What is solid mechanics ?

- It is the science of forces and motions of any solid material

- Solid materials have
  - → definite shape
  - → can resist tensile forces to some extent

If you apply some force or moment on a solid body, you would get some movements or motions. To understand the physics of the motions, you need solid mechanics.

This course also has several others names such as

→ Mechanics of Solids

→ Mechanics of Materials

We need to study this course

a) to understand the solid body behavior under the action of forces/moments

b) to develop rational rules for the safe ign of a system

What are the steps involved in analyzing a mechanical system?

(a mechanical system could be a building, bridge, aircraft, etc)

We know that mechanics deals with forces and motions

Therefore, we must study

a) Forces / Moments (you want to know the cause)

b) Motion / Deformation (if you consider any system under some force, you will see some motion)

→ study of motion involves change of geometry

Motion / movement

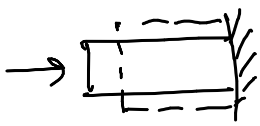
(Two types)

Overall changes in position of the body with time (with no change in shape of body)

Change in shape of the body

— termed as deformation

Translation
(or rotation)



We will consider this type of motion in this course

What are the steps involved in analyzing a mechanical system?

a) Forces / Moments

b) Motion / Deformation

c) Application of laws relating the forces to the motion

There are some laws that govern the relation between the cause and the effect, so here we try to correlate what would be the effect

In this course we will mostly focus on statics and we would not be talking about motion of the system, or systems that are not moving or vibrating

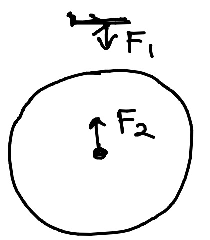
So we will be following these basic three steps but will exclude bodies that are static

Study of forces

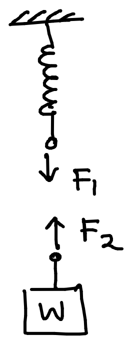
What is the concept of force?

It is an effective means to describe a very complex interaction between bodies

Ex:



Physically separated force interactions : electric
magnetic
gravitational



Direct contact force interactions

From Newton's 3rd law :

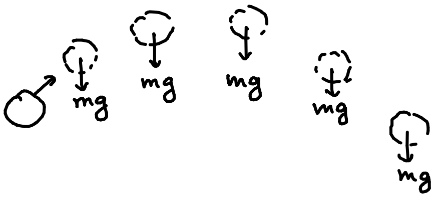
$F_1 = F_2$ (equal & opposite effect of forces along the same line of action)

Study of forces

- It is a vector interaction ⇒ each force has → a magnitude
  → a direction

- There are two principal effects of force:

  - tends to alter the motion of the system involved

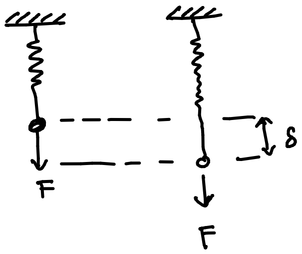


[You throw a ball in the upward direction, but it will eventually fall down]

[Gravitational force alters the motion of the ball]

  - tends to deform the shape of the system

Ex :



The force elongates the spring length

Let's say you are applying some force at the end.

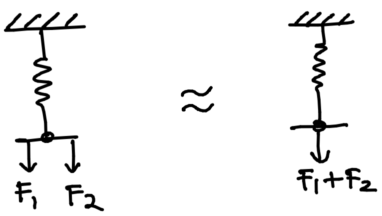
- The magnitude of a force must be established in terms of a standardized experiment and the direction must be known

  You must know how much force you are applying

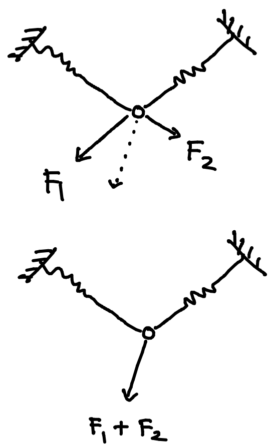
  This may be quantified using standard units of force like Newton (N)

- When two or more forces act simultaneously, at one point, the effect is same as if a single force equal to the vector sum of the individual forces were acting

Ex 1 :



Ex 2 :



