In the previous lectures, we looked at study of forces and equilibrium conditions. We saw how to draw FBDs to determine unknown reactions and internal forces using eq^m conditions.

For deformable bodies, we usually need additional requirements to be met. We will see these conditions today.

In rigid bodies, the distance between two pts remains same before and after applying external forces/moments. This is not true for Deformable bodies

Analysis of deformable bodies

There are three steps for analyzing a deformable body

Step 1) Study of forces and equilibrium requirements

(what are the different forces acting, draw FBD, satisfy the equilibrium conditions, find reactions)

Step 2) Study of deformation and conditions of geometric compatibility

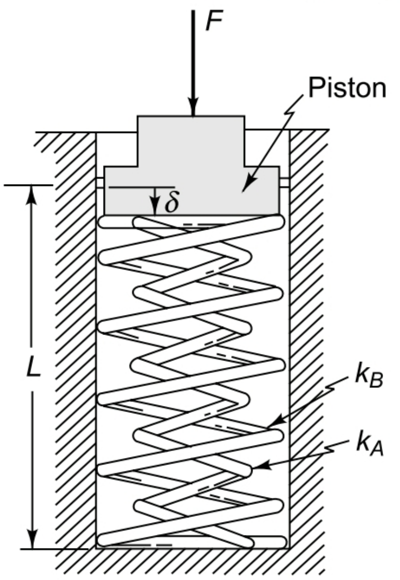
(Deformation cannot be arbitrary; it will follow some rules such that the deformation is compatible with the whole system)

Step 3) Application of force-deformation relations

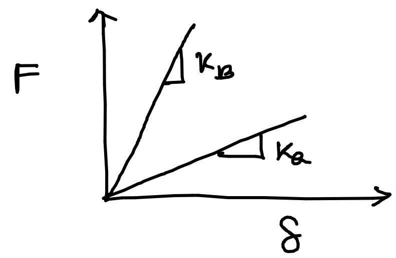
(In the first step, you study the forces which is the cause, In the second step, you study the deformation which is the effect, and in the 3rd step you study the relation betn the cause and effect)

Lets take an example to understand these 3 steps

A cylindrical cavity has two springs whose axes are aligned along the same line.



- Assumed to be linear springs



linear force-deflection relation

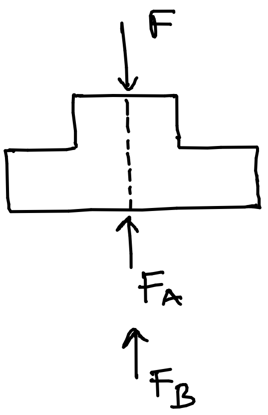
- What is the load carried in each spring?

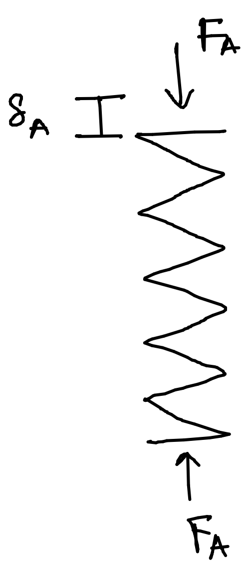
To determine this, we follow the three steps:

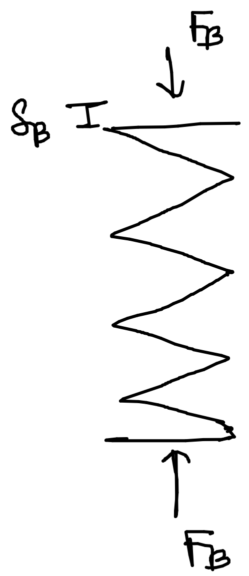
* Study of forces and equilibrium requirements

When you apply a force F on the top of the piston, the piston will try to push down the springs. If you draw the FBD of piston and springs, we get

FBD







2D problem

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

For the piston

$\xrightarrow{+} \sum F_x = 0$ (No forces in x-direction)

$\circlearrowleft + \sum M = 0$ (Since all forces are assumed to have the same line of action)

$+\uparrow \sum F_y = 0$

$$\Rightarrow F_A + F_B - F = 0$$

$$\Rightarrow F = F_A + F_B \quad \text{—— (1)}$$

For the springs

$\left.\begin{array}{l} \sum F_x = 0 \\ \sum M = 0 \end{array}\right]$ (automatically satisfied)

$\sum F_y = 0$ (equal & opposite forces at each ends)

* Study of deformation and conditions of geometric compatibility

We know that the deformations cannot be arbitrary, and there are constraints or restrictions on how the deformations must happen.

