Tutorial 8: Extension-torsion-inflation

APL 104 - 2024 (Solid Mechanics)

Q1. Suppose a solid disk of radius 'R' and a hollow disk of inner radius R^1 and outer radius R^2 are shrunk fit together (assume R $> R^1$). Further, assume that no external pressure is applied on the outer hollow disk and that no axial displacement is allowed in the two disks. Let the two disks be made of the same material. Only radial displacement u_r is generated in this case.

(a) Write down all the boundary conditions/ interface conditions required to obtain variation in radial stress σ_{rr} in the two disks.

(b) Solve the governing equations and obtain expression for σ_{rr} in the two disks.

(c) Obtain an expression for circumferential/hoop stress too.

(d) Draw plots for variation of both radial and circumferential stress.

Q2. Think of an isotropic solid cylinder that is glued to a rigid plate at both ends. The two rigid plates are then pulled apart along the axis of the cylinder such the normals of the rigid plates remain aligned with the axis of the cylinder. It turns out that u_θ is zero in this case but u_r and u_z do arise. Furthermore, (u_r, u_z) are not functions of θ coordinate. Assume that the deformation of the cylinder is such that every planar cross-section of the cylinder (z-plane or axial planes) remains planar even after deformation but it does change its radius.

(a) What can you say about the dependence of u_r and u_z on radial and axial coordinates (r, z)?

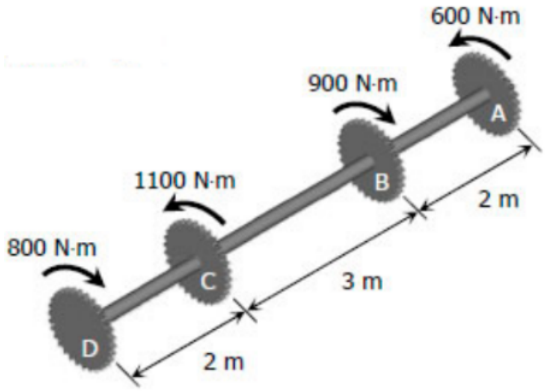
(b) Obtain the strain matrix and stress matrix for the above problem.

(c) Show that the θ-component of the stress equilibrium equation is automatically satisfied.

(d) What boundary condition will be used in order to solve the above deformation problem?

Q3. A 50.8mm diameter steel tube with a wall thickness of 1.27mm just fits in a rigid hole. Find the hoop stress if an axial compressive load of 1424kg is applied.

Q4. An aluminum shaft with a constant diameter of 50mm is loaded by torques applied to gears attached to it as shown. Using G = 28GPa, determine the relative angle of twist of gear relative to gear A.



Q5. A hollow bronze shaft of 76.2mm outer diameter and 50.8mm inner diameter is slipped observer a solid steel shaft 50.8mm in diameter and of the same length as the hollow shaft. The two shafts are then fastened rigidly together at their ends. For bronze $G_B = 7\text{kN/mm}^2$, and for steel, $G_S = 12\text{kN/mm}^2$. What torque can be applied to the composite shaft without exceeding a shearing stress of 55N/mm^2 in the bronze or 82N/mm^2 in the steel?

Q6. The compound shaft shown in the figure below is attached to rigid supports. For the bronze segment AB, the diameter is 75mm, $\tau \leq 60\text{MPa}$, and $G = 35\text{GPa}$. For the steel segment BC, the diameter is 50mm, $\tau \leq 80\text{MPa}$, and $G = 83\text{GPa}$ IF $a = 2\text{m}$ and $b = 1.5\text{m}$, compute the maximum torque T that can be applied.