- Point of application of force

If you apply a force at one point, you get one response

If you apply a force at some another point, you get a different response

Usually the following things must be known

1) Magnitude and direction of the force

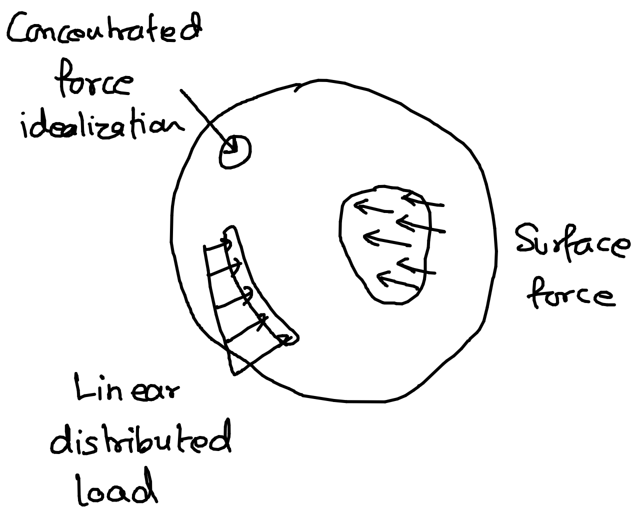
2) Vector summation of force system

3) Point of application of the forces

Types of forces

- EXTERNAL LOADS → Surface forces
- EXTERNAL LOADS → Body forces

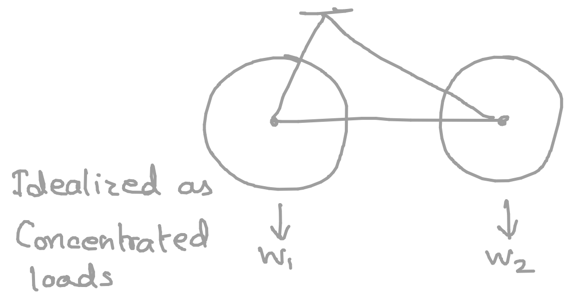
→ Surface forces : Forces caused by direct contact of one body with the surface of another



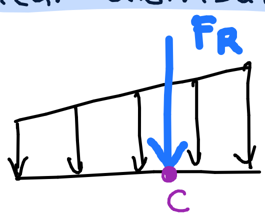
- Surface forces are distributed over the <u>area of contact</u> between the bodies

- If the area is small in comparison to total surface area the surface force may be idealized as a single concentrated force applied to a point on the body

Ex



- If the surface loading is applied along a narrow strip of area, the external load can be idealized as a linear distributed load



The resultant force F_R = Area under the distributed loading curve

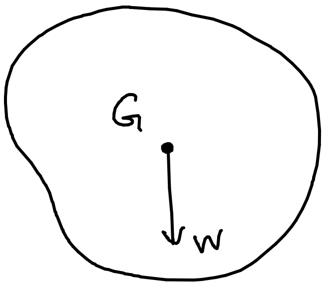
↓

Point of application of F_R ⟹ Geometric center of the area

Types of forces

- **External loads:** → Surface forces
  → Body forces

→ Body forces → These are developed when a body exerts a force on another body without direct physical contact between the bodies



Ex: Gravitational force
Electromagnetic force

Body forces — usually act on each particle composing the body
— can be denoted by a single concentrated force acting on the body

Ex: Weight due to gravity acts through the body's center of gravity

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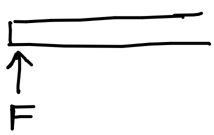
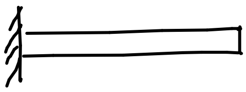
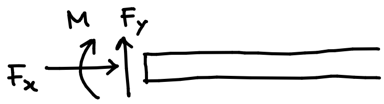
- **Support reactions:** The surface forces that develop at the supports or points of connections between bodies

Connection	Reaction
	
Roller	One unknown

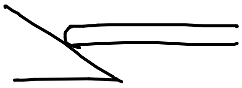
General rule: If support prevents translation in a given direction, then a force must be developed on the member in that direction



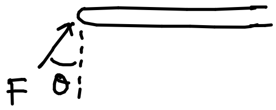
Fixed support



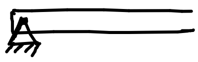
Three unknowns : F_x, F_y, M



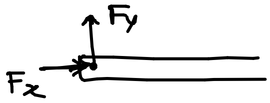
Smooth support



One unknown : F



Hinged support



Two unknowns : F_x, F_y

General rule : If rotation is prevented, a couple moment must be acting on the member

