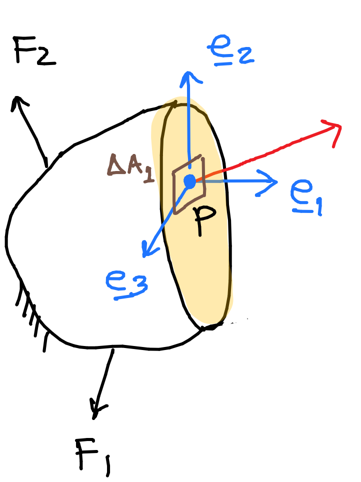
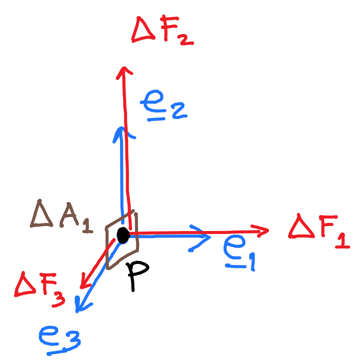
We have now got tractions on $\underline{e}_1, \underline{e}_2, \underline{e}_3$ planes $\rightarrow \underline{T}^1, \underline{T}^2, \underline{T}^3$
What do they depict?

Traction $\underline{T}^1$ acts on a plane with outward normal $\underline{e}_1$

If we cut the body with a $\underline{e}_1$-plane through pt P then we get a traction $\underline{T}^1$ whose components in the three directions of the coordinate system are given by T_1^1, T_2^1, and T_3^1



$\Rightarrow$



$\Rightarrow$

$$T_1^1 = \lim_{\Delta A_1 \to 0} \frac{\Delta F_1}{\Delta A_1}$$

$$T_2^1 = \lim_{\Delta A_1 \to 0} \frac{\Delta F_2}{\Delta A_1}$$

$$T_3^1 = \lim_{\Delta A_1 \to 0} \frac{\Delta F_3}{\Delta A_1}$$

NORMAL and SHEAR stresses

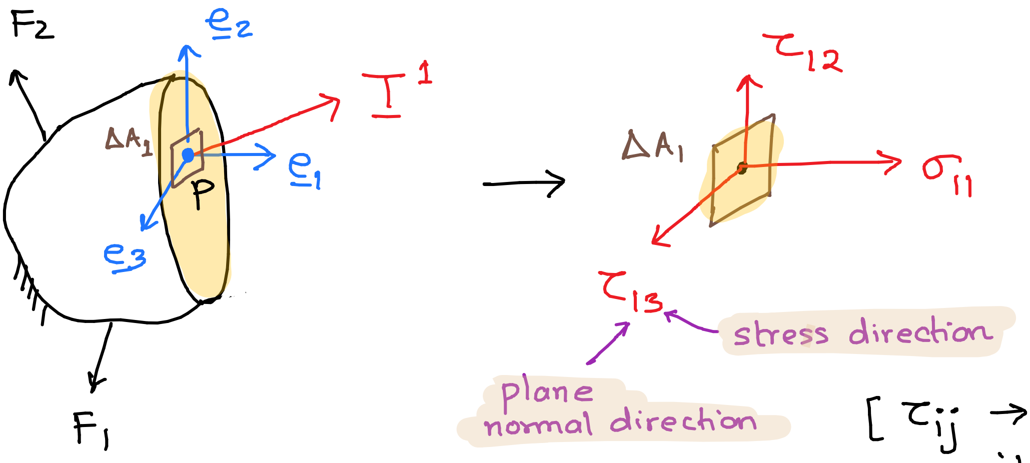
For defining these quantities, lets consider a plane with outward normal in the +ve $\underline{e}_1$-direction. With traction $\underline{T}^1$ acting on the $\underline{e}_1$-plane, we can use its components to define:

Normal stress, $\sigma_{11} = T_1^1$ } tendency to pull or push

Shear stress, $\tau_{12} = T_2^1$
Shear stress, $\tau_{13} = T_3^1$ } tendency to slide between two surfaces

