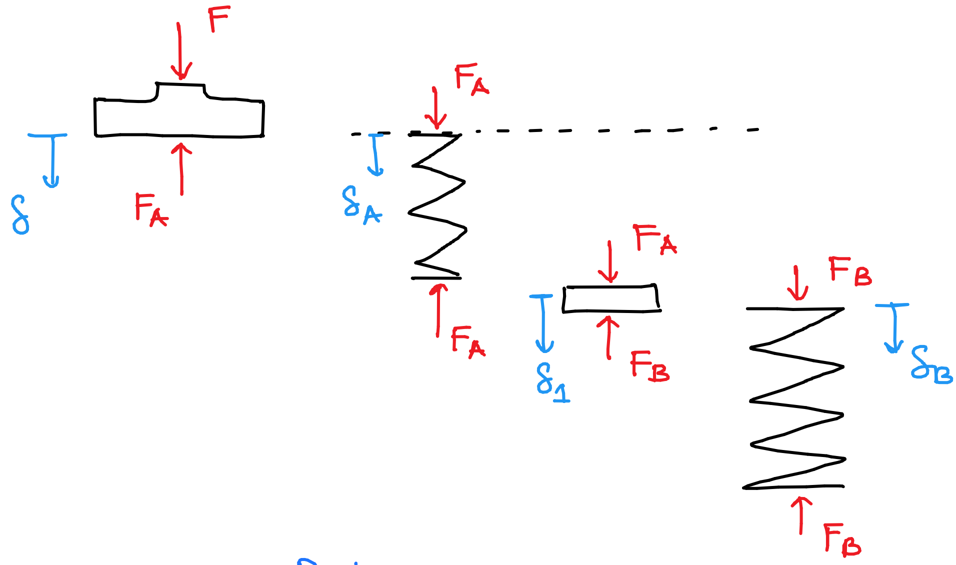
1) Draw FBDs



Force equilibrium

Piston 1

$+\uparrow \sum F_y = 0$

$\Rightarrow F_A - F = 0 \Rightarrow F_A = F$ (1)

Piston 2

$+\uparrow \sum F_y = 0$

$\Rightarrow F_B - F_A = 0 \Rightarrow F_B = F_A$ (2)

Using (1) & (2), $F_A = F_B = F$

Deformation and geometric compatibility

Compression of spring A

$\delta_A = \delta - \delta_1$ (3)

Compression of spring B

$\delta_B = \delta_1$ (4)

Using (3) & (4), $\delta = \delta_A + \delta_B$

Force-deformation relations

Spring A

$\delta_A = F/k_A$ (5)

Spring B

$\delta_B = F/k_B$ (6)

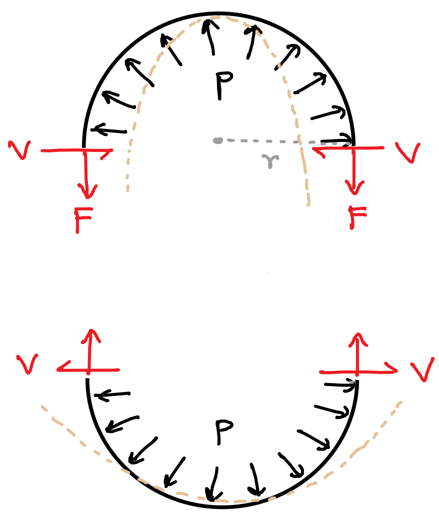
So, $\delta = F\left(\frac{1}{k_A} + \frac{1}{k_B}\right) \Rightarrow F = \frac{1}{\left(\frac{1}{k_A} + \frac{1}{k_B}\right)}\delta = \overbrace{\left(\frac{k_A k_B}{k_A + k_B}\right)}^{\text{effective stiffness}}\delta$

$$\boxed{\delta_B = F/k_B \Rightarrow \delta_B = \left(\frac{k_A}{k_A + k_B}\right)\delta}$$

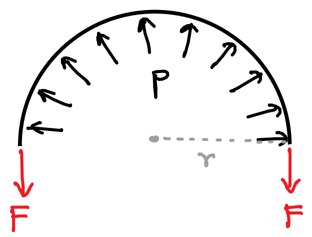
Similarly, find δ_A

2) See Example 2.3 (a) of Crandal & Dahl book

3) Lets draw an FBD of the ring by cutting a section through its diameter



- Due to Newton's 3rd law the shear forces must act equally and oppositely on the two halves
- For the top half the shear forces V has a tendency to push the ring inside, while those on the lower half tends to push it outside
- To maintain continuity and symmetry, the shear forces V must be equal to ZERO



