


Figure 1.3: metal grains

If one is interested in the gross deformation of a moderately sized component of this metal, it would be sufficient to consider deformations that are averaged over volumes which are large compared to individual grains, but small compared to the whole component. A minimum dimension of, say, $h^* = 0.5\text{mm}$ for the metal of Fig. 1.3 would seem to suffice, and this would be the macro/micro-scale boundary, with a minimum surface area of dimension $(h^*)^2$ for the definition of stress.

When one measures physical properties of the metal "at a point", for example the density, one need only measure an average quantity over an element of the order, say, $(0.5\text{mm})^3$ or higher. It is not necessary to consider the individual grains of metal – these are inside the "black box". The model will return valuable information about the deformation of the gross material, but it will not be able to furnish any information about movement of individual grains.

It was shown how to evaluate the Young's Modulus and other properties of a metal. The test specimens used for such tests are vastly larger than the continuum elements discussed above. Thus the test data is perfectly adequate to describe the response of the metal, on the macroscale.

What if the response of individual grains to applied loads is required? In that case a model would have to be constructed which accounted for the different mechanical properties of each grain. The metal could no longer be considered to be a uniform material, but a complex one with many individual grains, each with different properties and orientation. The macro/micro boundary could be set at about $h^* = 0.1\mu m$. There are now two problems which need to be dealt with: (1) experiments such as the tensile test would have to be conducted on specimens much smaller than the grain size in order to provide data for any mathematical model, and (2) the mathematical model will be more complex and difficult to solve.

Micromechanical Models

Consider the schematic of a continuum model shown in Fig. 1.4 below. One can determine the material's properties, such as the Young's modulus *E*, through experimentation, and the resulting mathematical continuum model can be used to make predictions about the material's response. With the improved power of computers, especially since the 1990s, it has now become possible to complement continuum models with **micromechanical models**. These models take into account more fine detail of the material's structure (for example of the individual grains of the metal discussed earlier). Usually, one will have a micromechanical model of a small (typical) RVE of material. This then provides information regarding the properties of the RVE to be included in a continuum model (rather than having a micromechanical model of the *complete* material, which is in most cases still not practical). The means by which the properties at the micro scale are averaged (for example into a "smeared out" single *E* value) and passed "up" to the continuum model is through **homogenization theory**. Such micromechanical models can provide further insight into material behavior than the simpler continuum model.

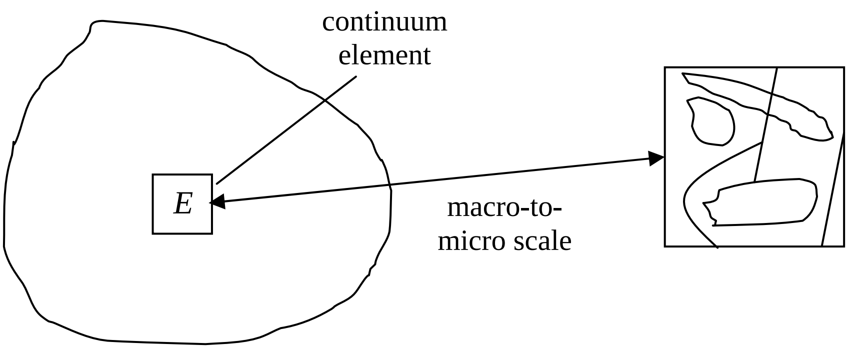


Figure 1.4: continuum model and micromechanical model