What is Solid Mechanics?

- It is the study of the deformation and motion of solid materials under the action of forces / moments

- Solid materials have
  - definite shape
  - can resist tensile forces to some extent

- Knowledge gained from solid mechanics is used to describe, explain, and predict many physical phenomena of solid materials

- Some of the questions that solid mechanics tries to answer :

How thick should be a dam?

How to design SAFE submersibles?

Collapse Of Rail Bridge On Delhi-Jammu Route

Kangra, Himachal Pradesh: A horrifying tragedy was narrowed by a train carryi
the bridge of Chakki river, in Himachal Pradesh's Kangra while at the same the
due to flooding in the river. Fortunately, there was no accidents and the train qu
bridge.

Rutunjay Dole | **Updated:** Monday, July 21, 2025, 03:48 PM IST

How should you build a bridge that does not collapse?

How to design a ship to withstand wave slamming loads?

When could this cliff collapse?

Analysis of a Mechanical / Civil Structure

The structure (or system) could be a building, bridge, aircraft, etc.)

We know that mechanics deals with the study of forces and motions, therefore, for an analysis of a system, we must study

a) Generalized Forces (i.e. forces / moments)

↳ these cause motion/deformation of a body

b) Motion / Deformation (KINEMATICS)

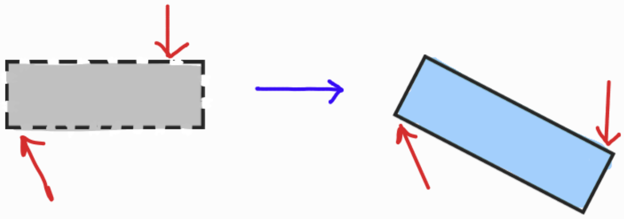
↳ study of geometrical aspects of motion of material bodies

described by position vector, velocity, acceleration which are used to describe change of geometry (with time)

Motion vs. Deformation

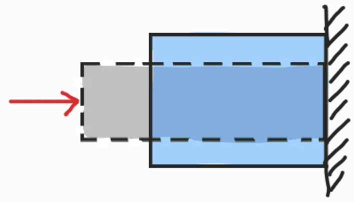
Overall changes in position of the body with time (with no change in the shap

Changes in the shape of the body



MOTION

(Translation / Rotation / both)



DEFORMATION

(we will consider this type of geometry changes in this course)

Steps involved in analyzing a mechanical system

a) Analysis of Generalized forces (forces/moments)

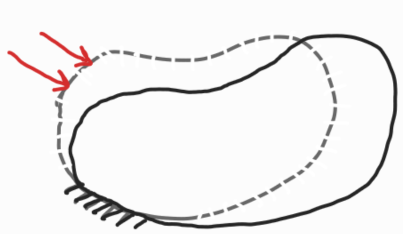
b) Motion / Deformation

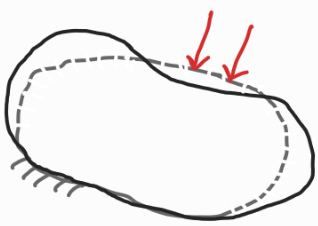
c) Application of laws relating the forces to the motion/deformation

Study of forces

- Force is a vector → a magnitude, → a direction } both need to be known for an analysis of a system

- Point of application of force must also be established

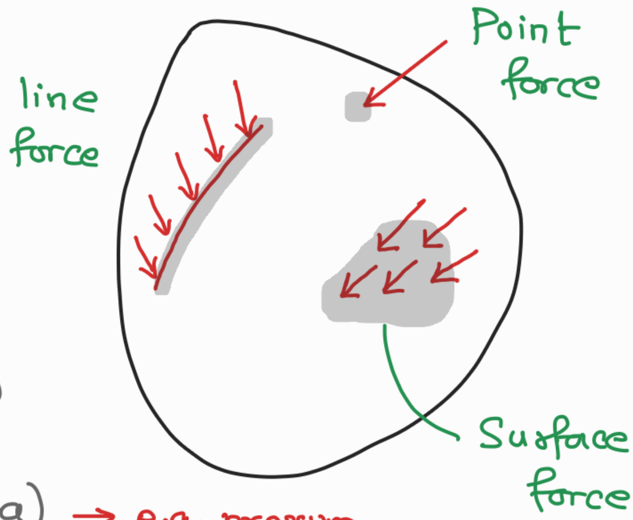




- Equivalent force system

When multiple forces act simultaneously, the effect is same as an equivalent force acting at the centroid of the multiple forces

