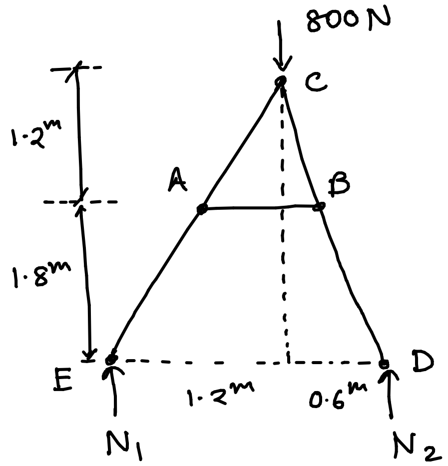
Tutorial 1 solutions

1) NO SURFACE FRICTION

→ Draw FBD of the entire ladder frame



Use eq^m conditions

$\circlearrowleft + \sum M_D = 0$

$\Rightarrow \quad 800\,(0.6) - N_1\,(1.8) = 0$

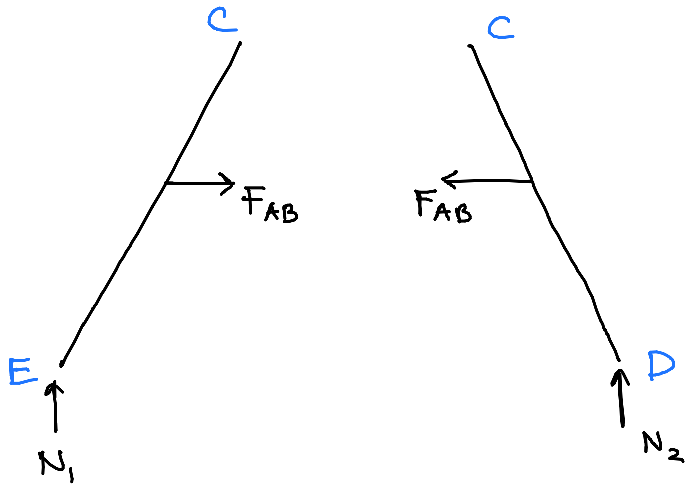
$\Rightarrow \quad N_1 = 266.67\text{ N}$

$+\uparrow \sum F_y = 0$

$\Rightarrow \quad N_1 + N_2 - 800 = 0$

$\Rightarrow \quad N_2 = 533.3\text{ N}$

→ Draw FBD of isolated members

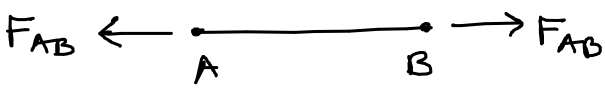


Apply eq^m condition for CE

$\circlearrowleft + \sum M_C = 0$

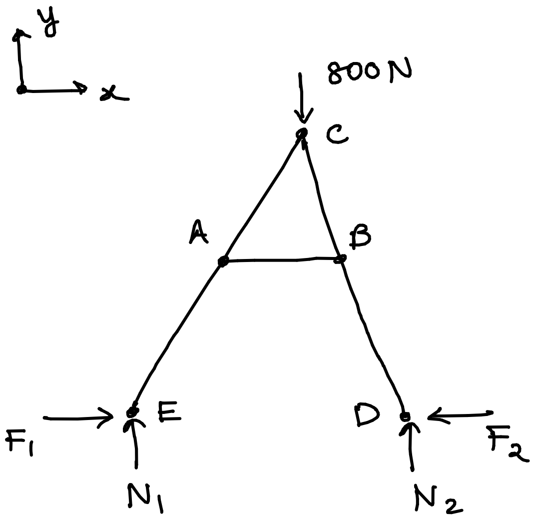
$\Rightarrow F_{AB}\,(1.2) - N_1\,(1.2) = 0$

$\Rightarrow F_{AB} = 266.67\text{ N}$



(under tension)

