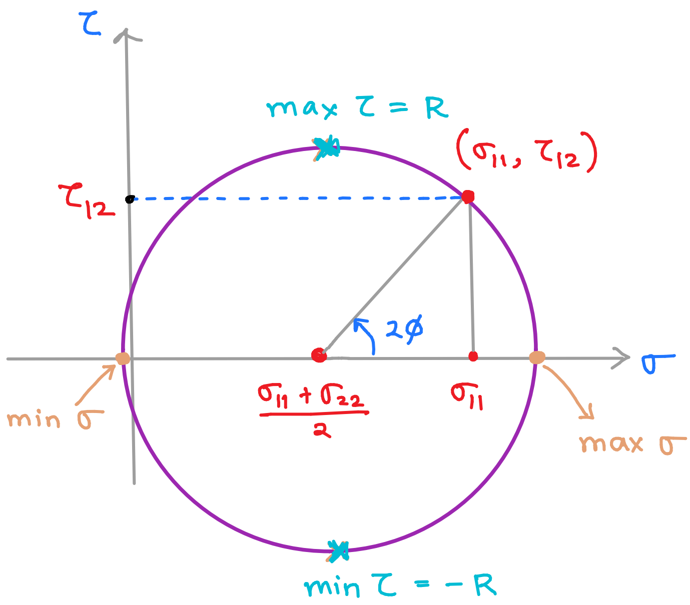
- The max/min values of shear are equal to the radius of the Mohr's circle and occur on the τ-axis

$$\tau_{max} = R = \frac{\lambda_1 - \lambda_2}{2} \quad \text{(top)}$$

$$\tau_{min} = -R = -\frac{\lambda_1 - \lambda_2}{2} \quad \text{(bottom)}$$



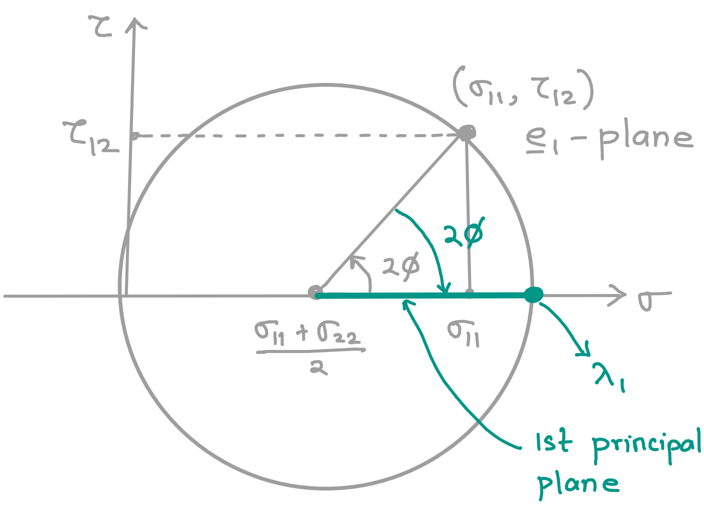
Also, note that the normal stress on the planes having maximum shear stress (topmost and bottommost points of the circle) was the σ corresponding to the center of the circle i.e. $\frac{\sigma_{11}+\sigma_{22}}{2}$ or $\frac{\lambda_1+\lambda_2}{2}$

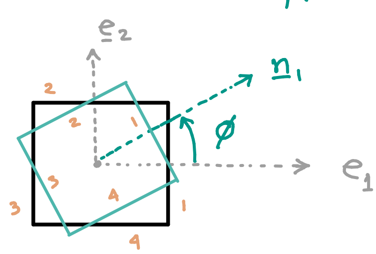
So the coordinates of the topmost pt of the circle is $\left(\frac{\lambda_1+\lambda_2}{2}, \frac{\lambda_1-\lambda_2}{2}\right)$

How do you find the planes of principal stresses from Mohr's circle?