Similarly, we can define $\underline{T}^2$ and $\underline{T}^3$

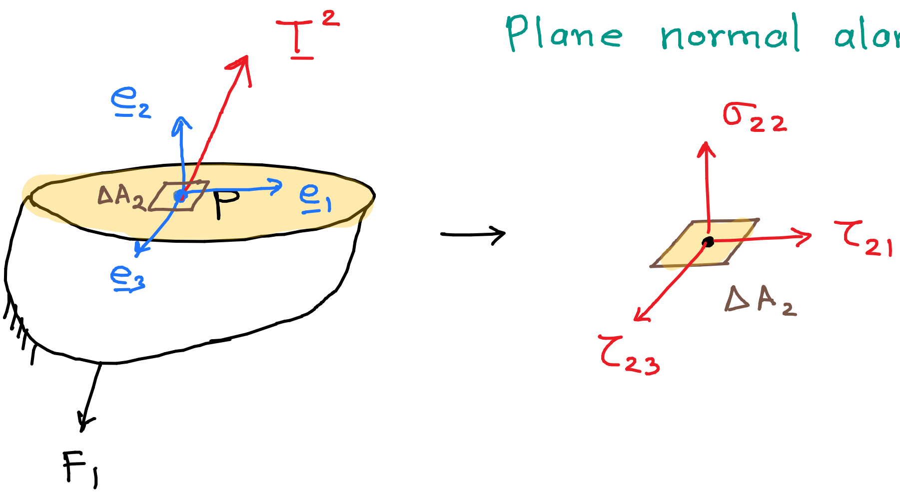
Plane normal along $\underline{e}_1$



$$\underline{T}^1 = \begin{bmatrix} T_1^1 \\ T_2^1 \\ T_3^1 \end{bmatrix} = \begin{bmatrix} \sigma_{11} \\ \tau_{12} \\ \tau_{13} \end{bmatrix}$$

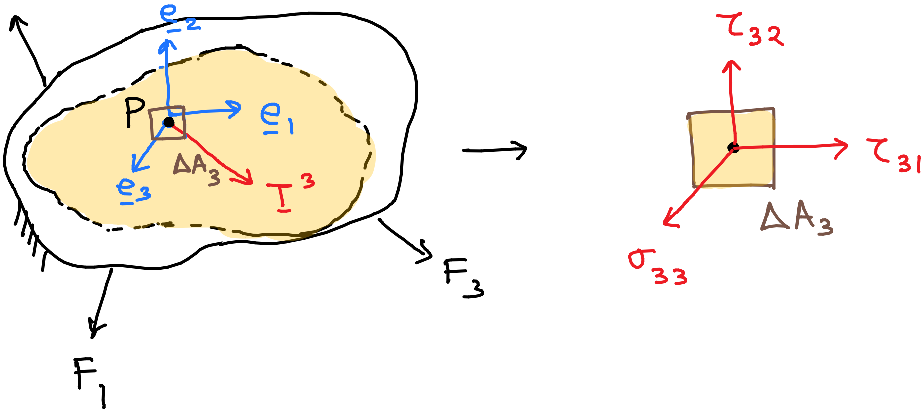
[τ_{ij} → represents the jth component of traction on the ith plane]

Plane normal along $\underline{e}_2$



$$\underline{T} = \begin{bmatrix} T_1^2 \\ T_2^2 \\ T_3^2 \end{bmatrix} = \begin{bmatrix} \tau_{21} \\ \sigma_{22} \\ \tau_{23} \end{bmatrix}$$

