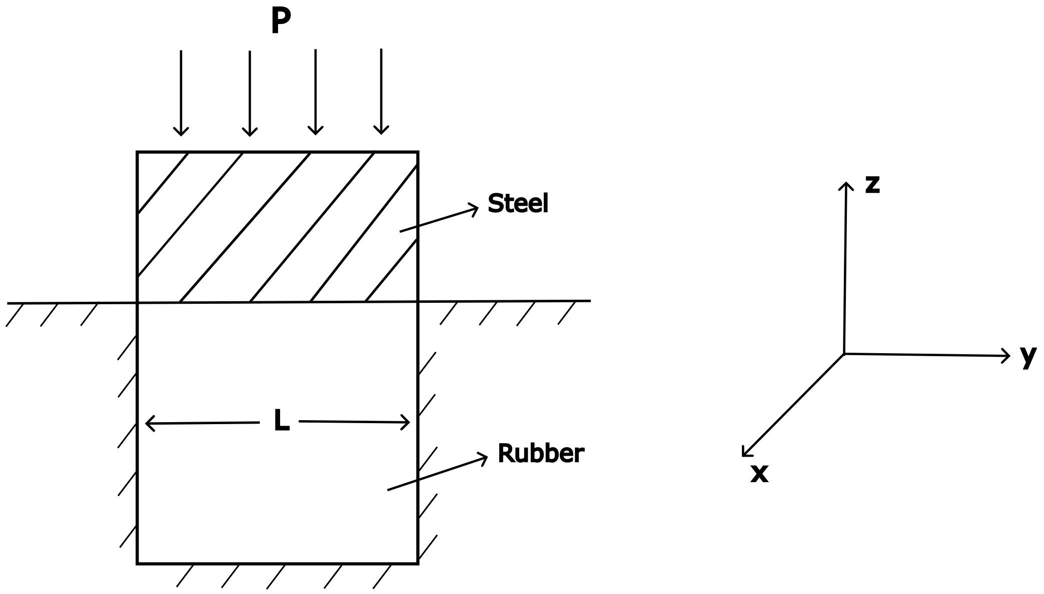
Tutorial 7: Complete equations of elasticity

APL 104 - 2024 (Solid Mechanics)

1. Think of a rubber cube that is inserted within a cavity in a steel block - the cavity has the same form and size as that of a rubber cube (as shown below). The top surface of the rubber cube is pressed by another steel block with a pressure of p.
   Assume the steel to be rigid and that there is no friction between steel and rubber. The steel block is rigid and the weight of the rubber block is negligible compared to the steel block. Treat rubber as an isotropic linear elastic material. The mass density of the steel block is ρ.



   (a) Find the relation between normal stresses in the x and y directions in this problem.

   (b) Find the volumetric strain.

2. Think of a thin rectangular plate being compressed along its four edges but not allowed to expand or contract in its thickness direction. Assume the thickness direction is along the z-axis. The following components of stress and strain matrices are given:

$$\sigma_{xx} = \sigma_{yy} = -p, \tau_{xy} = \tau_{yz} = \tau_{zx} = 0, \qquad \epsilon_{zz} = 0.$$

   Assuming the material to be isotropic, find out the remaining components of the strain matrix. Also, obtain the change in the area divided by the original area of the face of the plate (z-plane).

3. Show that in the case of isotropic bodies, the stress tensor and the strain tensor will both have the same set of principal directions. Further, show that the set of planes whose normals are parallel to one of the principal directions do not slide relative to each other.

4. Think of a solid beam having a square cross-section of side length h and axial length L. The beam's axis lies along the z axis while its cross-section's sides lie along x and y axes. Suppose the beam is stretched by applying axial force P to it such that its cross-section remains square and planar even after deformation. Also, assume the deformation to be axially homogeneous. Let us think of using a Cartesian coordinate system.

   (a) What coordinates (x, y, z) will the displacement functions $(u_x,\ u_y,\ u_z)$ depend on? Give reasons for your answer.

   (b) Find out the strain matrix and the stress matrix in terms of displacement functions and the material parameters $(\lambda,\ \mu)$ in the Cartesian coordinate system.

   (c) Substitute the expressions for stress components in the equilibrium equation (assume no body force) and obtain the equations. Write down the boundary conditions too. Solve them to obtain the three displacement functions.