With surface friction



From eqm of the entire system in the x-direction,

$$\xrightarrow{+} \sum F_x = 0$$

$$\Rightarrow \quad F_1 = F_2$$

$$\circlearrowleft + \sum M_E = 0$$

$$\Rightarrow \quad N_1 = 266.67 \text{ N}$$

$$+\uparrow \sum F_y = 0$$

$$\Rightarrow \quad N_2 = 533.33 \text{ N}$$

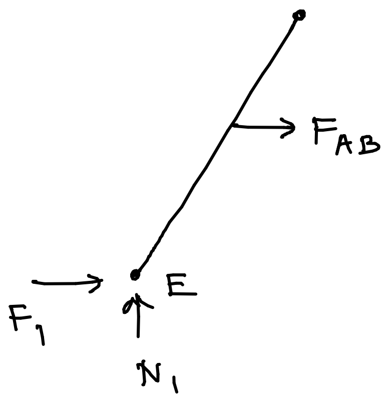
The friction forces developed could be

$$|F_1| \leq 0.2\,(266.67) = 53.33 \text{ N}$$

$$|F_2| \leq 0.2\,(533.33) = 106.67 \text{ N}$$

Since $F_1 = F_2$, it must be that $F_1 = F_2 = 53.33$ N

To obtain F_{AB}, isolate member CE



$$\circlearrowleft + \sum M_c = 0$$

$$\Rightarrow \quad F_{AB}\,(1.2) + F_1\,(3) - N_1\,(1.2) = 0$$

$$\Rightarrow \quad F_{AB} = 133.3 \text{ N}$$

However, if F_1 is directed opposite, then

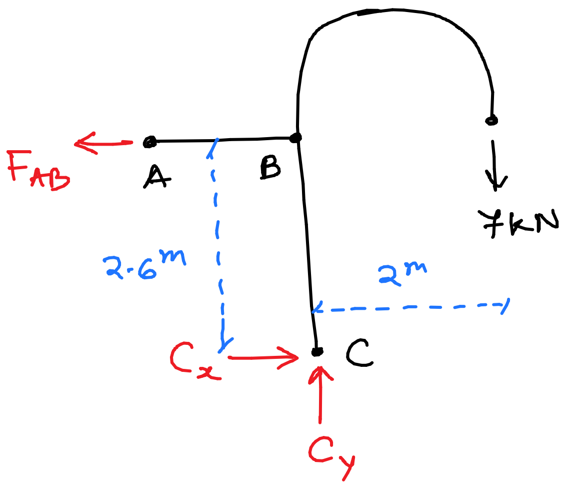
$$F_{AB} = F_1\,(3) + N_1\,(1.2)$$

$$= 400 \text{ N}$$

The designer should account for the maximum force F_{AB}

2)

FBD of the entire system



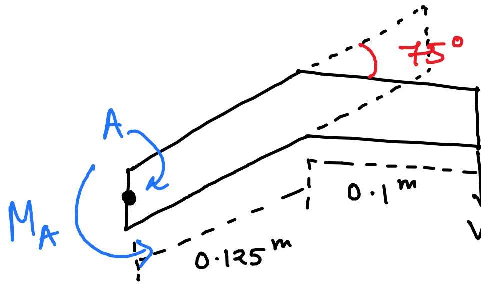
Apply eq^m condition

$\circlearrowleft + \sum M_c = 0$

$\Rightarrow F_{AB}(2.6) - 7(2) = 0$

$\Rightarrow F_{AB} = 5.385$ kN

3)



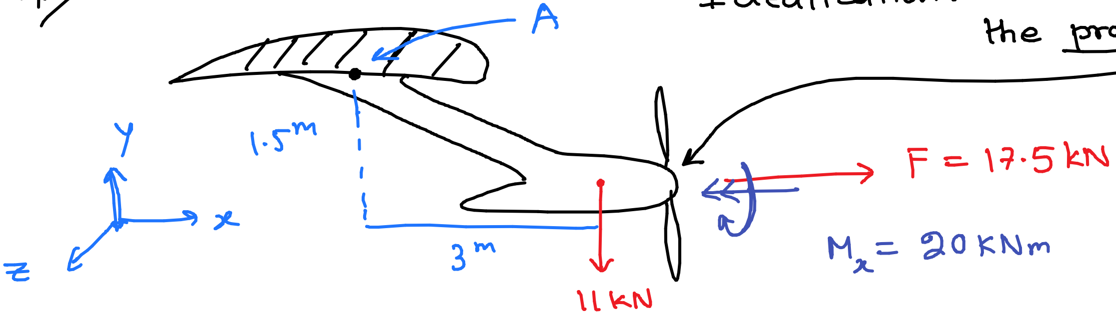
Maximum moment at weld

$M_A = W(0.1\cos 75° + 0.125)$

$\Rightarrow 100 = W(0.1\cos 75° + 0.125)$

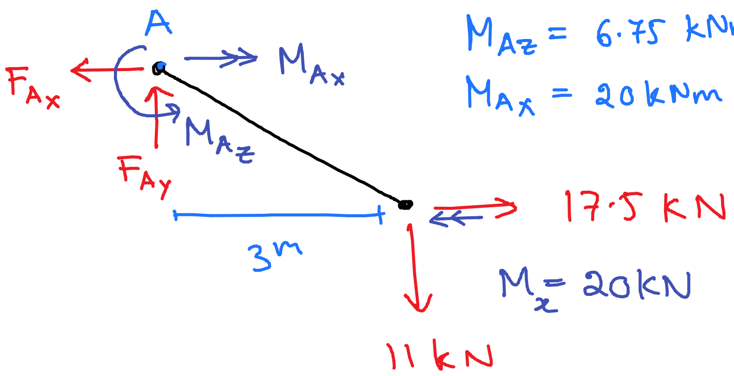
$\Rightarrow W = 632.8$ N

4)



