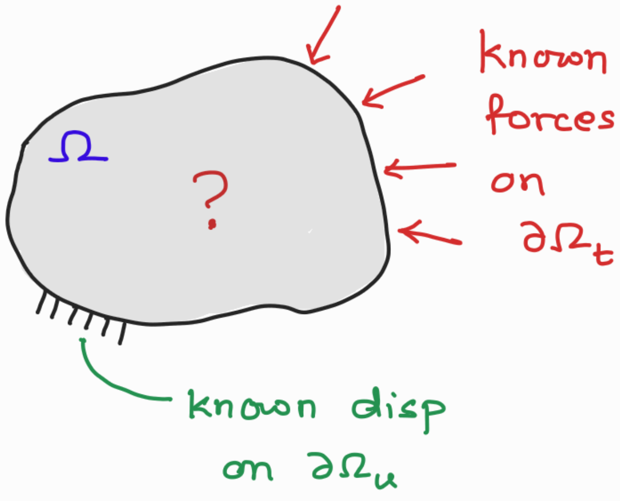
Material Behavior and Modeling

In this new module, the real physical response of various types of materials to different loading conditions will be examined. We will also consider mathematical models to predict such real responses.

The Mechanics Problem!

In most problems, one knows (some of) the forces acting on a material component, be it wind, water pressure, weight of the human body, a moving train, etc.



One also knows something about the displacements along some portion of the body (i.e. $\partial\Omega_u$), for example, it may be fixed to the ground and so disp. there are zero

The basic problem in mechanics is to determine what is happening inside the material body (Ω)

This means :

What are the stresses and strains in Ω ?

With this information, one can answer further questions:

a) Where are the stresses high ?

b) Where will the material first fail?

c) What can we change to make the component function better ?

d) Where will the component move to (i.e. displace) ?

e) What is going on inside the material, at the microscopic level ?

One can relate the external loads on the component to the stresses inside the body through equilibrium relations

3 PDEs $\longleftrightarrow$ 6 independent stress components

(stress equilibrium equations)

σ_{ij}

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = \rho \ddot{u}_i$$

$\underline{\underline{\sigma}}\, \underline{n} = \underline{T}^n$ on $\partial \Omega_t$ $\leftarrow$ Traction BCs

One can relate the displacements to internal strains using kinematic relations

6 strain-disp relations $\longleftrightarrow$ 6 ind. strain comp. + 3 ind. disp comp.

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \epsilon_{ij} \qquad u_i$$

$\underline{u} = \underline{u}^0$ on $\partial\Omega_u$ ← Displacement BC

Some remarks:

1) In all our discussion up until now, there has been no mention of any particular material under study be it metallic, polymeric, or biological.

2) The concept of stress and the theory of stress transformation, principal stresses, etc. are based on physical principles (Newton's law), which apply to ALL materials

3) The concept of strain is based on geometry and trigonometry and again applies to all materials, with the requirement that the strain be small for engg. strain

4) It is the relationship between stress and strain which differs from material to material.

