From the 2D plane stress case, we found that the normal and shear stress components acting on an inclined plane with angle θ from $\underset{\sim}{e}_1$-axis are given by:

$$\sigma_n = \frac{\sigma_{11} + \sigma_{22}}{2} + \frac{\sigma_{11} - \sigma_{22}}{2} \cos 2\theta + \tau_{12} \sin 2\theta$$

$$\tau_n = -\frac{\sigma_{11} - \sigma_{22}}{2} \sin 2\theta + \tau_{12} \cos 2\theta$$

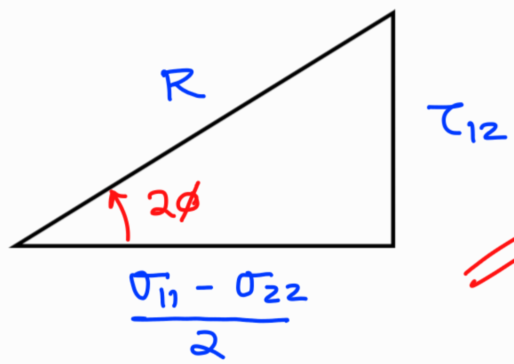
Let's define a scalar $R = \underbrace{\sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \tau_{12}^2}}$

magnitude of the hypotenuse of a right-angled triangle

From trigonometry,

$$\sin 2\phi = \frac{\tau_{12}}{R}$$

$$\cos 2\phi = \frac{\sigma_{11} - \sigma_{22}}{2R}$$



Using R, we can rewrite σ_n and τ_n as:

$$\sigma_n = \frac{\sigma_{11} + \sigma_{22}}{2} + \overset{R\cos 2\phi}{\cancel{\frac{\sigma_{11} - \sigma_{22}}{2}}} \cos 2\theta + \overset{R\sin 2\phi}{\cancel{\tau_{12}}} \sin 2\theta$$

$$= \frac{\sigma_{11} + \sigma_{22}}{2} + R\cos 2\phi \cos 2\theta + R \sin 2\phi \sin 2\theta$$

$$= \frac{\sigma_{11} + \sigma_{22}}{2} + R \cos (2\phi - 2\theta)$$

$$\tau_n = -\underbrace{\cancel{\frac{\sigma_{11}-\sigma_{22}}{2}}}_{R\cos 2\phi} \sin 2\theta + \underbrace{\cancel{\tau_{12}}}_{R\sin 2\phi} \cos 2\theta$$