So here, what are the requirements for geometric compatibility

Since the springs are in touch with the piston, the springs and the piston must experience the same amt of deformation

Geometric compatibility

$$\therefore \quad \delta_A = \delta_B = \delta \quad \text{—— (2)}$$

* Application of force-deformation relations

Here the force-deformation relation is linear, that is, the force in each spring is linearly proportional to the deflection of the spring:

$$F_A = k_A \delta_A \quad \text{— (3)}$$

$$F_B = k_B \delta_B \quad \text{— (4)}$$

Adding (3) & (4)

$$F_A + F_B = k_A \delta_A + k_A \delta_B \quad \text{— (5)}$$

From (1), $F = F_A + F_B$

From (2), $\delta_A = \delta_B = \delta$

$$F = (k_A + k_B)\,\delta \quad \text{— (6)}$$

From (3) & (5), we can rewrite

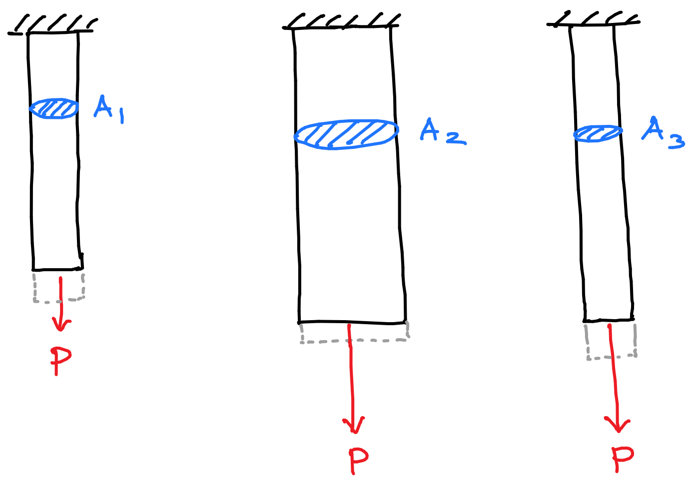
$$\frac{F_A}{F} = \frac{k_A}{k_A + k_B} \Rightarrow F_A = \frac{k_A}{k_A + k_B} F$$

Similarly, $$F_B = \frac{k_B}{k_A + k_B} F$$

An additional observation is that the stiffer of the two springs takes greater load

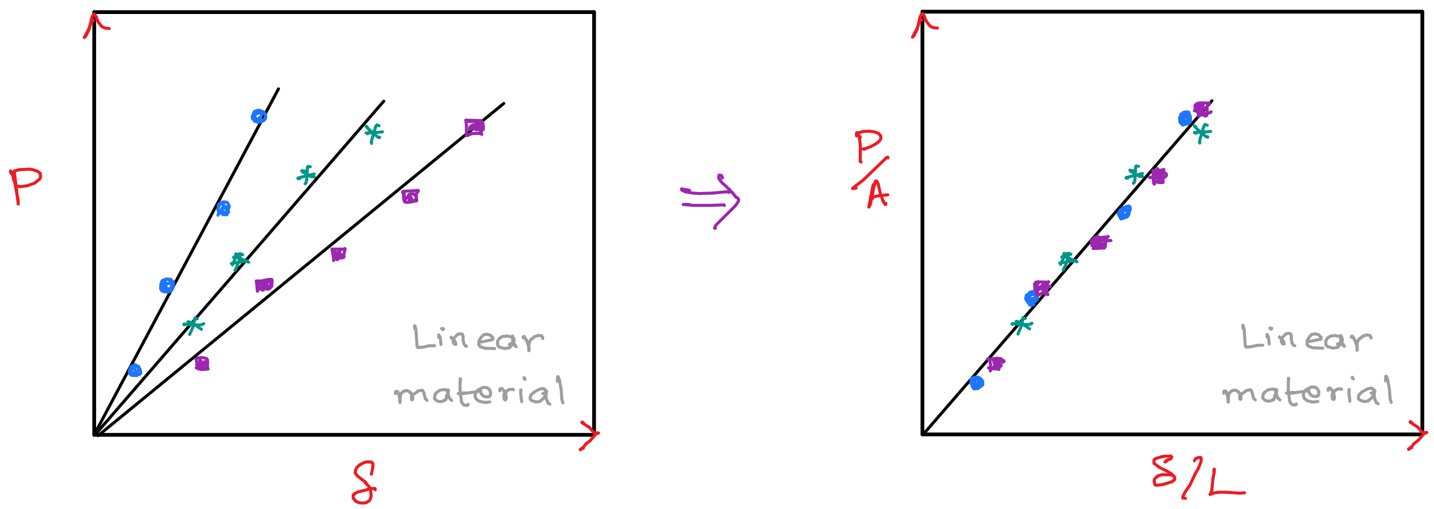
Uniaxial loading and deformation

If we are interested in the relative deformation of one end of a bar/rod w.r.t the other end, under the action of a tensile load, then we can do a uniaxial tensile test.



