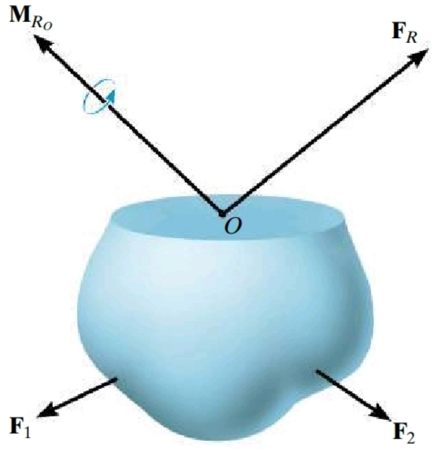
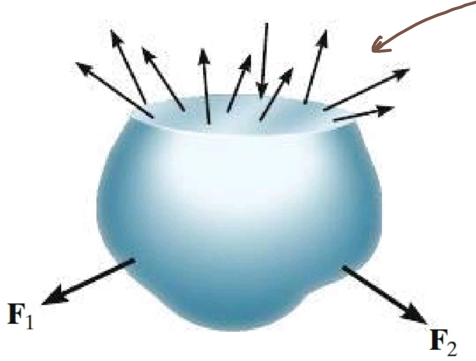
Traction vector



Earlier we assumed a resultant force and a moment at the exposed area of the cut body. Now we look at the distribution of internal forces at the cut face of the body



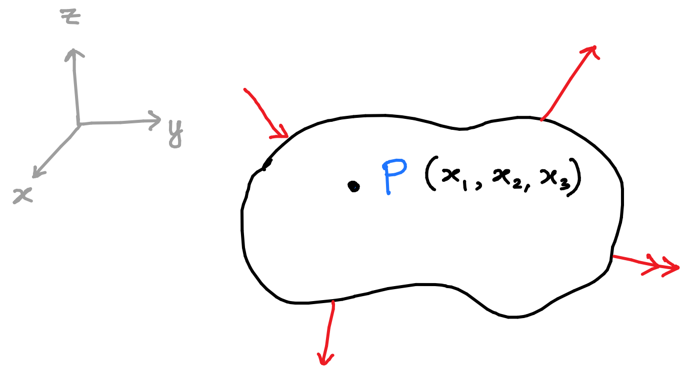
this distribution of internal forces is usually not uniform and varies from point to point

Obtaining the distribution of internal resistance in a deformable body is very important in solid mechanics

To get this distribution, it is necessary to establish the concept of TRACTION & STRESS. Both these quantities vary with the location of point in the body. Therefore, if one asks to find out the traction or stress in a body, you must ask for the exact coordinate of the point in the body

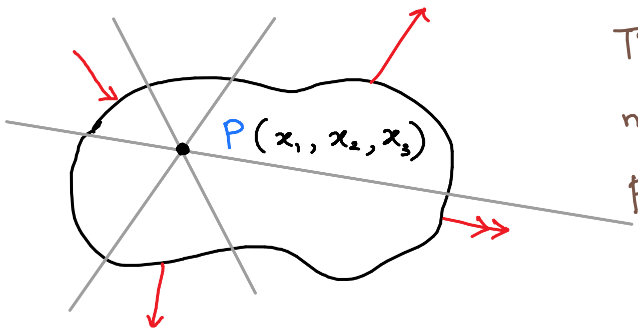
Lets consider a 3D body

a point P in the body



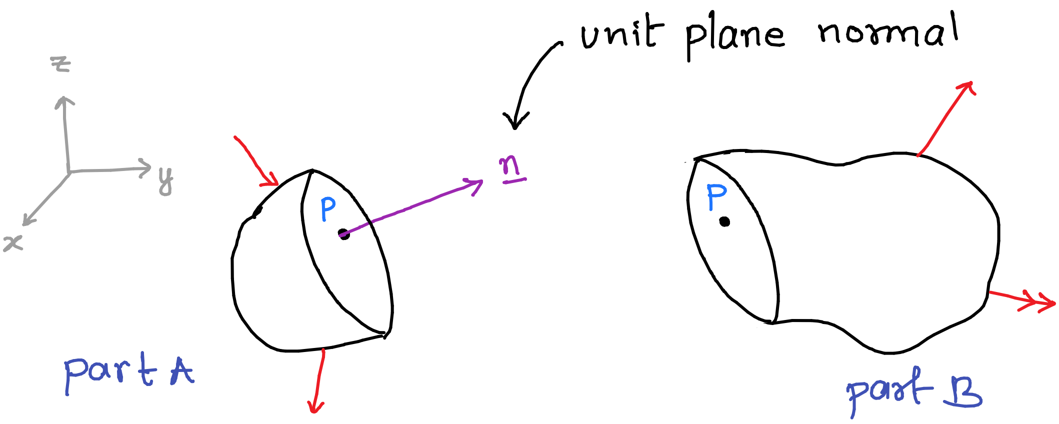
Coordinates of P $\rightarrow \underline{x} = \{x_1, x_2, x_3\}$