$$= -R\cos 2\phi \sin 2\theta + R\sin 2\phi \cos 2\theta$$

$$= R\sin(2\phi - 2\theta)$$

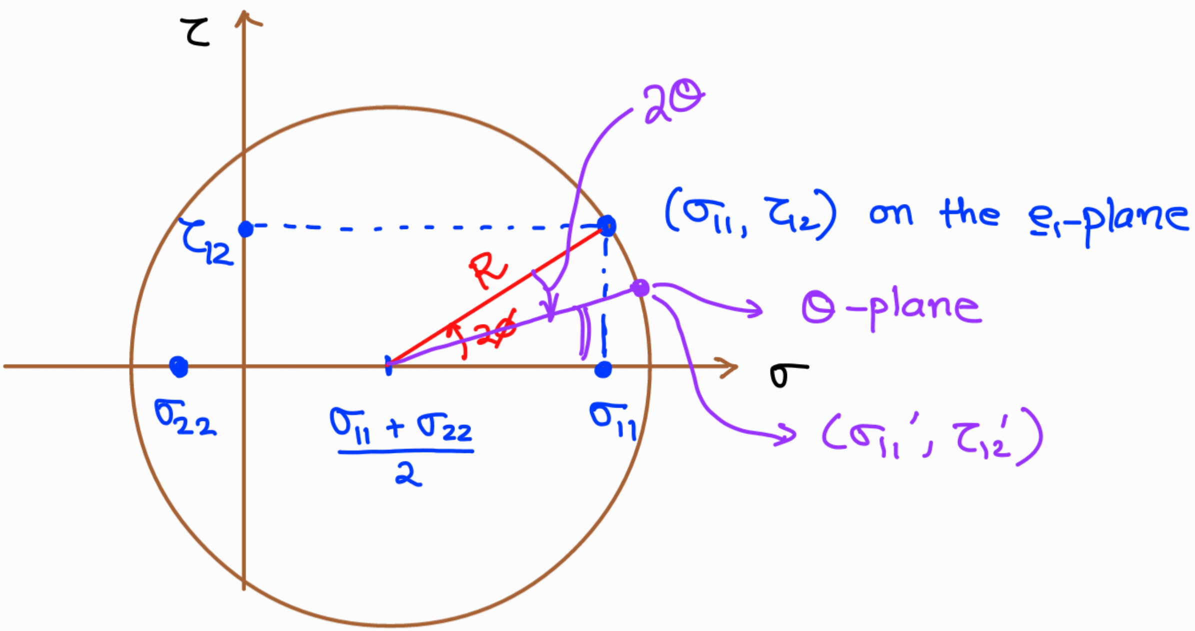
Mohr's Circle

$$\sigma_n = \frac{\sigma_{11}+\sigma_{22}}{2} + R\cos(2\phi - 2\theta)$$

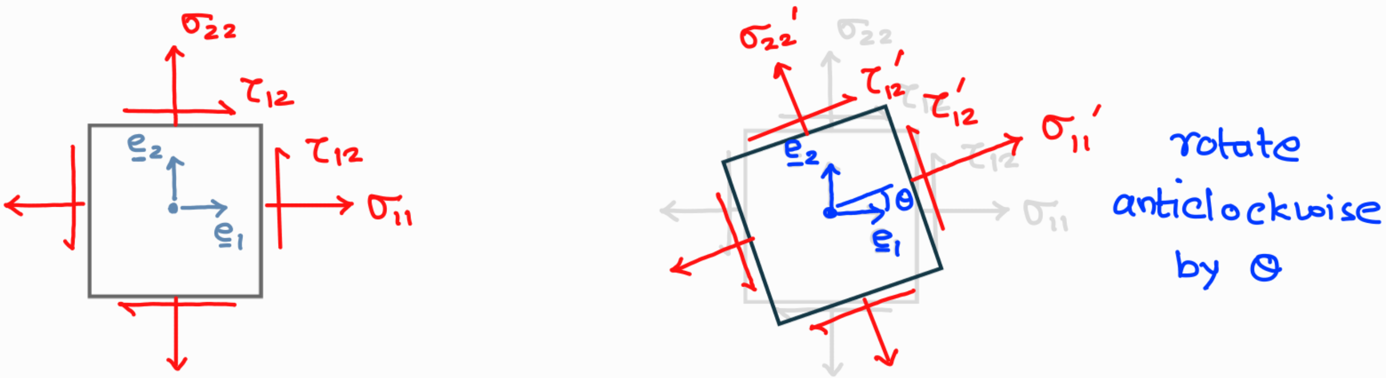
$$\tau_n = R\sin(2\phi - 2\theta)$$

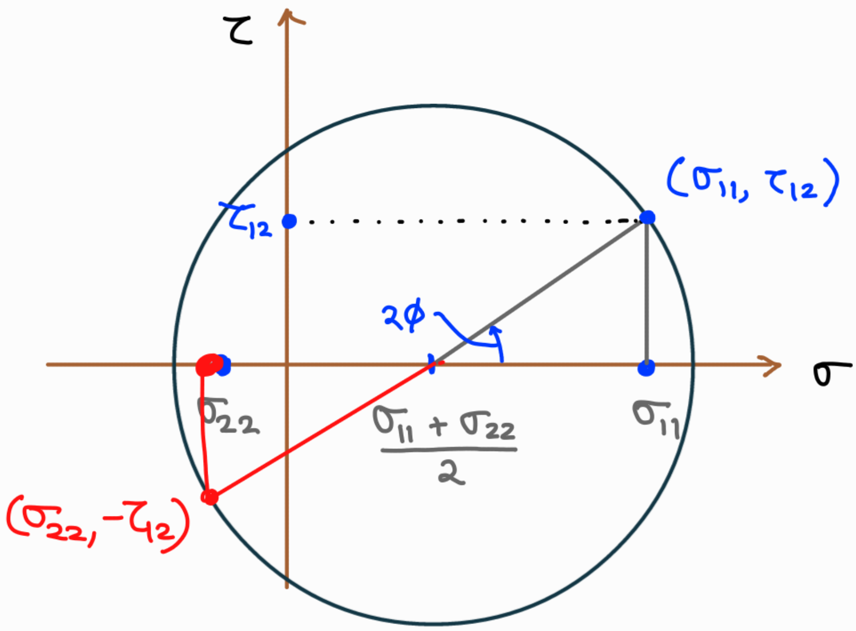
Based on these formulae, lets try to obtain the locus of σ_n and τ_n for all values of θ

Think of a $\sigma-\tau$ plane and plot σ_n and τ_n for each value of θ in this plane. The plane has σ on the x-axis and τ on the y-axis.