For each bar, the load is gradually increased from zero and, at several values of the load, a measurement is made of elongation 'δ'

If the maximum elongation is very small, then a linear relation between load and displacement is usually obtained



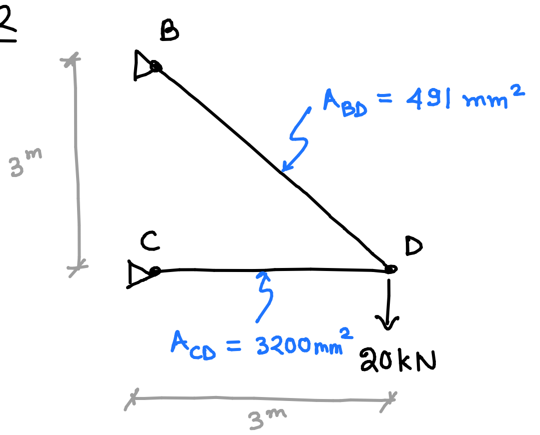
If the uniaxial load-elongation relation of a material is LINEAR $\Rightarrow$ slope of straight line gives modulus of elasticity

$$E = \frac{P/A}{\delta/L} \Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

For most materials with small deformations, the material behavior is similar under compression and tension. So the formula for δ will be assumed to hold for both cases

Let us now analyze deformable uniaxially loaded members

Ex 2



Find displacement at point D

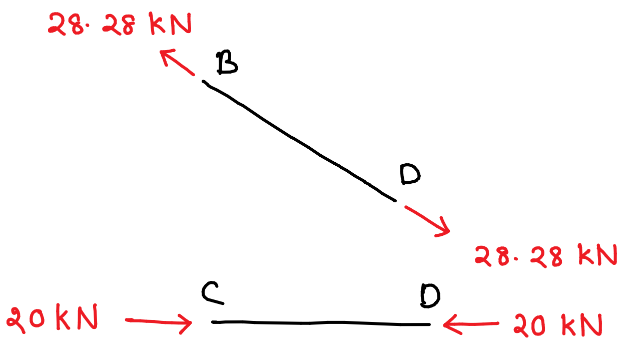
Given : $E = 205\ GPa$ (steel)

$= 205 \times 10^6\ kN/m^2$

$= 205 \times 10^6 \times 10^{-6}\ kN/mm^2$

Solu :

a) Force equilibrium



b) Force - deformation relation

$$\delta_{BD} = \frac{F_{BD}\, L_{BD}}{A_{BD}\, E_{BD}} = \frac{(28.28\ kN)(4.242 \times 10^3\ mm)}{(491\ mm^2)(205\ kN/mm^2)} = 1.19\ mm$$

(Elongation)

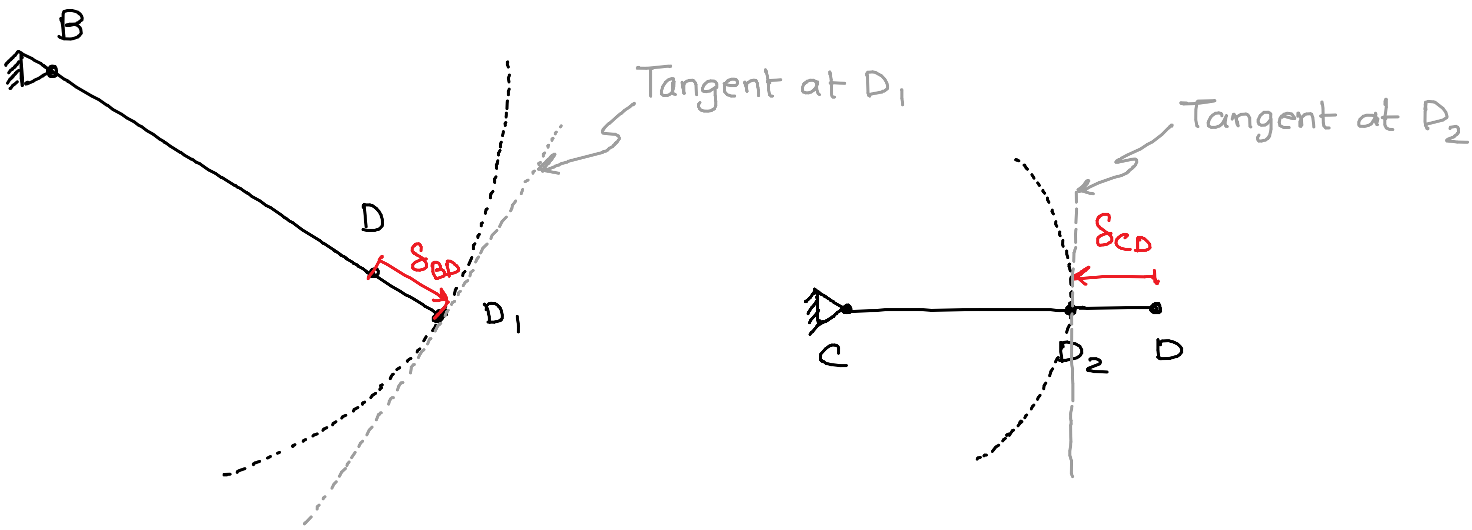
$$\delta_{CD} = \frac{F_{CD}\, L_{CD}}{A_{CD}\, E_{CD}} = \frac{(20\ kN)(3 \times 10^3\ mm)}{(3200\ mm^2)(205\ kN/mm^2)} = 0.0915\ mm$$