Types of Forces



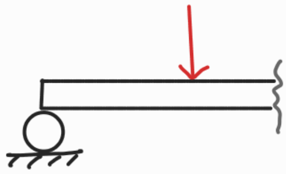
- Point force idealization

- Distributed force

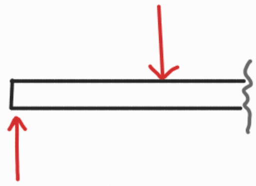
Contact forces
- ↳ line force (force/length)
- ↳ surface force (force/area) → e.g. pressure
- ↳ body / volumetric force (force/vol)
  - ↳ e.g. gravitational pull, magnetic pull/push
  - ↓ Usually non-contact force

* **EXTERNALLY APPLIED FORCES** : Forces applied to a material body by some external agent (could be point force, surface force, or body force)

* **SUPPORT REACTIONS :** These are contact forces which are generated at supports or points of connections betn bodies



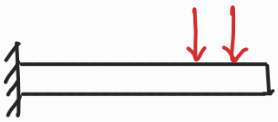
Roller



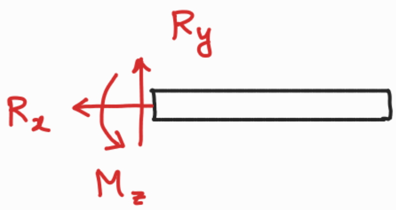
Support Reaction

General rule : If support prevents translation in a given direction, then a force must be developed on the member in the (opposite) direction

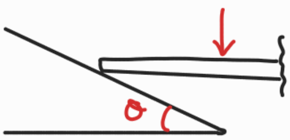
(similarly, for rotation constraint, a moment arises)



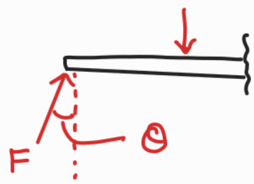
Fixed support



Three support reactions



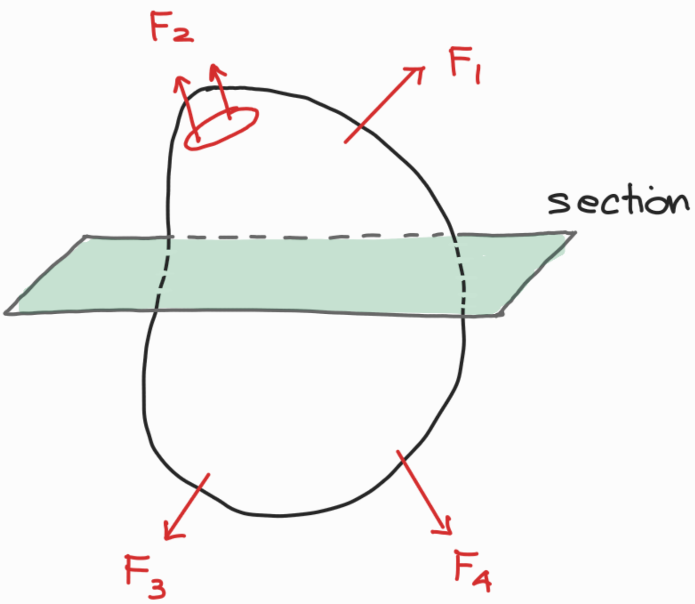
Smooth support



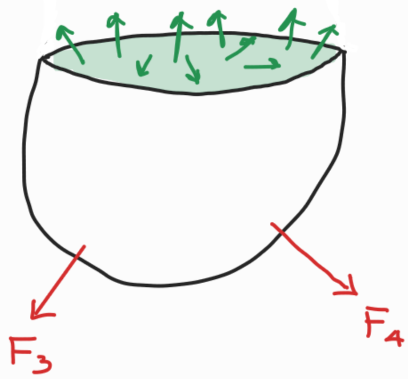
One unknown supp. reaction

* **INTERNAL RESISTIVE FORCES:** These are surface / line forces that are developed INSIDE a material body in resistance to externally applied forces