- We need to go 2ϕ clockwise from the $\underline{e}_1$-plane to reach the principal plane, where λ_1 is acting, on the Mohr's circle

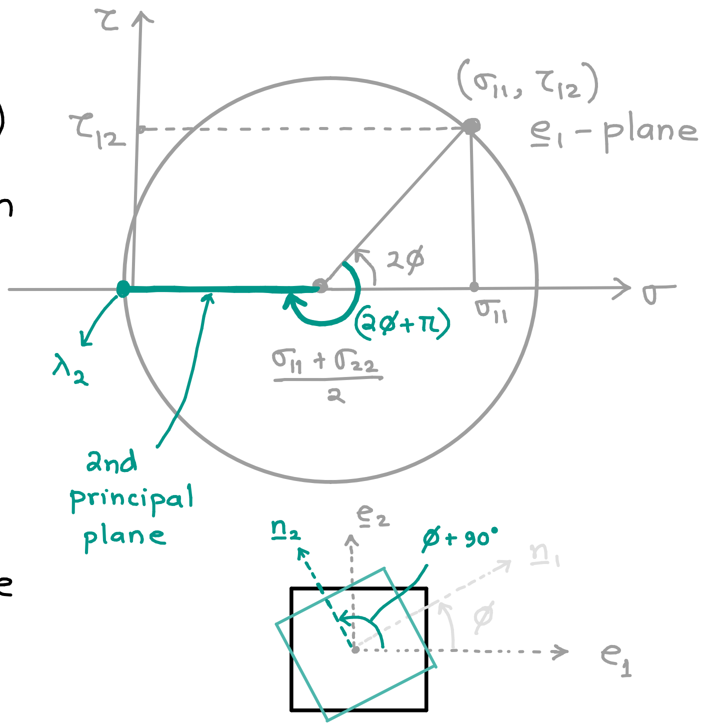
- That means in the original coordinate system, we need to go by an angle of ϕ in the CCW direction from the $\underline{e}_1$-plane to get to the first principal plane





- Similarly, for the 2nd principal plane, we have to go $(2\phi + 180°)$ clockwise in the Mohr's circle from e_1 - plane.

- In actual coordinate system, we need to go by an angle of $(\phi + 90°)$ in the CCW direction from e_1-plane

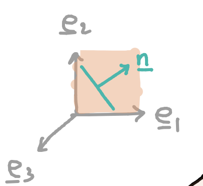
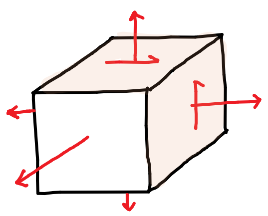


Limitation of 2D Mohr's circle

Mohr's circle is only applicable to finding normal and shear components on planes that are perpendicular to one of the principal directions. As such, we must consider a coordinate system s.t. the third coordinate axis is along one of the principal directions. The stress matrix in this coordinate system will have zero shear components in the third row and column

$$[\underline{\sigma}] = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{bmatrix}$$

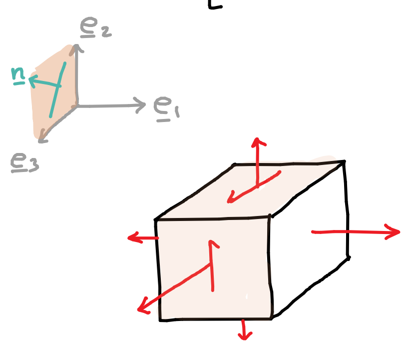
represents this case

$\Downarrow$

You can draw 2D Mohr's circle for e_1-e_2 plane

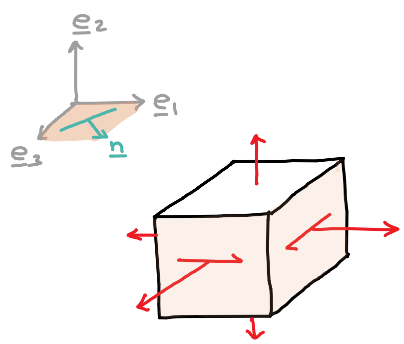
$\underline{n} \perp \underline{e_3}$



$\Downarrow$

You can draw 2D Mohr's circle for e_2-e_3 plane

$\underline{n} \perp \underline{e_1}$



$\Downarrow$

You can draw 2D Mohr's circle for e_1-e_3 plane

$\underline{n} \perp \underline{e_2}$

Example : Consider the following stress matrix

$$[\underline{\sigma}] = \begin{bmatrix} 4\sqrt{2} & 2\sqrt{2} & 0 \\ 2\sqrt{2} & 8\sqrt{2} & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