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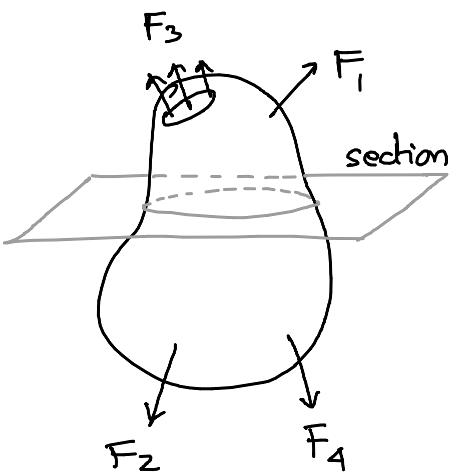
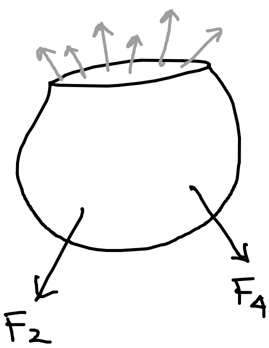
- INTERNAL RESISTIVE FORCES : These are surface forces that are developed inside a body in resistance to the externally applied forces.

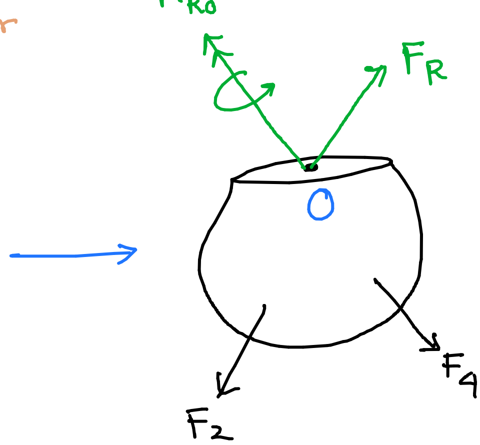
In order to obtain internal resistive forces acting on a specific region, cut an imaginary section where the internal forces are to be determined



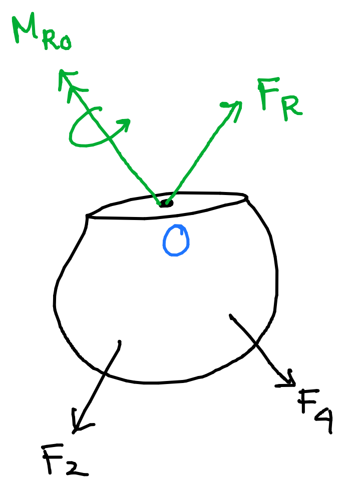
There will be a distribution of internal force over the exposed area

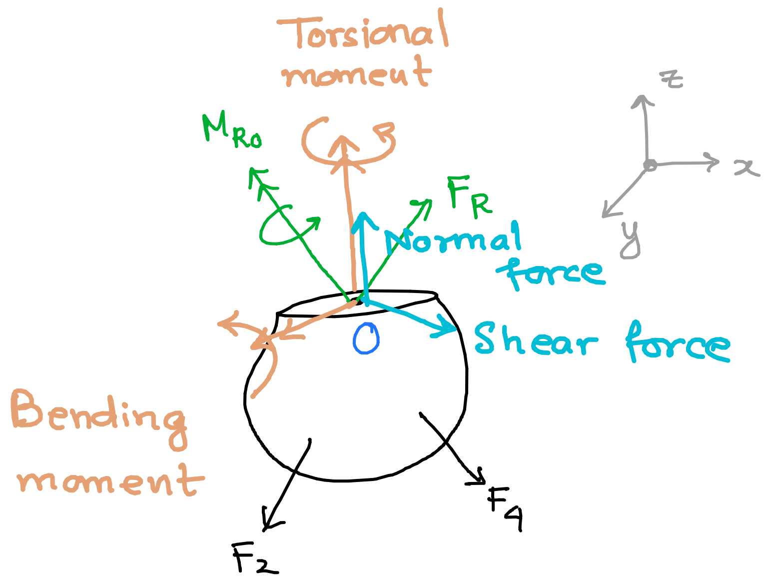


We often take resultants of the distribution acting at the centroid O



Upon resolving the resultant forces along normal and tangent directions to the exposed area, we obtain four different types of internal resistive forces