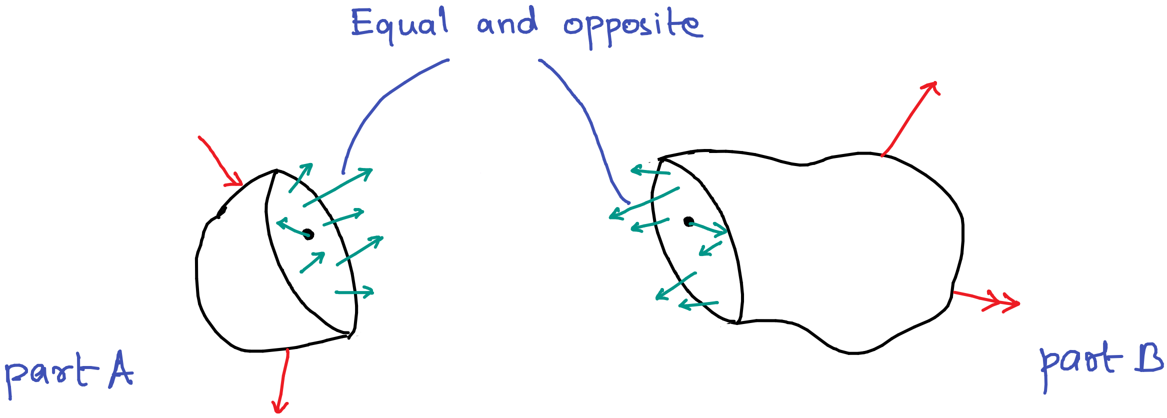
To examine the internal forces at point P in the interior of the body, we cut the body into two halves by passing a plane through pt P.



There are infinitely many planes that pass through pt P

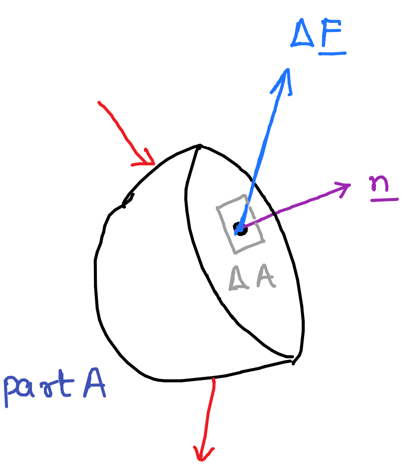
We consider a plane with normal vector $\underline{n}$ passing through point P and cutting the body into two halves





After cutting the body, if you look at the part A of the body, it is brought under equilibrium by a distribution of internal forces. The distribution of internal forces on part B must be equal & opposite according to Newton's 3rd law

Traction vector is defined as the intensity of the force or force per unit area with which part B is pulling or pushing part A



Lets concentrate on the pt P and draw a small area ΔA around it

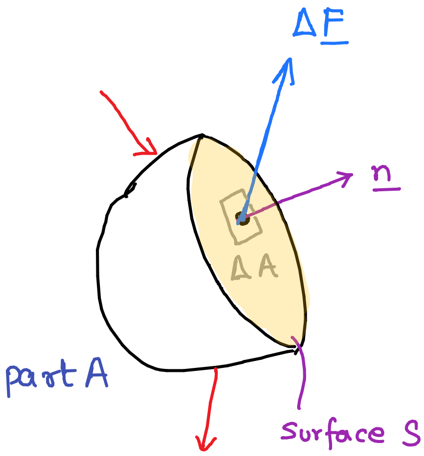
There will be a small force $\Delta \underline{F}$

Traction vector (also called stress vector) acting at point P on a plane with unit normal $\underline{n}$ is defined

plane normal

$$\boxed{\underline{T}^{n}(\underline{x}) = \lim_{\Delta A \to 0} \frac{\Delta \underline{F}}{\Delta A}}$$

location in body



Since traction is defined as force per unit area, total surface force can be obtained from integration