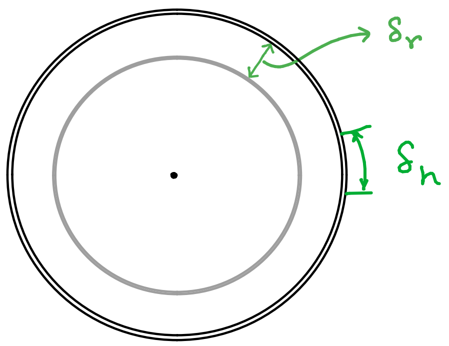
The effective vertical force due to the internal pressure $= (pb)(2r) = 2pbr$

This force is resisted by $2F$

From force equilibrium in vertical direction, we get:

$$\boxed{F = pbr}$$

Deformation and Geometric Compatibility



Let the increase in the circumference of ring be δ_h and the deformation in the radial direction be δ_r

The increase in circumference is accompanied by a radial expansion $\boxed{\delta_r = \delta_h / 2\pi}$

Force-deformation relation

Increase in circumference of ring

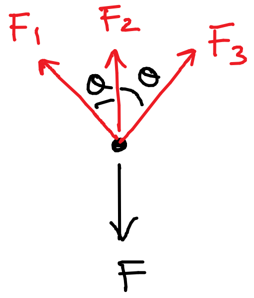
$$\delta_h = \frac{FL}{AE} = \frac{(p\cancel{b}r)\left(2\pi\left(r+\frac{t}{2}\right)\right)}{(t\cancel{b})\ E} \qquad \therefore\ \delta_r = \frac{pr^2}{tE}\left(1+\frac{t}{2r}\right)$$

$$= \frac{2\pi p r^2}{tE}\left(1+\frac{t}{2r}\right)$$

Thin ring assumption $t \ll r$, then $\frac{t}{r} \ll 1$

$$\delta_h = \frac{2\pi p r^2}{tE}, \qquad \delta_r = \frac{pr^2}{tE}$$

4)



Force equilibrium

$$\xrightarrow{+} \sum F_x = 0$$

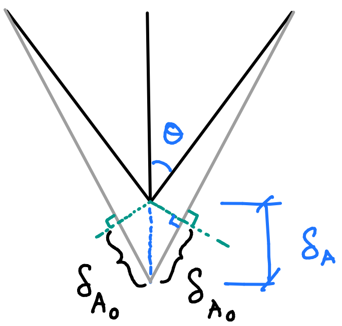
$$\Rightarrow -F_1 \sin\theta + F_3 \sin\theta = 0$$

$$\Rightarrow F_1 = F_3$$

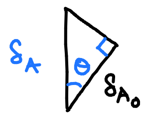
$$+\uparrow \sum F_y = 0$$

$$\Rightarrow 2F_1 \cos\theta + F_2 = F$$

<u>Deformation & Geometric compatibility</u>



Due to the symmetry of the system, there would only be vertical deformation δ_A at the free end



From small deformation, we can utilize geometric compatibility as

$$\delta_{A_0} = \delta_A \cos\theta$$

<u>Force-deformation relation</u>

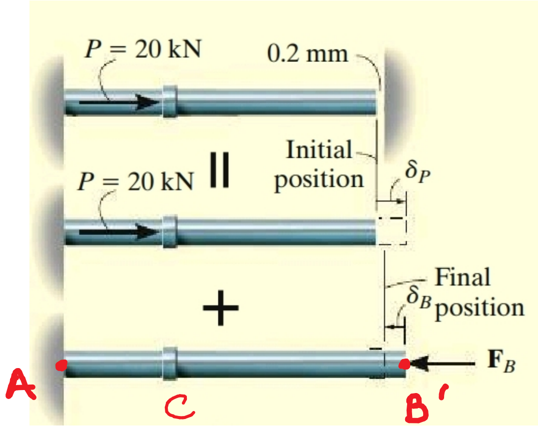
$$\frac{F_1\,(\cancel{L}/\cos\theta)}{A_0\,\cancel{E}} = \frac{F_2\,\cancel{L}}{A\,\cancel{E}}\cos\theta$$