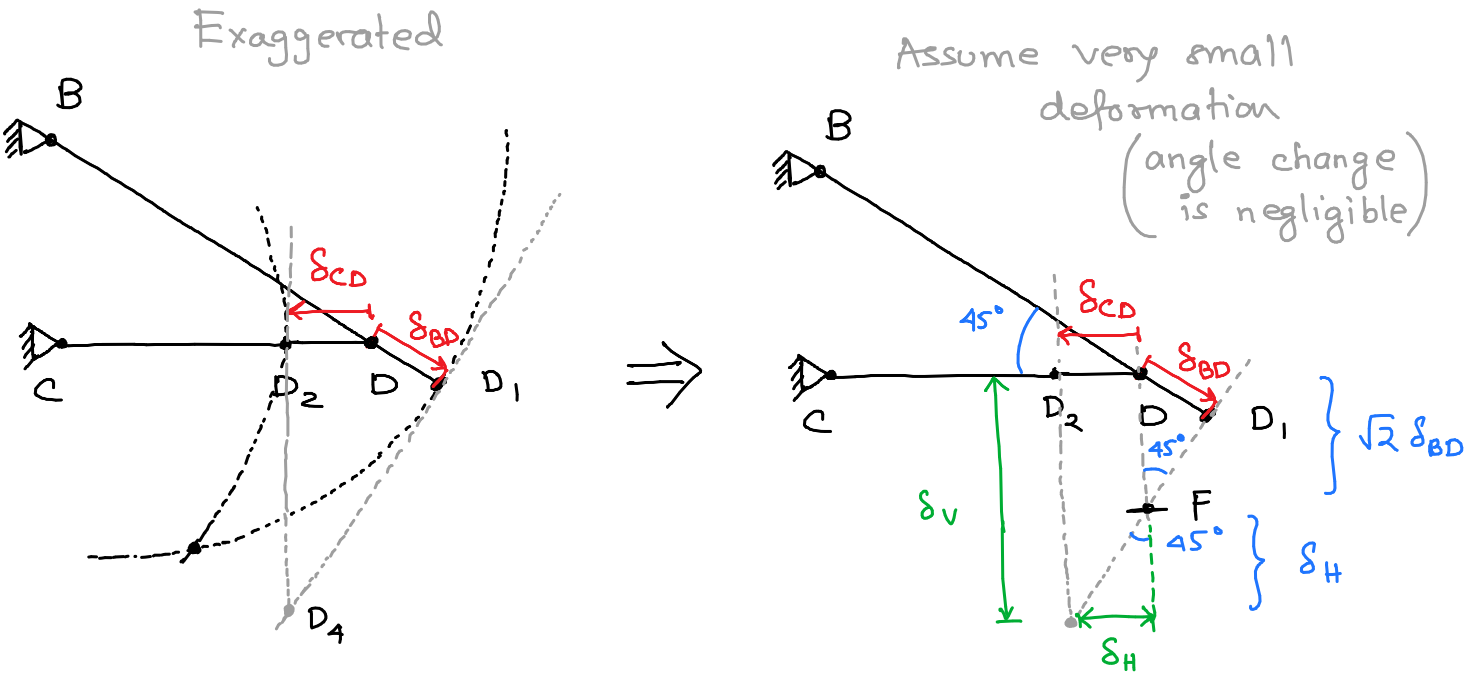
(Compression)

c) Geometric compatibility

Compatibility requires that bars BD and CD move in such a way that they remain straight and connected together at D.

The bars are allowed to change lengths by δ_{BD} and δ_{CD}





$$\delta_H = \delta_{CD} = 0.0915 \text{ mm}$$

$$\delta_V = \sqrt{2}\,\delta_{BD} + \delta_{CD} = 1.77 \text{ mm}$$