Idealization: The thrust acts through the propeller hub

FBD of strut only



$M_{Az} = 6.75$ kNm

$M_{Ax} = 20$ kNm

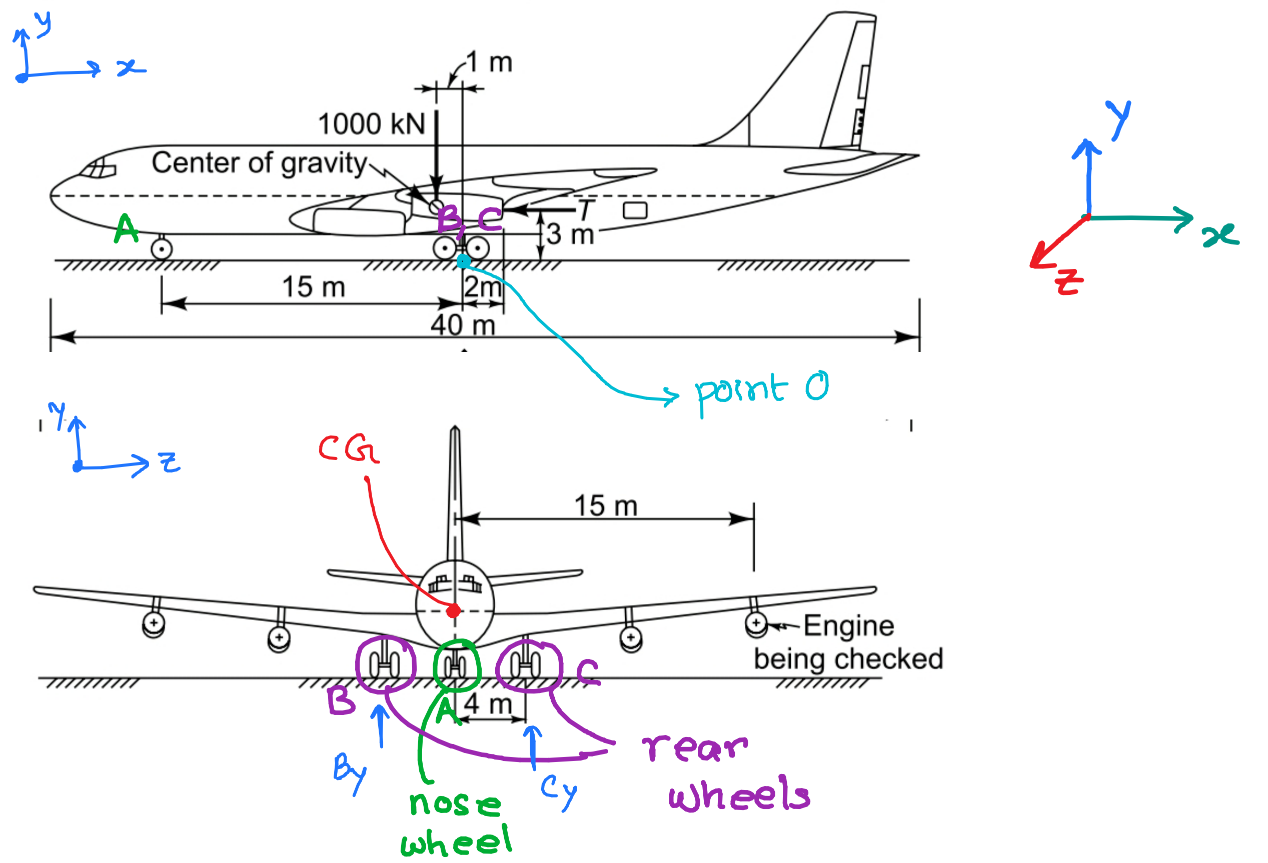
$$\begin{cases} F_A = -17.5 \text{ kN } \hat{i} + 11 \text{ kN } \hat{j} \\ M_A = 20 \text{ kNm } \hat{i} + 6.75\hat{k} \text{ kNm} \end{cases}$$