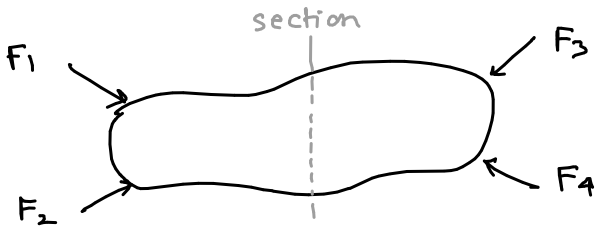
Normal force : - This force acts perpendicular to the area
- Developed when external forces pull or push

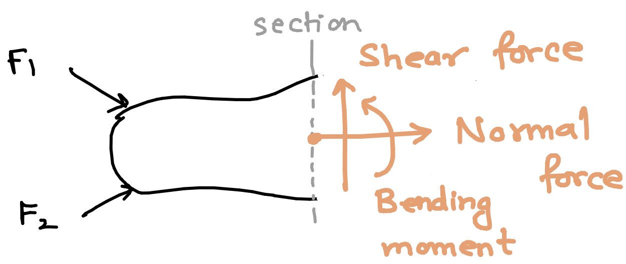
Shear force : - This force lies in the plane of the area.
- Developed when external forces cause sliding

Torsional moment : - Caused when external loads tend to twist one segment of body w.r.t other about an axis perpendicular to the area

Bending moment : - Caused by external loads trying to bend the body about an axis lying within the plane of the area

For <u>COPLANAR LOADS</u>





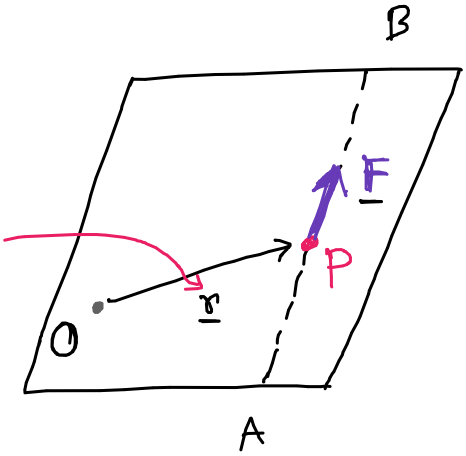
Moment of a force

underbar F → represents vector F

It is an effect of applying a force

So lets consider a force $\underline{F}$ acting along the line of action AB

displacement vector from O to P



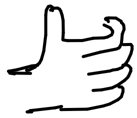
- the point of application of the load/ force is important
- the line of action is very important in this case

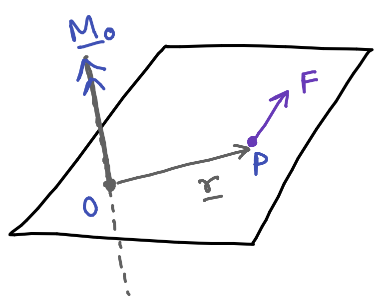
Now, there is another point O, which is just an arbitrary fixed point. Now, if you want to calculate the moment of F about point O, it will be the vector (cross) product

$$\underline{M}_O = \underline{r} \times \underline{F}$$

What is $\underline{r}$? ⟶ Displacement vector from O to P

- So knowing the point of application of force is important to establish the displacement vector
- This moment M_o is a vector quantity, and it will act in a direction perpendicular to the plane given by OP and force vector $\underline{F}$
- Sense of direction is given by Right-hand thumb rule

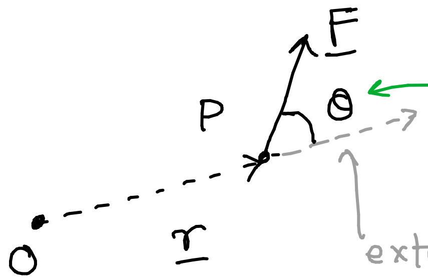




Now how will you calculate the magnitude of the moment?