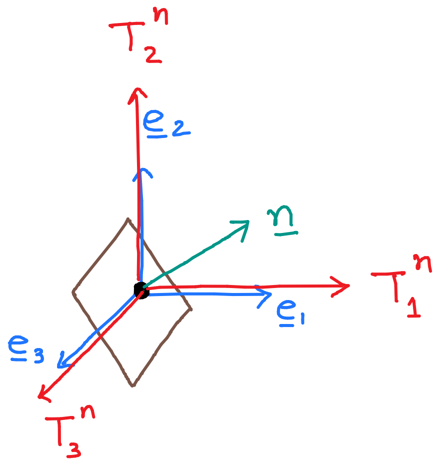
Plane normal along $\underline{e}_3$



$$\underline{T}^3 = \begin{bmatrix} T_1^3 \\ T_2^3 \\ T_3^3 \end{bmatrix} = \begin{bmatrix} \tau_{31} \\ \tau_{32} \\ \sigma_{33} \end{bmatrix}$$

$$\underline{T}^{n} = \underline{T}^{1} n_1 + \underline{T}^{2} n_2 + \underline{T}^{3} n_3$$

$$\begin{bmatrix} T_1^n \\ T_2^n \\ T_3^n \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_{11} \\ \tau_{12} \\ \tau_{21} \end{bmatrix}}_{\text{Traction on plane with normal } \underline{e}_1} n_1 + \begin{bmatrix} \tau_{21} \\ \sigma_{22} \\ \tau_{23} \end{bmatrix} n_2 + \underbrace{\begin{bmatrix} \tau_{31} \\ \tau_{32} \\ \sigma_{33} \end{bmatrix}}_{\text{Traction on plane with normal } \underline{e}_3} n_3$$



Componentwise

$$T_1^n = \sigma_{11} n_1 + \tau_{21} n_2 + \tau_{31} n_3$$

$$T_2^n = \tau_{12} n_1 + \sigma_{22} n_2 + \tau_{32} n_3$$

$$T_3^n = \tau_{13} n_1 + \tau_{23} n_2 + \sigma_{33} n_3$$

Representing in the form of a matrix:

$$\begin{bmatrix} T_1^n \\ T_2^n \\ T_3^n \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_{11} & \tau_{21} & \tau_{31} \\ \tau_{12} & \sigma_{22} & \tau_{32} \\ \tau_{13} & \tau_{23} & \sigma_{33} \end{bmatrix}}_{[\underline{\underline{\sigma}}]} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

Stress tensor

$$\boxed{\underline{T}^{n} = \underline{\underline{\sigma}}(\underline{x})\, \underline{n}}$$

* NOTE: The stress tensor is independent of the plane normal $\underline{n}$

State of stress at a point

An infinite number of traction vectors act at a given point. The totality of all traction vectors acting on every possible plane through a point is defined to be the STATE of STRESS at the point.

Since traction along any plane can be obtained from info of tractions on mutually perpendicular planes, therefore it is enough to know tractions on three mutually perpendicular planes to define the totality of all tractions acting at a pt.

The tractions on three mutually perpendicular planes can be further resolved into normal and tangential directions, which lead to the normal and shear stresses on each plane.

Thus the state of stress at a point is completely defined by the nine stress components acting on three mutually perpendicular planes (say $\underset{\sim}{e}_1$, $\underset{\sim}{e}_2$, $\underset{\sim}{e}_3$ planes)