Force + moment exerted on wing

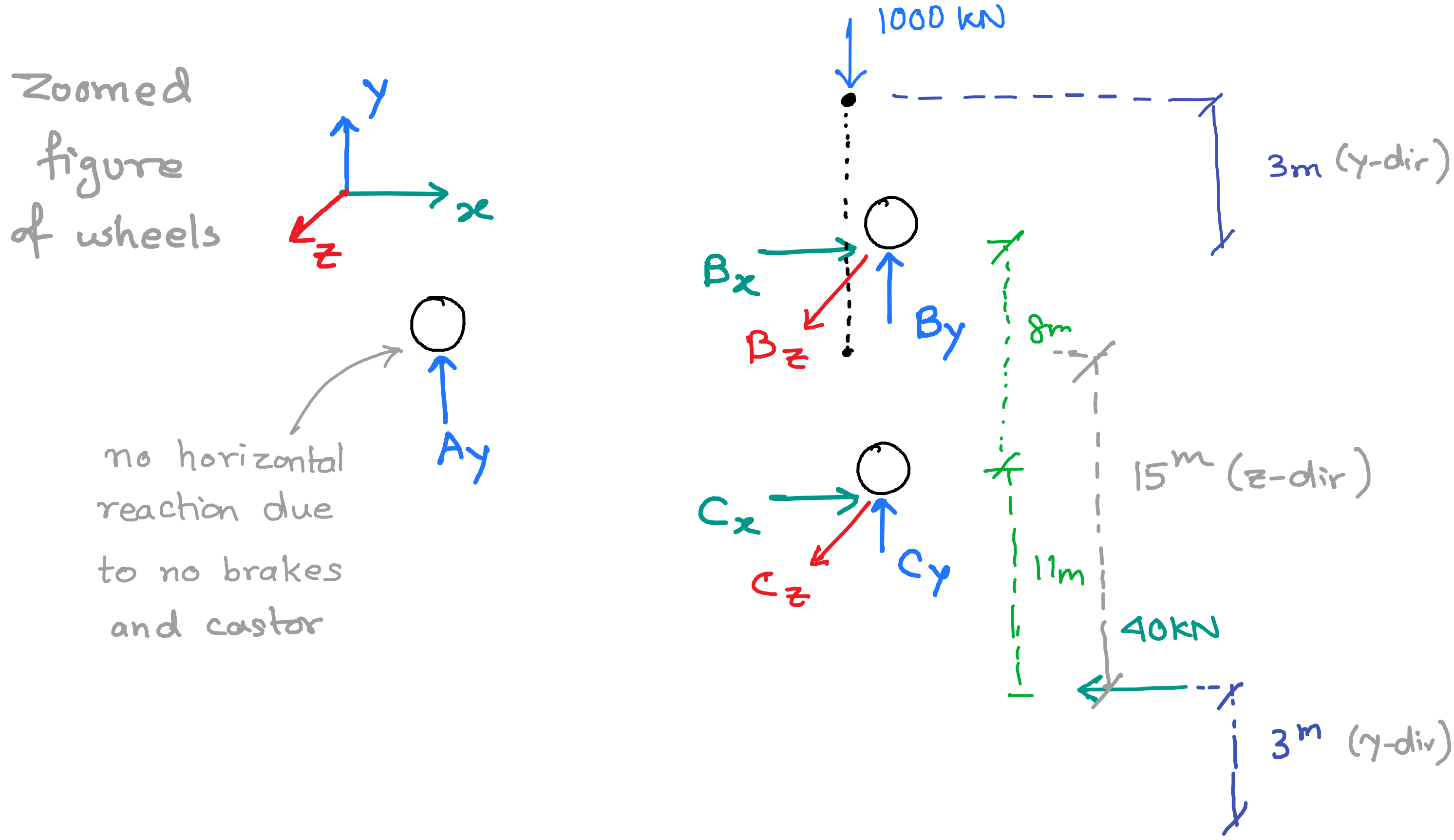
$F_A = 17.5\hat{i} - 11\hat{j}$ kN

$M_A = -(20\hat{i} + 6.75\hat{k})$ kNm

5)



Idealize the two wheels on one side as one wheel



Applying eq^m equations

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow C_x + B_x - 40\text{ kN} = 0$$

$$\Rightarrow C_x + B_x = 40\text{ kN} \quad — (1)$$

$$+\uparrow \sum F_y = 0 \Rightarrow A_y + C_y + B_y - 1000\text{ kN} = 0 \quad — (2)$$

$$\overset{+}{\swarrow} \sum F_z = 0 \Rightarrow C_z + B_z = 0$$

$$\Rightarrow C_z = -B_z \quad — (3)$$

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$$\overset{+}{\twoheadrightarrow} \sum M_{x|O} = 0 \Rightarrow -B_y(4^m) + C_y(4^m) = 0$$

$$\Rightarrow B_y = C_y \quad — (4)$$

$$+\Uparrow \sum M_{y|O} = 0 \Rightarrow C_x(4^m) - B_x(4^m) - (40\text{ kN})(15^m) = 0$$

$$\Rightarrow C_x - B_x = 150\text{ kN} \quad — (5)$$

$$\overset{+}{\swarrow} \sum M_{z|O} = 0 \Rightarrow (40\text{kN})(3^m) - A_y(15m) + (1000\text{ kN})(1^m) = 0$$

$$\Rightarrow A_y = 1200/15 = 80\text{ kN} \quad — (6)$$

7 Unknowns: $B_x, C_x, A_y, B_y, C_y, B_z, C_z$

Using (1) & (5) $\Rightarrow$ $C_x = 95\text{kN}$, $B_x = -55\text{kN}$

Using (2), (4) & (5) $\Rightarrow$ $C_y = 460\text{ kN}$, $B_y = 460\text{ kN}$

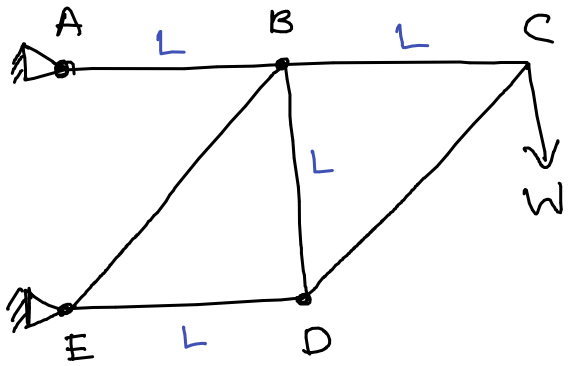
$A_y = 80\text{ kN}$

(b) To prevent slipping

$$f_s \geq \max\left\{\frac{B_x}{B_y}, \frac{C_x}{C_y}\right\} = \max\left\{\frac{55}{460}, \frac{95}{460}\right\} = \max\{0.12, 0.2\}$$