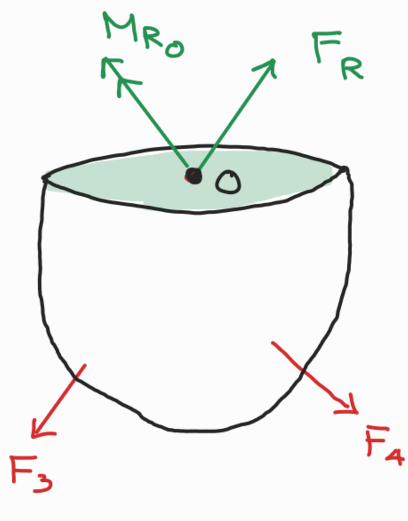
In order to obtain internal resistive forces acting on a specific region, we cut an imaginary section where the internal forces are to be determined!



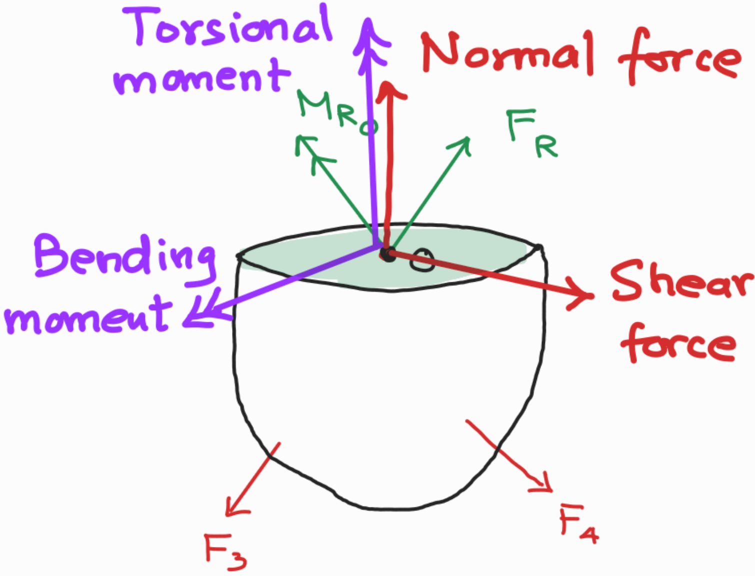
There will be a dist. of internal force over the exposed area



We take equivalent force system at C/s centroid O



Normal force : Force acts ⊥ to C/s area



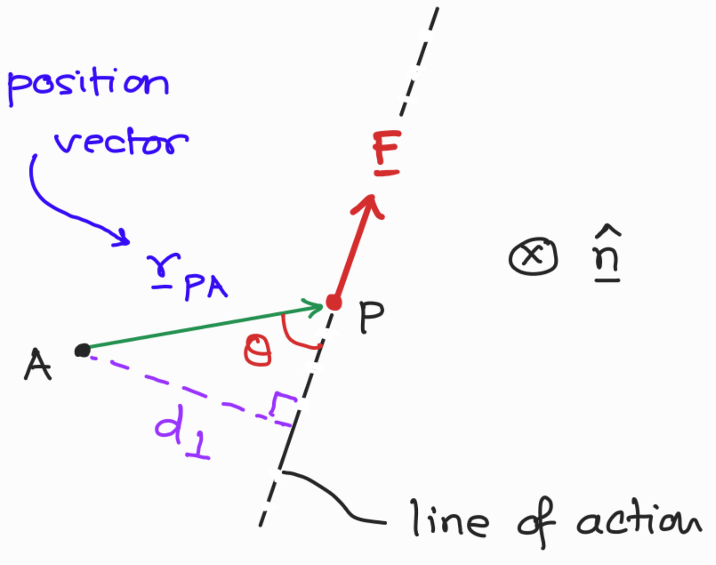
Shear force : Force lies in the plane of C/s area