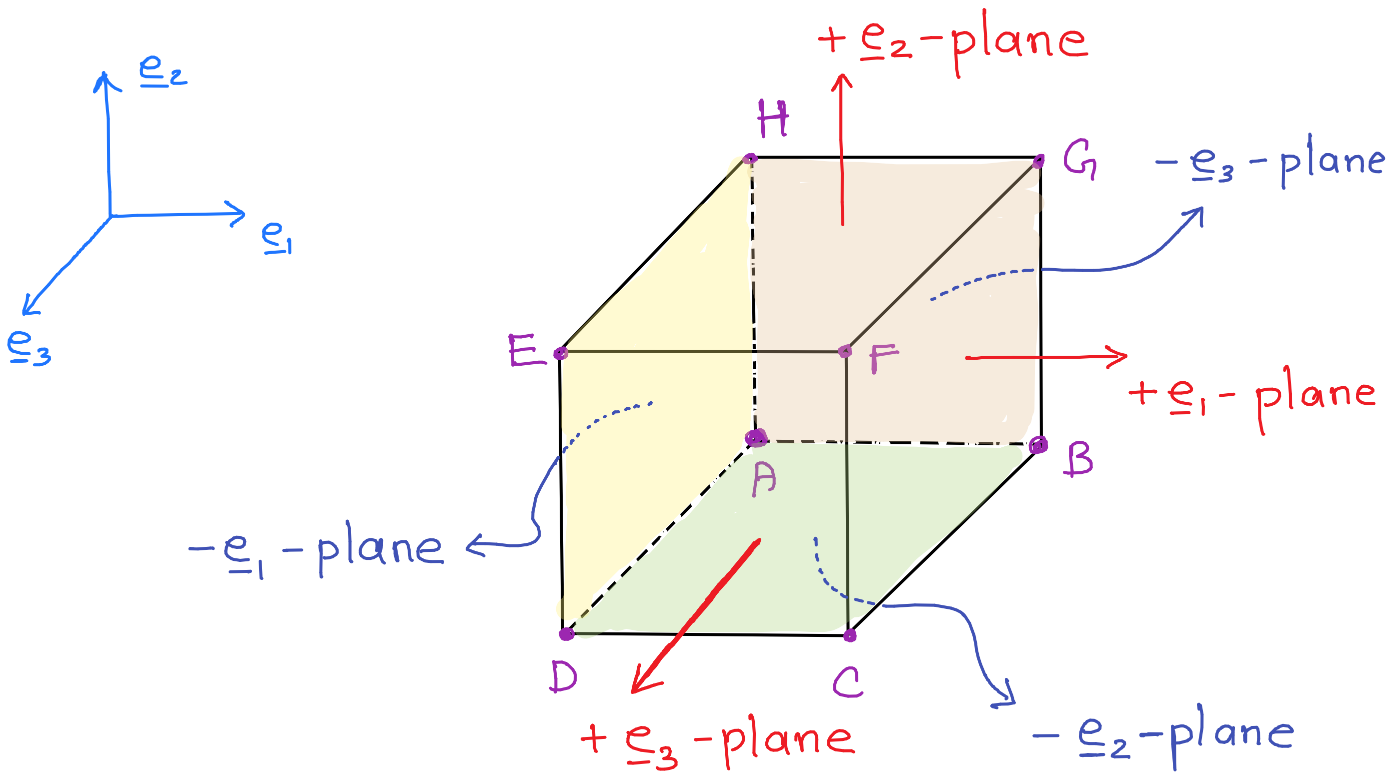
$$[\underset{\sim}{\sigma}]_{(e_1-e_2-e_3)} = \begin{array}{c} \begin{bmatrix} \sigma_{11} & \tau_{21} & \tau_{31} \\ \tau_{12} & \sigma_{22} & \tau_{32} \\ \tau_{13} & \tau_{23} & \sigma_{33} \end{bmatrix}_{(e_1-e_2-e_3)} \\ \begin{matrix} \underset{\sim}{T}^1 & \underset{\sim}{T}^2 & \underset{\sim}{T}^3 \end{matrix} \end{array}$$

$\underset{\sim}{\sigma}$ → STRESS TENSOR

$[\underset{\sim}{\sigma}]_{(e_1-e_2-e_3)}$ → Representation of stress tensor in $(e_1-e_2-e_3)$ Coor. Sys.

STRESS tensor depends only on the point $\underset{\sim}{x}$

Sign convention for stress components



- A face is +ve if the outward normal vector points in the direction of the +ve coordinate axis
- A face is -ve if the outward normal vector points in the direction of the -ve coordinate axis
- The stress component is positive when a positively directed force component acts on a positive face
- The stress component is positive when a negatively directed force component acts on a negative face
- When a positively directed force component acts on a negative face or a negatively directed force component acts on a positive face, the stress component is negative