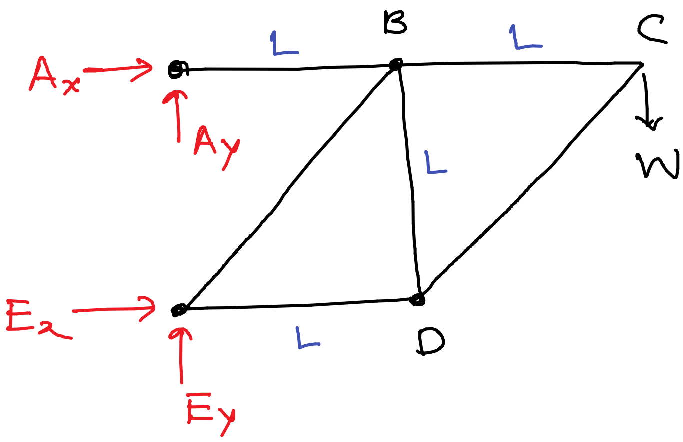
$$= 0.2$$

6)



A truss consists of members which resist axial forces such as tension or compression

FBD of entire truss



$\circlearrowleft + \sum M_E = 0$

$\Rightarrow -A_x(L) - W(2L) = 0$

$\Rightarrow A_x = -2W$

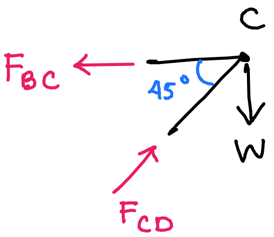
$+\uparrow \sum F_y = 0$

$\Rightarrow E_y + A_y - W = 0$

$\xrightarrow{+} \sum F_x = 0$

$\Rightarrow A_x + E_x = 0$

FBD of point C



$+\uparrow \sum F_y = 0$

$\Rightarrow F_{CD} \cos 45° - W = 0$

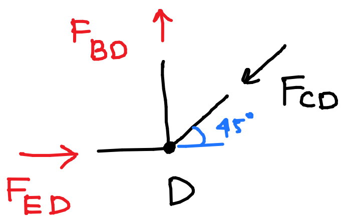
$\Rightarrow F_{CD} = \sqrt{2}\, W$

$\xrightarrow{+} \sum F_x = 0$

$\Rightarrow -F_{BC} + F_{CD} \sin 45° = 0$

$\Rightarrow F_{BC} = W$

FBD of point D



$+\uparrow \sum F_y = 0$

$\Rightarrow -F_{CD} \sin 45° + F_{BD} = 0$

$\Rightarrow F_{BD} = W$

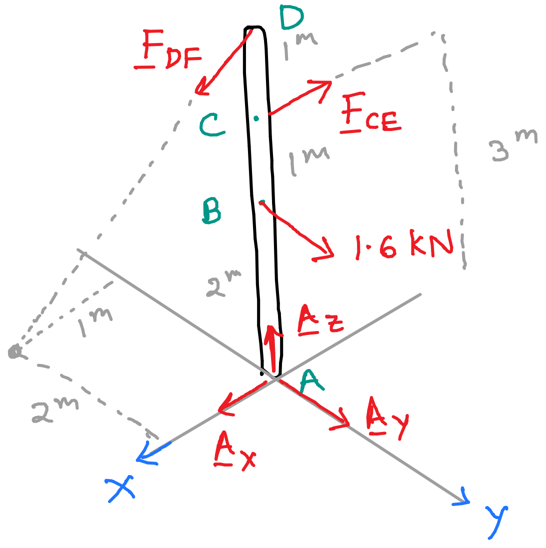
$\xrightarrow{+} \sum F_x = 0$

$\Rightarrow F_{ED} - F_{CD} \cos 45° = 0$

$\Rightarrow F_{ED} = W$

Proceed similarly!

7)



In this 3D case, it is better to use vector notation to denote the forces

- Unit vector along F_{DF}

$$= \frac{(\underline{i} - 2\underline{j} - 4\underline{k})}{\sqrt{1+2^2+4^2}}$$

$$\underline{F}_{DF} = F_{DF}\,(\underline{i} - 2\underline{j} - 4\underline{k})/\sqrt{21}$$

- Unit vector along F_{CE}

$= -\underline{i}$ $\Rightarrow$ $\underline{F}_{CE} = F_{CE}\,(-\underline{i})$

- Reaction force

$$\underline{A} = A_x\,\underline{i} + A_y\,\underline{j} + A_z\,\underline{k}$$

Using equilibrium of forces on the rod AD

$$\sum \underline{F} = \underline{0}$$

$$\Rightarrow \underline{F}_{DF} + \underline{F}_{CE} + \underline{A} = 0$$

Componentwise forces:

x-dir: $\dfrac{F_{DF}}{\sqrt{21}} - F_{CE} + A_x = 0$ — (1)