Torsional moment : Tends to twist one segment of the body w.r.t other half about an axis ⊥ to C/s area

Bending moment : Tends to bend the body abt an axis lying within the C/s area plane

Moment of Forces

Moment of force $\underline{F}$ about pt A



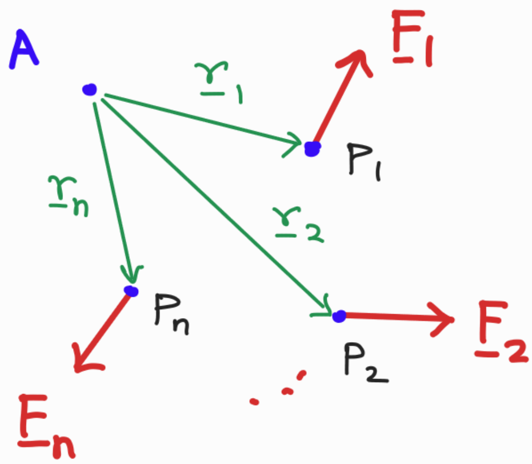
$$\underline{M}_A = \underline{r}_{PA} \times \underline{F}$$

$$= |\underline{F}| \underbrace{|\underline{r}_{PA}| \sin\theta}_{d_\perp} \; \hat{\underline{n}}$$

perpendicular dist betn pt A and line of action of force $\underline{F}$

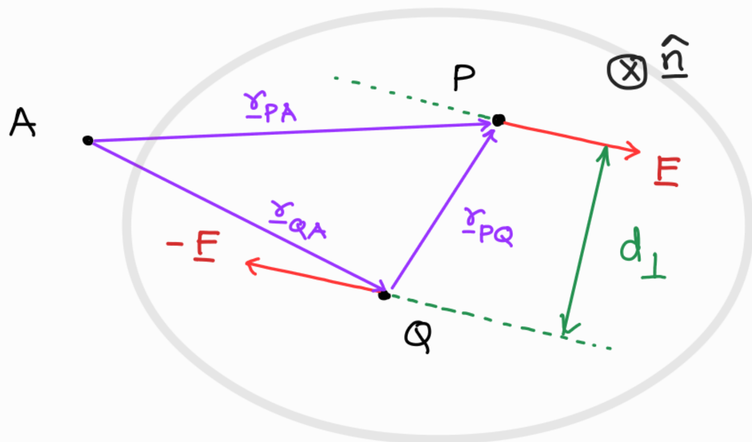
Moment due to several forces can be summed up:

$$\underline{M}_A = \sum_{i=1}^{n} \underline{r}_i \times \underline{F}_i$$



Couple : Set of forces $\underline{F}$ and $-\underline{F}$ acting along different lines of action

$\underline{C}$ ← symbol of couple vector



$$\underline{M}_A = \underline{r}_{PA} \times \underline{F}$$

abt pt A

$$+ \underline{r}_{QA} \times (-\underline{F})$$

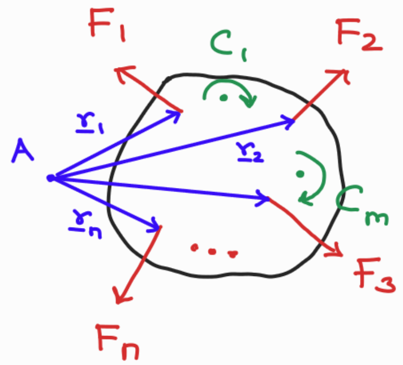
$$= (\underline{r}_{PA} - \underline{r}_{QA}) \times \underline{F}$$

$$= \underline{r}_{PQ} \times \underline{F}$$

$$= |F| \, d_\perp \, \hat{\underline{n}} \overset{def}{\equiv} \underline{C}$$ → does NOT depend upon pt A (and stays the same abt ANY point in space)