The above circle is called the 2D Mohr's circle. It can be used to find the 2D state of stress on any plane oriented at an angle Θ



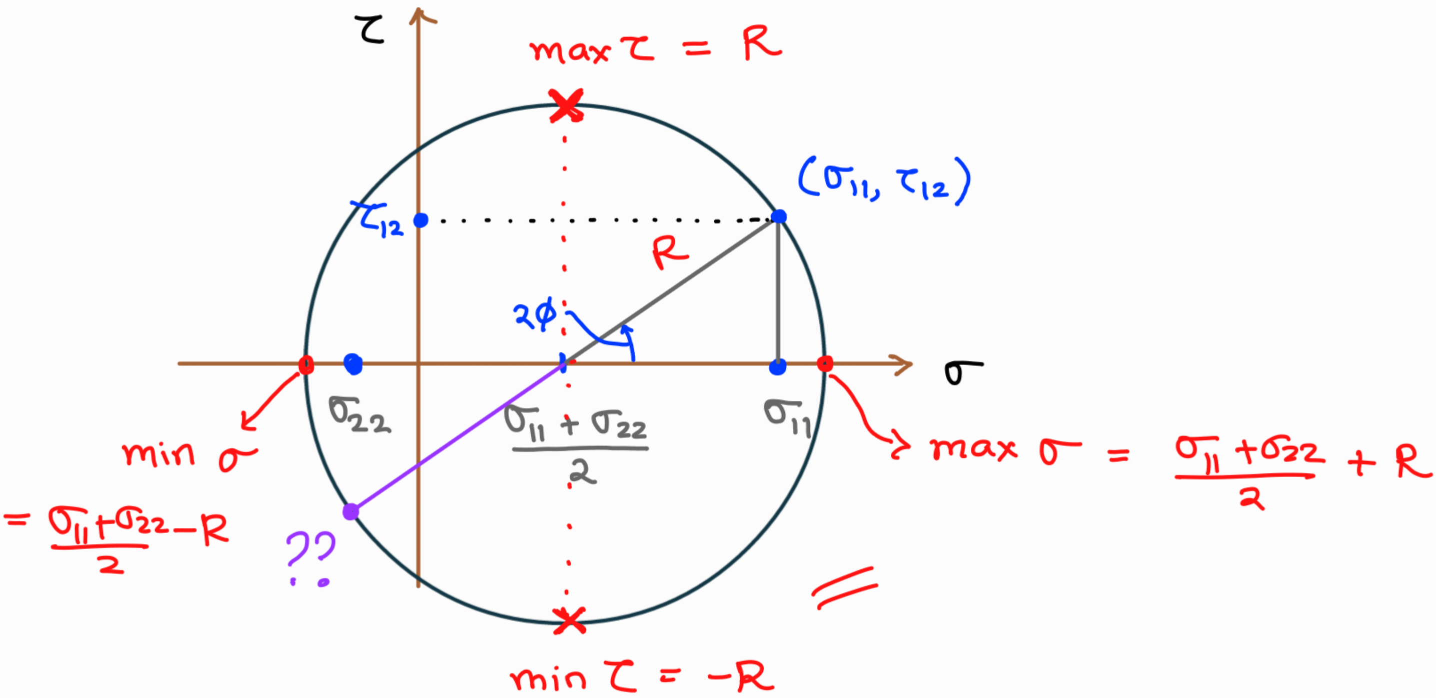


$$\sigma_n = \frac{\sigma_{11} + \sigma_{22}}{2} + R\cos(\underbrace{2\phi}_{\text{clockwise}} \underbrace{- 2\Theta}_{\text{anticlockwise}})$$