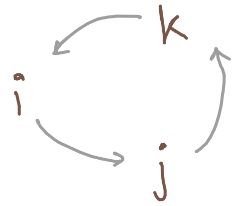
y-dir: $-\dfrac{2}{\sqrt{21}} F_{DF} + A_y - 1{\cdot}6\text{ kN} = 0$ — (2)

z-dir: $-\dfrac{4}{\sqrt{21}} F_{DF} + A_z = 0$ — (3)

Now, using moment equilibrium equations

Taking moment about pt A,

$$\Sigma \underline{M}_A = \underline{0}$$



$$\underline{M}_{DF} = \underline{r}_{D|A} \times \underline{F}_{DF} = (4\underline{k}) \times \frac{F_{DF}}{\sqrt{21}}(\underline{i} - 2\underline{j} - 4\underline{k})$$

$$= \frac{1}{\sqrt{21}}(4\underline{j} + 8\underline{i})\, F_{DF}$$

$$\underline{M}_{CE} = \underline{r}_{C|A} \times \underline{F}_{CE} = (3\underline{k}) \times F_{CE}(-\underline{i})$$

$$= -3F_{CE}\,\underline{j}$$

$$\underline{M}_{applied} = \underline{r}_{B|A} \times F_{applied} = (2k) \times (1.6\text{ kN }\underline{j})$$

$$= -3.2\,\underline{i} \text{ kNm}$$

Total sum of moments should be zero:

$$\underline{M}_{DF} + \underline{M}_{CE} + \underline{M}_{applied} = 0$$

Componentwise moments:

x - dir: $\frac{8}{\sqrt{21}} F_{DF} - 3.2\text{ kNm} = 0$ — (4)

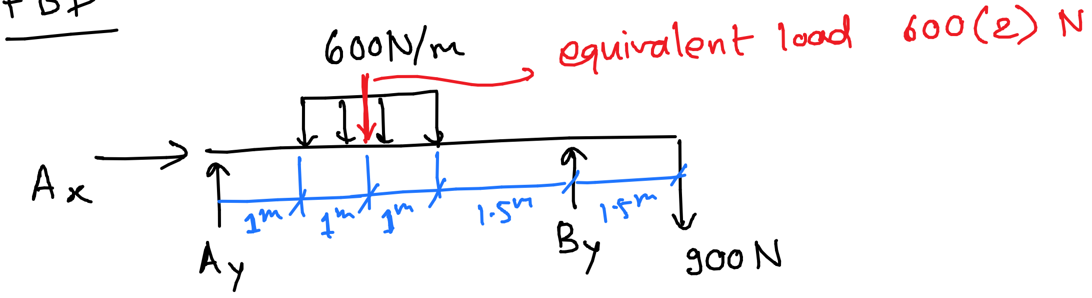
Y-dir: $\frac{4}{\sqrt{21}} F_{DF} - 3F_{CE} = 0$ — (5)

Solve (1), (2), (3), (4) & (5), get the values of

F_{DF}, F_{CE}, A_x, A_y, A_z

8) Smooth journal bearings → only prevents translation vertically

FBD



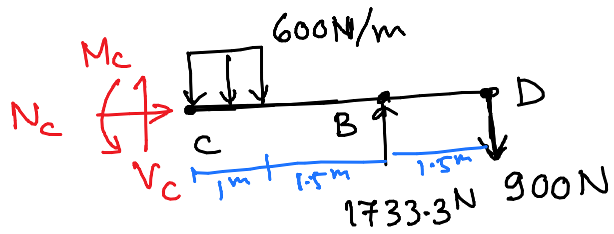
Support reactions

$\circlearrowleft + \sum M_A = 0$

$\Rightarrow \quad B_y(4.5) - (600(2))(2) - 900(6) = 0$

$\Rightarrow \quad B_y = 1733.3 \text{ N}$

Cutting a section and isolating only the RHS of C



$\overset{+}{\rightarrow} \sum F_x = 0 \quad \Rightarrow \quad N_c = 0$

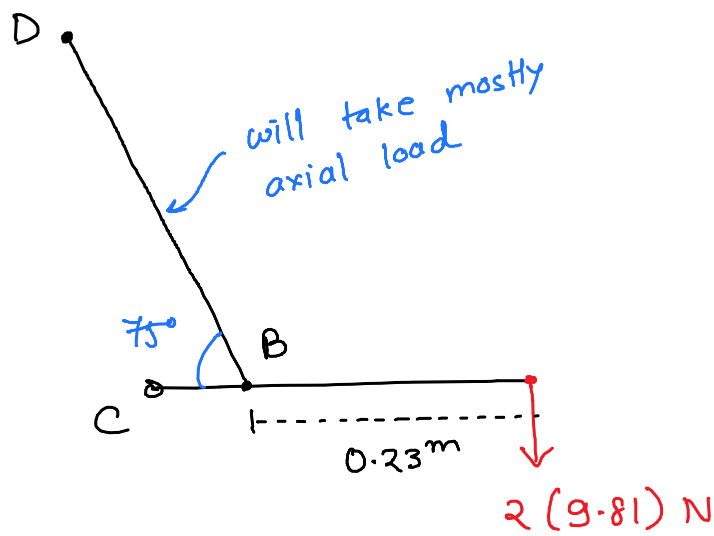
$+\uparrow \sum F_y = 0 \quad \Rightarrow \quad V_c - 600(1) + 1733.3 - 900 = 0$