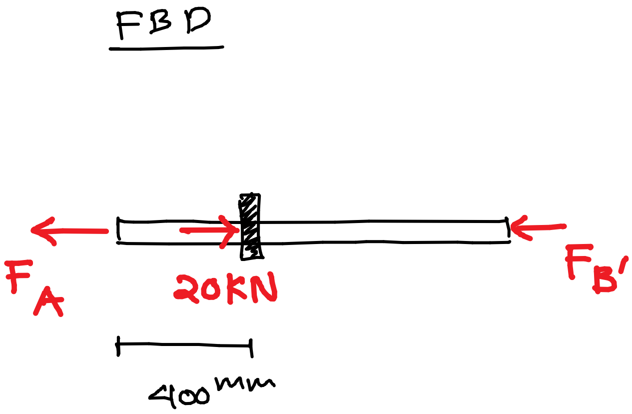
$$\Rightarrow F_1 = F_2\left(\frac{A_0}{A}\right)\cos^2\theta$$

Therefore, $2F_2\left(\frac{A_0}{A}\right)\cos^3\theta + F_2 = F \Rightarrow F_2 = \dfrac{F}{\left(1 + 2\frac{A_0}{A}\cos^3\theta\right)}$

and $F_1 = \dfrac{F}{\left(\frac{A}{A_0\cos^2\theta} + 2\cos\theta\right)}$

5)





Force equilibrium

$$\overset{+}{\rightarrow} \sum F_x = 0 \quad \Rightarrow \quad F_A + F_B - 20 = 0 \quad \rightarrow F_A = 15.95 \text{ kN}$$

Deformation & Compatibility

If there was no support at B', then the displacement would have been $\delta_P = \dfrac{P L_{AC}}{A E}$

$$\delta_P = \frac{(20 \times 10^3 \text{ N})(0.4 \text{ m})}{(\pi \times 0.005^2 \text{ m}^2)(200 \times 10^9 \text{ N/m}^2)}$$

$$= 0.5093 \times 10^{-3} \text{ m}$$

Due to the load $F_{B'}$ acting on the right hand wall, the portion CB' would be compressed by amount say δ_B

From compatibility, it must be that

$$\delta_P - \delta_B = 0.2 \times 10^{-3} \text{ m}$$

$$\Rightarrow \quad \delta_B = 0.3093 \times 10^{-3} \text{ m}$$

Force-deformation relation

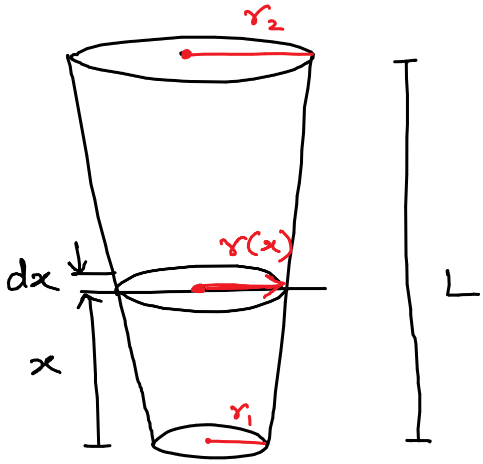
$$\delta_B = \frac{F_B L_{AB}}{A E} \quad \Rightarrow \quad F_B = \frac{(\pi \times 0.005 \text{ m}^2)(200 \times 0^9 \text{ N/m}^2)}{(1.2002 \text{ m})} \delta_B = 4.05 \text{ kN}$$

6)

$$r(x) = r_1 + \left(\frac{r_2 - r_1}{L}\right) x$$

$$= \frac{r_1 L + (r_2 - r_1) x}{L}$$

$$A(x) = \frac{\pi}{L^2} \left(r_1 L + (r_2 - r_1) x\right)^2$$



$$\delta = \int_0^L \frac{P\,dx}{A(x)\,E} = \frac{PL^2}{\pi E} \int_0^L \frac{dx}{[r_1 L + (r_2 - r_1) x]^2}$$

$$= -\frac{PL^2}{\pi E} \left[\frac{1}{(r_2 - r_1)[r_1 L + (r_2 - r_1) x]} \right] \Bigg|_0^L$$

$$= -\frac{PL^2}{\pi E (r_2 - r_1)} \left[\frac{1}{r_2 L} - \frac{1}{r_1 L} \right]$$

$$= \frac{\cancel{-} P L^{\cancel{2}}}{\pi E \cancel{(r_2 - r_1)}} \left[\frac{\cancel{r_1 - r_2}}{r_1 r_2 \cancel{L}} \right]$$

$$= \frac{PL}{\pi E\, r_1 r_2}$$