$$\tau_n = R\sin(2\phi - 2\Theta)$$

2Θ ACW in Mohr's circle $\Leftrightarrow$ Θ CW in physical plane

Inferences drawn from a 2D Mohr's circle

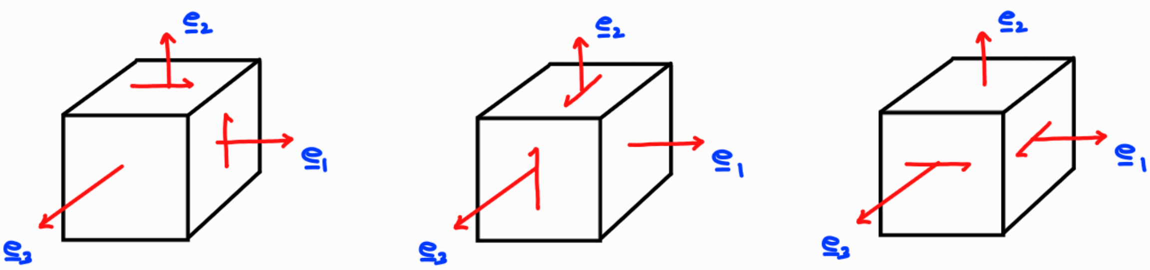


Limitation of 2D Mohr's circle

2D Mohr's circle is only applicable for finding normal and shear stress components on planes that are perpendicular to one of the principal directions. This means we must consider a coordinate system s.t. the third coordinate axis is along one of the principal directions.

So the stress states that are amenable for drawing 2D Mohr's circles must have the following forms:

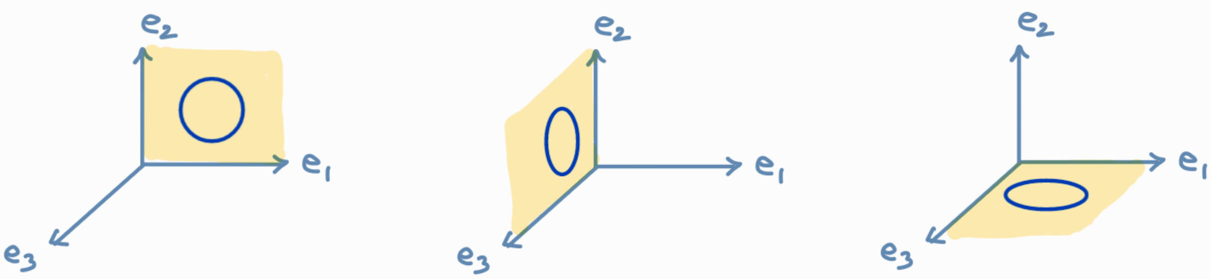
$$[\underline{\sigma}] = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{bmatrix}, \begin{bmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{bmatrix}, \begin{bmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{bmatrix}$$

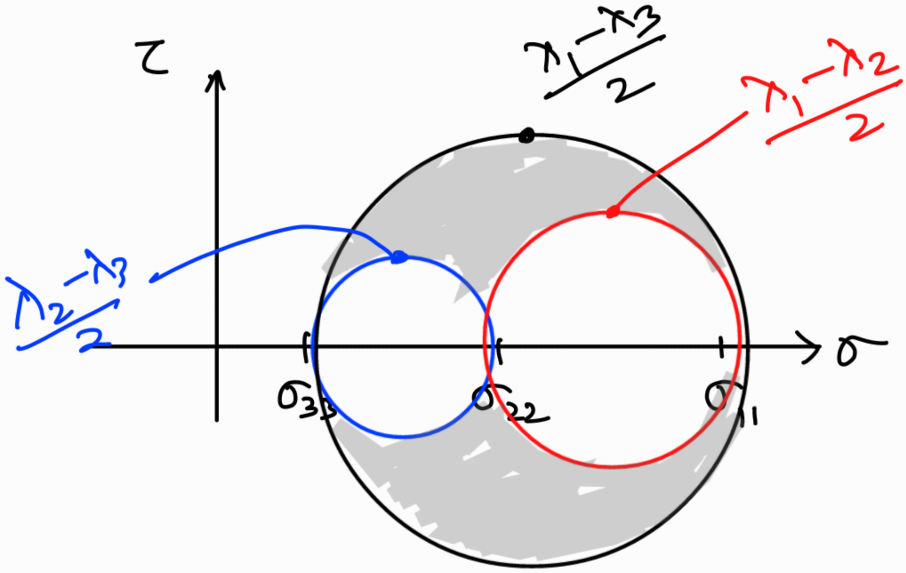


$\Downarrow$ Mohr's circle in $\underline{e}_1$-$\underline{e}_2$ plane

$\Downarrow$ Mohr's circle in $\underline{e}_2$-$\underline{e}_3$ plane

$\Downarrow$ Mohr's circle in $\underline{e}_1$-$\underline{e}_3$ plane





$\sigma_{11} > \sigma_{22} > \sigma_{33}$