$\Rightarrow \quad V_c = -233 \text{ N}$

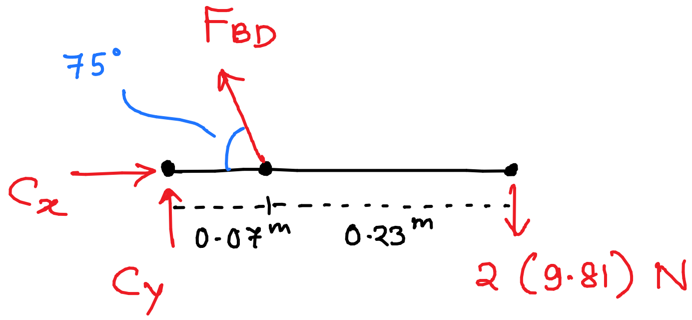
$\circlearrowleft + \sum M_c = 0 \quad \Rightarrow \quad M_c - (600(1))(0.5) + 1733.3(2.5) - 900(4) = 0$

$\Rightarrow \quad M_c = 433 \text{ N}$

9) Idealize the forearm and biceps as follows:



Draw FBD



Apply equilibrium equation,

$$\circlearrowleft + \sum M_c = 0 \quad \Rightarrow \quad F_{BD} \sin 75^\circ \, (0.07) - 2\,(9.81)\,(0.3) = 0$$

$$\Rightarrow \quad F_{BD} = 87.05 \text{ N}$$

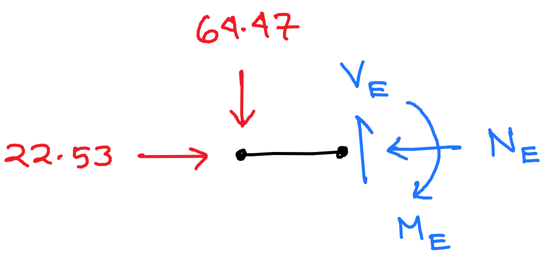
$$\xrightarrow{+} \sum F_x = 0 \quad \Rightarrow \quad C_x - F_{BD} \cos 75^\circ = 0$$

$$\Rightarrow \quad C_x = 22.53 \text{ N}$$

$$+\uparrow \sum F_y = 0 \quad \Rightarrow \quad C_y - 2\,(9.81) + 87.05 \cos 75^\circ = 0$$

$$\Rightarrow \quad C_y = -64.47 \text{ N}$$

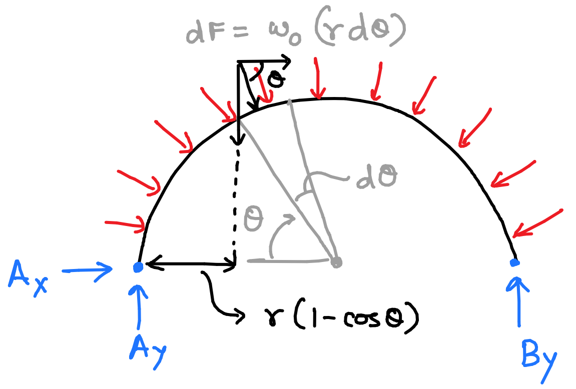
The internal resistive forces in the bone are:



$N_E = 22.53$ N

$V_E =$

10) FBD of entire structure



$\circlearrowleft + \sum M_A = 0$

$$\Rightarrow B_y(2r) - \int_0^{\pi} \left[(w_o\, r\, d\theta)\sin\theta\right] r(1-\cancel{\cos\theta})$$

$$- \cancel{\int_0^{\pi} \left[(w_o\, r\, d\theta)\cos\theta\right] r\sin\theta}$$

$$\Rightarrow B_y(2r) - w_o r^2 \int_0^{\pi} \sin\theta\, d\theta = 0$$

$$\Rightarrow B_y = \frac{w_o r^2}{2r}\left[-\cos\theta\right]_0^{\pi} = w_o r$$

(You can also find the resultant directly by looking at the symmetry and seeing that forces cancel out in x-dir)