Equilibrium Conditions

From engineering mechanics, we have learned that a body is in equilibrium if



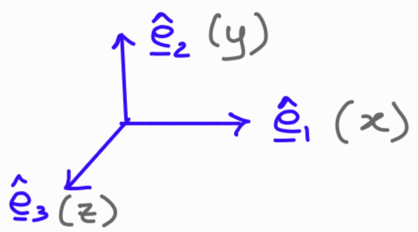
1) Net resultant force is ZERO

$$\Rightarrow \quad \underline{F}_R = \sum_{j=1}^{n} \underline{F}_j = \underline{0}$$

2) Net resultant moment due to all forces and couples about any arbitrary point is ZERO

$$\underline{M}_A = \sum_i \underline{r}_i \times \underline{F}_i + \sum_j \underline{C}_j = 0$$

Assuming an $\hat{\underline{e}}_1 - \hat{\underline{e}}_2 - \hat{\underline{e}}_3$ coordinate system (or x-y-z coor. sys)



Six scalar equilibrium conditions

$$\boxed{\begin{aligned} \sum F_1 &= 0 \quad (\text{or } \sum F_x = 0) \\ \sum F_2 &= 0 \quad (\text{or } \sum F_y = 0) \\ \sum F_3 &= 0 \quad (\text{or } \sum F_z = 0) \end{aligned}}$$

A

$$\boxed{\begin{aligned} \sum M_1 &= 0 \quad (\text{or } \sum M_x = 0) \\ \sum M_2 &= 0 \quad (\text{or } \sum M_y = 0) \\ \sum M_3 &= 0 \quad (\text{or } \sum M_z = 0) \end{aligned}}$$

B

Equations (A) and (B) are NECESSARY conditions of equilibrium for a deformable body

$\Rightarrow$ If the body is in equilibrium, the (A) and (B) would be satisfied. If a body is known to be in eqb^m then one can use (A) and (B) to obtain unknown support reactions

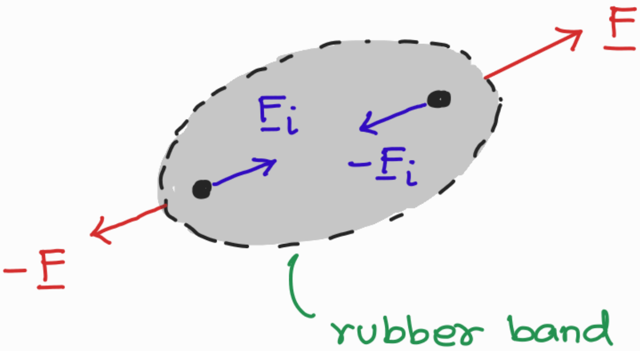
In this course, we will deal with STATIC EQUILIBRIUM

Therefore, we shall know that our system of interest is in equilibrium (usually from the fact that the sys is at rest), and we shall then use (A) and (B) to obtain information about support reactions

Converse problem: Suppose you know that the external forces acting on a system (of particles) satisfy (A) and (B), then can you conclude that every particle of the system is in equilibrium?

Ans: In general, NO, for deformable bodies

Ex. Consider stretching of a rubber band



* The resultant of all externally applied forces $\underline{F}$ and $-\underline{F}$ is zero

* The internal forces also occur in self-cancelling pair (due to Newton's 3rd law)

The particles will be in eqb^m only when $\underline{F} = -\underline{F}_i$

Therefore, if equal and opposite forces are applied to the ends of an unstretched rubber band, it does not remain in equilibrium. The ends of the rubber band begin to accelerate away and the band begins to stretch

Necessary and Sufficient conditions for Equilibrium

RIGID BODY	DEFORMABLE BODY
Vector sum of all EXTERNAL forces $= \underline{0}$ Vector sum of all EXTERNAL moments + EXTERNAL couples $= \underline{0}$	External and internal forces on every possible subsystem isolated out of the original system should satisfy A and B