The magnitude of moment $|\underline{M}_0| = |\underline{F}|\,|\underline{r}| \sin\theta$

What is θ? If you come back to the figure

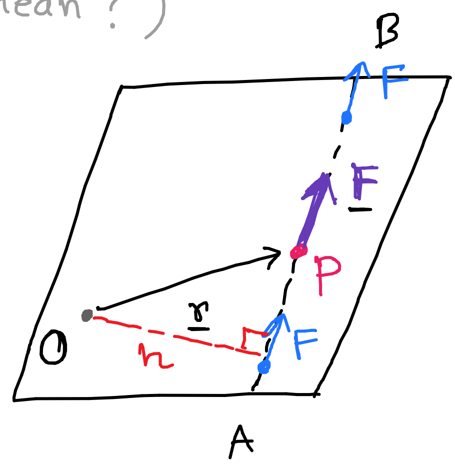


this angle between vectors $\underline{F}$ and $\underline{r}$ is θ

extend this $\underline{r}$

This magnitude of $\underline{M}_0$ is independent of the position of P along AB $\Leftrightarrow$ $|\underline{M}_0|$ of a force about a given pt is invariant under the operation of sliding the force along its line of action

(what does this mean?)



If you slide this force vector $\underline{F}$ along the line of action AB, and if you recalculate M_0, you will find M_0 remains same

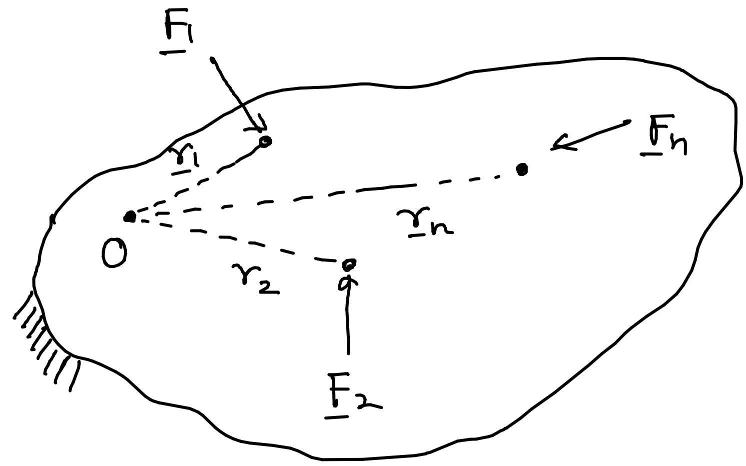
In simple terms, magnitude of moment

$|\underline{M}_0| = h\,|\underline{F}|$, $h \rightarrow$ length of perpendicular dropped from O to AB

$h = |\underline{r}| \sin\theta$

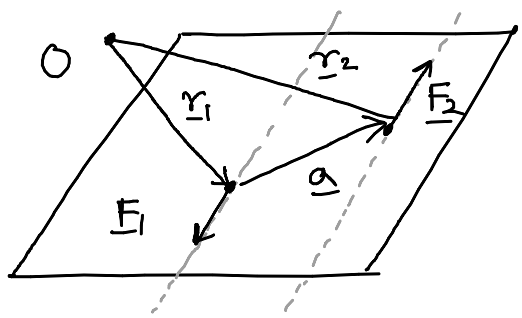
Moment due to several forces

When several forces $\underline{F}_1, \underline{F}_2, \cdots, \underline{F}_n$ act on a body, the total moment about a fixed point O is defined as



$$\underline{M}_o = \underline{r}_1 \times F_1 + \underline{r}_2 \times \underline{F}_2 + \cdots + \underline{r}_n \times \underline{F}_n$$

- When there are two equal and parallel forces that act in opposite sense, then the configuration is called a couple



$$M_o = \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2$$
$$= \underline{r}_1 \times \underline{F}_1 + (\underline{r}_1 + \underline{a})\, \underline{F}_2$$
$$= \underline{r}_1 \times (\underline{F}_1 + \underline{F}_2) + \underline{a}\, \underline{F}_2$$
$$= r_1 \times (\underline{F}_1 - \underline{F}_1) + \underline{a}\, \underline{F}_2$$
$$= \underline{a}\, \underline{F}_2$$

* the result is independent of the location of O

or

the moment of a couple is same about all pts in space