- Find principal planes, planes of maximum shear stress, and also their values

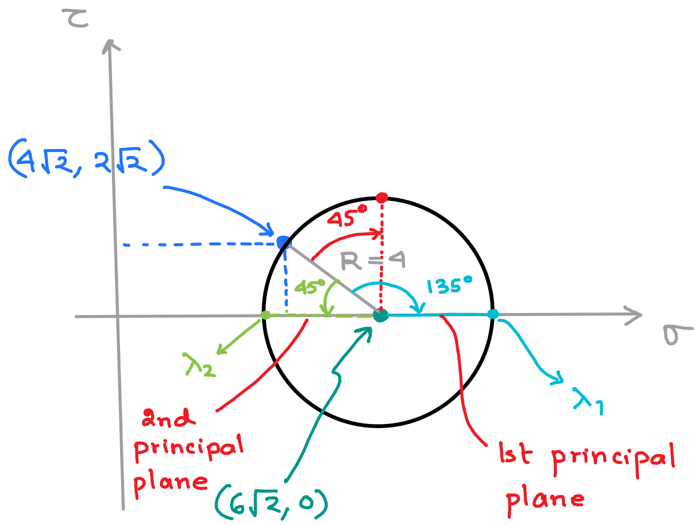
Soln : We can see from the stress matrix that the $\underline{e}_3$-axis is aligned with the principal axis and hence we can use 2D Mohr's circle for the plane spanned by $\underline{e}_1 - \underline{e}_2$.

- First plot (σ_{11}, τ_{12}) on σ-τ plot

- Next plot the center

$$\left(\frac{\sigma_{11}+\sigma_{22}}{2}, 0\right) = (6\sqrt{2}, 0)$$

- Radius $R = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4$



- After drawing the Mohr's circle, the principal stress components and τ_{max} are:

$$\lambda_{1,2} = (\sigma \text{ at center}) \pm R = 6\sqrt{2} \pm 4$$

$$\tau_{max} = R = 4$$

Plane direction from $\underline{e}_1$-plane	Mohr's circle	In actual coordinates
- 1st principal plane	135° CW	$\frac{135°}{2}$ CCW
- 2nd principal plane	45° CCW	22.5° CW
- Max shear plane	45° CW	22.5 CCW

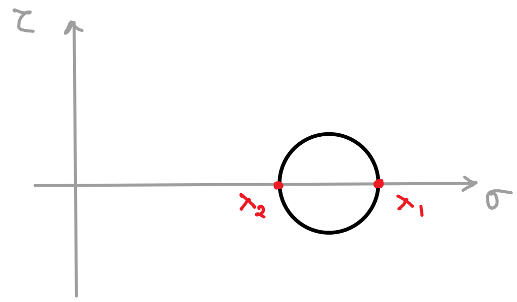
<u>Mohr's stress planes</u>

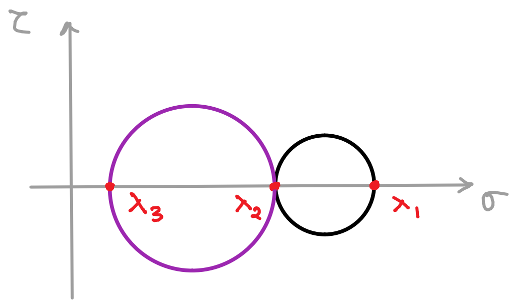
Suppose that we express the stress tensor using the principal directions. Then the stress matrix will be diagonal

$$[\underline{\sigma}] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \qquad \text{Assume} \quad \lambda_1 \geqslant \lambda_2 \geqslant \lambda_3$$

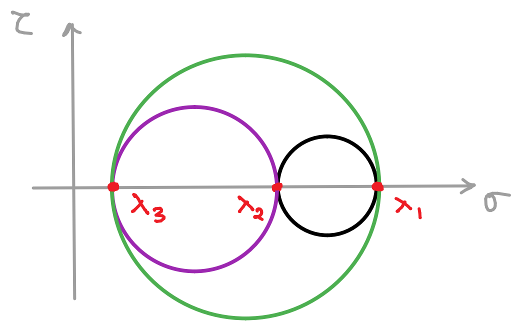
With this stress matrix, we will now think of arbitrary planes and plot $\sigma-\tau$ on those planes. We will NOT confine ourselves to planes whose normal is perpendicular to one of the principal axes.