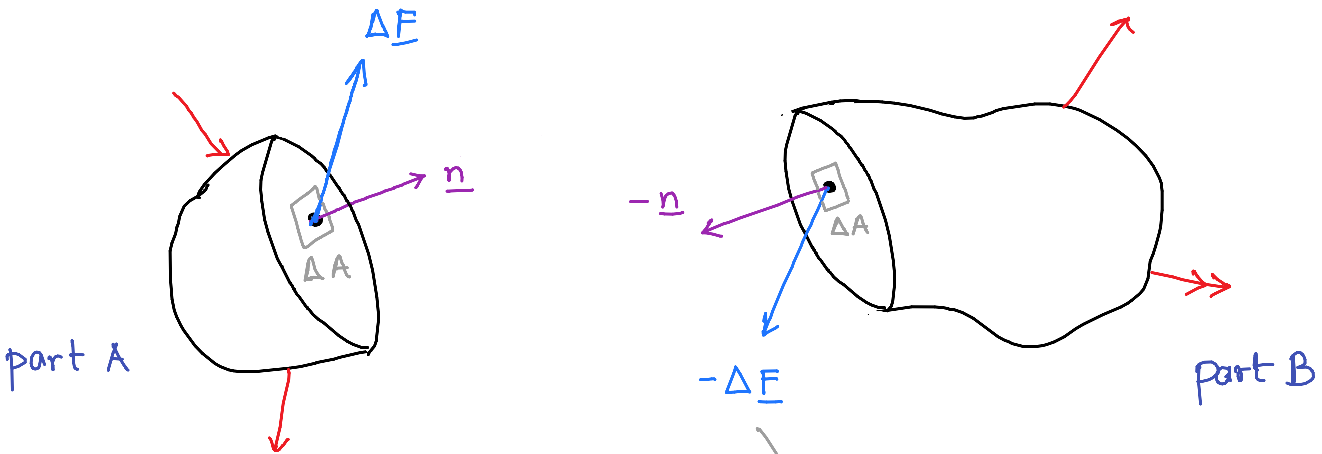
$$\underline{F}_R = \iint_S \underline{T}^n(\underline{x}) \, dS$$

resultant force

moment arm of pt P from O

$$\underline{M}_{R_O} = \iint_S \underline{r}_{P|O} \times \underline{T}^n(\underline{x}) \, dS$$

Furthermore, we note that the surface normal pointing out from part B has direction opposite to unit normal $\underline{n}$ and hence is written as $-\underline{n}$. Similarly, equal and opposite force must act on point P for part B $\rightarrow -\Delta \underline{F}$



$$\underline{T}^{-n}(\underline{x}) = \lim_{\Delta A \to 0} \frac{-\Delta \underline{F}}{\Delta A}$$

We find that the tractions at the same point in two opposite directions are related as:

$$\boxed{\underline{T}^n(\underline{x}) = -\underline{T}^{-n}(\underline{x})}$$

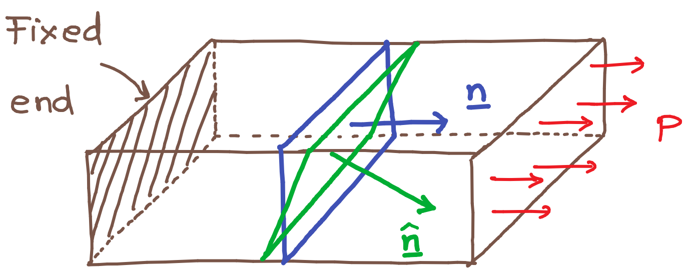
Remarks

- Traction vector has same units as pressure (e.g. N/m^2)

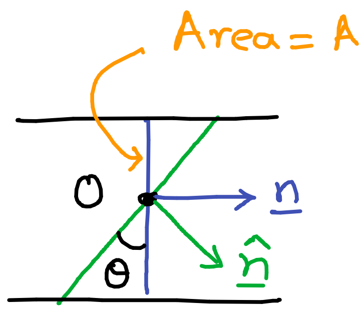
- Traction vector changes from point to point in the body

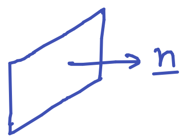
- Traction vector depends upon the plane orientation

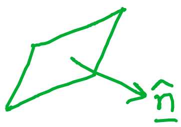
Ex.



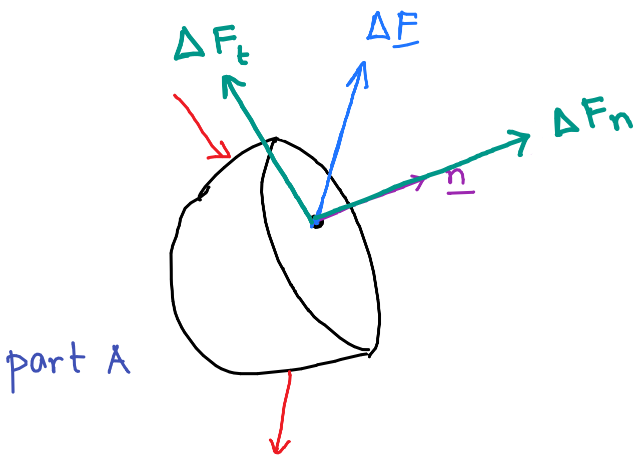
Uniform load P applied to a bar fixed at one end



Traction on  $= \dfrac{P}{A}$

Traction on  $= \dfrac{P}{\left(A/\cos\theta\right)}$

- Traction vector need not be just perpendicular or tangential to the surface, it can have both components



Why is traction important?

- If traction is greater than a threshold limit
  $\Rightarrow$ the body will fail

- Probability of failure is more along a plane on which there is more traction

Stress at a point

Let us consider a Cartesian coordinate system