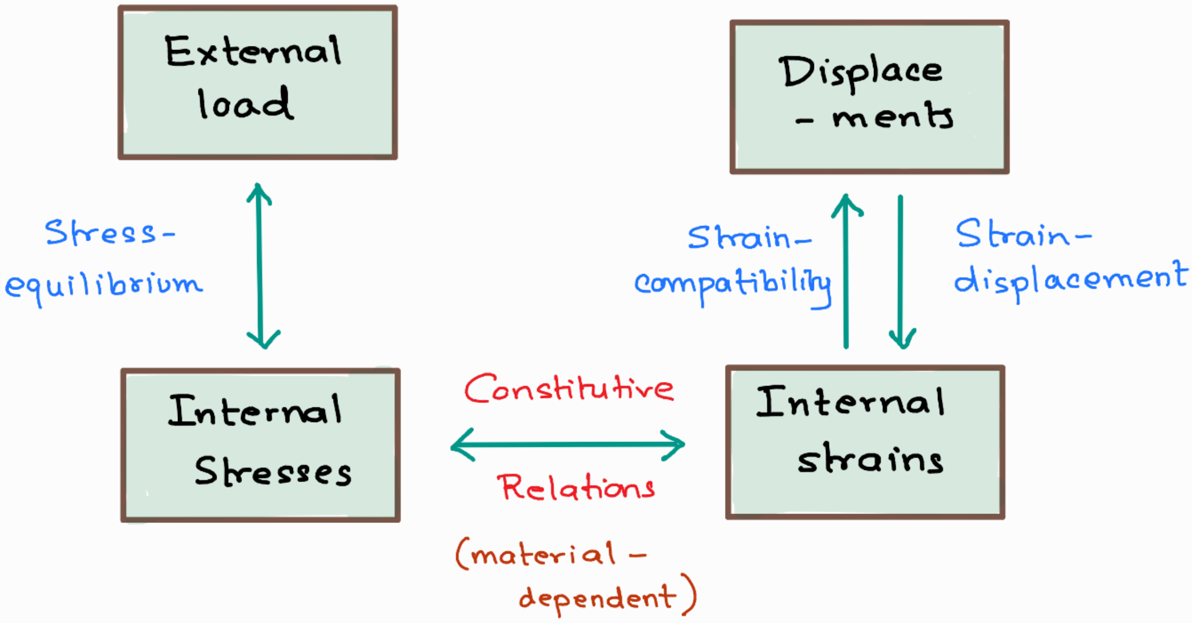
Constitutive relation

The relationship between the stress and strain at a material point for any material will depend upon the microstructure of the material — what constitutes that material. For this reason, the stress-strain relationship in solid mechanics is also called constitutive relation.

e.g.
- metals consist of a closely packed lattice of atoms ($\sim 10^{-7}$ mm) forming polycrystals.
- rubber consists of a tangled mass of long-chain polymer molecules
- wood is a cellular material with fibres and cells (width $\sim$ 0.1 mm and length $\sim$ 1 mm)
- concrete is a granular material composed of sand, cement and aggregate. Water is mixed to form a slurry. By a chemical process called hydration, the cement paste solidifies and hardens to bind the aggregate

For these reasons, the strain in a metal will be different to that in rubber/wood/concrete, when they are subjected to the same stress.

It is the constitutive relation that allows the mechanics problem to be solved, shown schematically below:



Three basic characteristics of material behavior:

1) Time-independent $\underline{\underline{\sigma}} - \underline{\underline{\epsilon}}$ relations donot depend on the time derivatives of $\underline{\underline{\sigma}}$ or $\underline{\underline{\epsilon}}$ $\left(\text{i.e. } \frac{d\underline{\underline{\sigma}}}{dt}, \frac{d\underline{\underline{\epsilon}}}{dt}\right)$

However, $\underline{\underline{\sigma}} - \underline{\underline{\epsilon}}$ can depend upon prior history of $\underline{\underline{\sigma}}$ & $\underline{\underline{\epsilon}}$

We will see, elastic and elasto-plastic behavior fall under this category

2) Time-dependent $\underline{\underline{\sigma}}-\underline{\underline{\epsilon}}$ relations involve $\frac{d\underline{\underline{\sigma}}}{dt}$ and/or $\frac{d\underline{\underline{\epsilon}}}{dt}$ as well (i.e. the rate at which material is being strained or stressed)

This type of material behavior falls under VISCOELASTIC and VISCOPLASTIC materials

3) Response to cyclic load which is called fatigue behavior

Need for a mathematical model for material behavior

Some of the questions asked earlier can be answered using experimentation. For example, one could use a car-crash test to determine the weakest pts in a car. However, carrying out multiple tests for each and every possible scenario — diff. car speeds, diff. obstacles, etc. would be too time-consuming and expensive.

The only practical way in which these questions can be answered is to develop a mathematical model. It will consist of (a) equilibrium eqns, (b) kinematics relations

(c) constitutive relations, (d) information related to geometry of the body, (e) boundary conditions, and so on.

The mathematical model will have certain approximations associated with it.

e.g. - it might as assume a body to be a perfect sphere, when in fact it might only resemble a sphere

- it might be assumed that a load is applied at a "point", when in fact it is applied over a region on the body's surface

- the constitutive relation itself is an approximation in most cases; the relation between stress and strain in any material can be extremely complex, and the constitutive relation is an approximation of the reality

Once a mathematical model is developed, all the equations can be solved and the model can be used to make a prediction. The prediction of the model can be tested against reality by carrying out a set of well-defined experiments

Simple models (e.g. simple constitutive relations) should be used as a first step. If the model predictions are wildly incorrect, the model can be made more complicated and the output tested again.

Mechanics equations associated with simple models can often be solved analytically, i.e. using a pen and paper. More complex models result in complex sets of equations which can only be solved using a computer.