However, let us first consider the planes whose normal is $\perp$ to $\underline{n}_3$ axis. If we start plotting $(\sigma-\tau)$ on such planes, we will get the Mohr's circle passing through λ_1 and λ_2

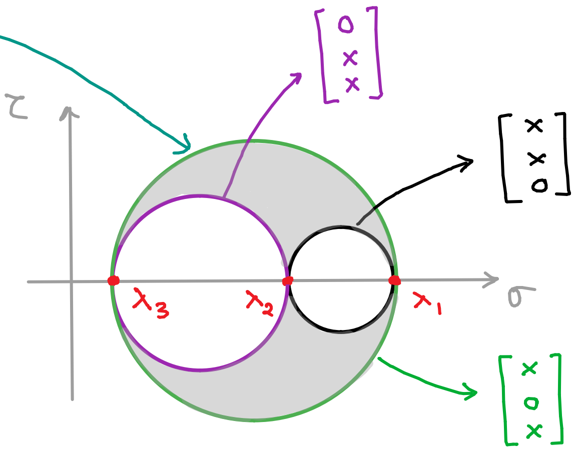




Similarly, we can draw the Mohr's circle corresponding to planes whose normals are perpendicular to 1st principal axis, $\underline{n}_1$: this circle passes through λ_2 and λ_3. Then we take the planes with normals perpendicular to the 2nd principal direction and we get Mohr's circle through λ_1 and λ_3



This will be the biggest circle of the three circles ($\because \lambda_1 \geq \lambda_2 \geq \lambda_3$)



These three circles correspond to very specific normal directions i.e., they have one of their components zero. If we now plot (σ, τ) for all planes with arbitrary normal directions, we will get the shaded region. This $\sigma - \tau$ plot is called the Mohr's stress plane or 3D Mohr's circle

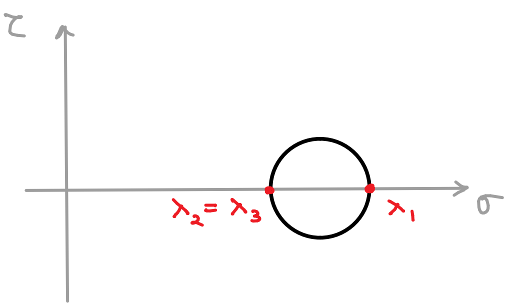
Absolute Maximum Shear Stress can be obtained from the 3D Mohr's circle:

$$\tau_{max} = \frac{\lambda_1 - \lambda_3}{2} \qquad (\because \lambda_1 \geq \lambda_2 \geq \lambda_3)$$

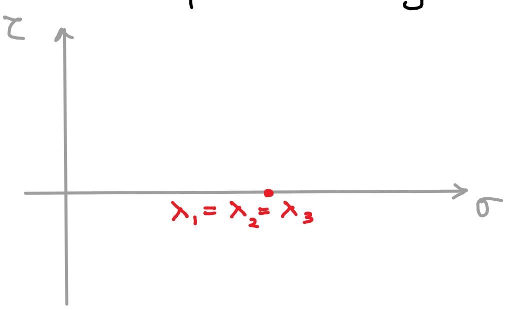
(the radius of largest, outer Mohr's circle)

Special case I: Two repeated eigenvalues (say $\lambda_1 > \lambda_2 = \lambda_3$)

Then the circle corresponding to (λ_2, λ_3) shrinks to a point



Special case II: Three repeated eigenvalues ($\lambda_1 = \lambda_2 = \lambda_3$)



Whole region shrinks to a point