$\overset{+}{\rightarrow} \sum F_x = 0$

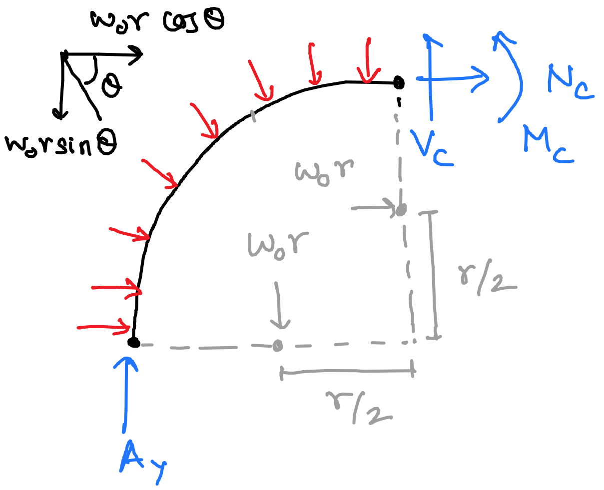
$\Rightarrow A_x = 0$

$+\uparrow \sum F_y = 0$

$\Rightarrow A_y + B_y - w_o(2r) = 0$

$\Rightarrow A_y = w_o r$

FBD of isolated part



$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N_c + \int_0^{\pi/2} w_o r \cos\theta\, d\theta = 0$$

$$\Rightarrow N_c = -w_o r\left[\sin\theta\right]_0^{\pi/2}$$

$$= -w_o r$$

$+\uparrow \sum F_y = 0$

$$\Rightarrow w_o r + V_c - w_o r \int_0^{\pi/2} \sin\theta\, d\theta = 0$$

$$\Rightarrow V_c = -w_o r + w_o r\left[-\cos\theta\right]_0^{\pi/2}$$

$$= -w_o r - 0 + w_o r$$

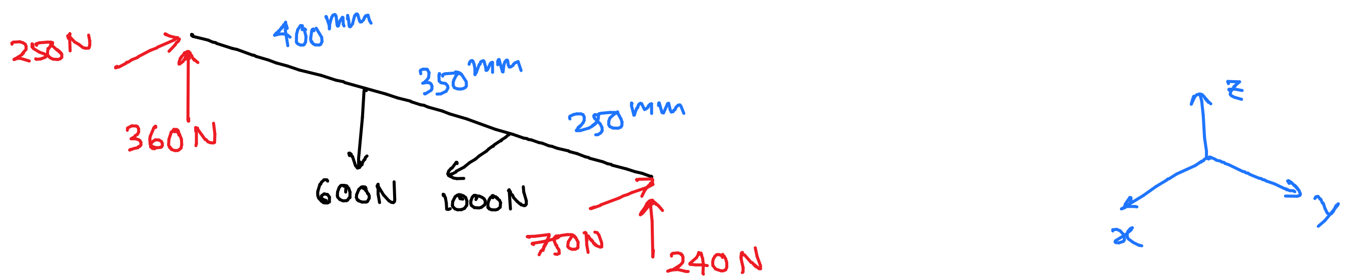
$$= 0$$

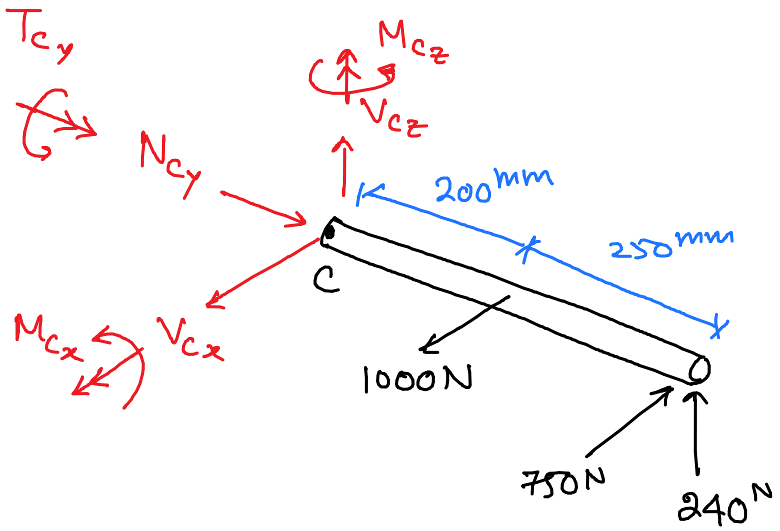
$\circlearrowleft + \sum M_c = 0$

$$\Rightarrow -A_y(r) + M_c + w_o r\left(\frac{r}{2}\right) + w_o r\left(\frac{r}{2}\right) = 0$$

$$\Rightarrow M_c = 0$$

11) Draw FBD





$\overset{+}{\swarrow} \Sigma F_x = 0$

$\Rightarrow V_{cx} + 1000 - 750 = 0$

$\Rightarrow V_{cx} = -250\ N$

$\overset{+}{\searrow} \Sigma F_y = 0$

$\Rightarrow N_{cy} = 0$

$+\uparrow \Sigma F_z = 0$

$\Rightarrow V_{cz} + 240 = 0$

$\Rightarrow V_{cz} = -240\ N$

$+ \Sigma M_x |_c = 0$

$\Rightarrow M_{cx} + 240\,(0.45) = 0$

$\Rightarrow M_{cx} = -108\ Nm$

$\Sigma M_y |_c = 0$

$\Rightarrow T_{cy} = 0$

$\Sigma M_z |_c = 0$

$\Rightarrow M_{cz} - 1000\,(0.2) + 750\,(0.45) = 0$

$\Rightarrow M_{cz} = -138\ Nm$