We have talked about force

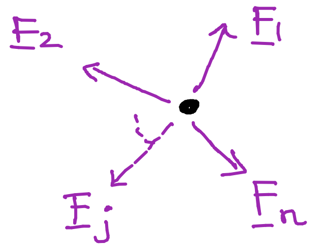
- how to calculate force,
- how to quantify force
- types of forces
- moments of forces

Now before going onto study deformation, we will look at equilibrium conditions

Equilibrium conditions

According to Newton's law, a particle having no resultant force acting on it will remain in equilibrium

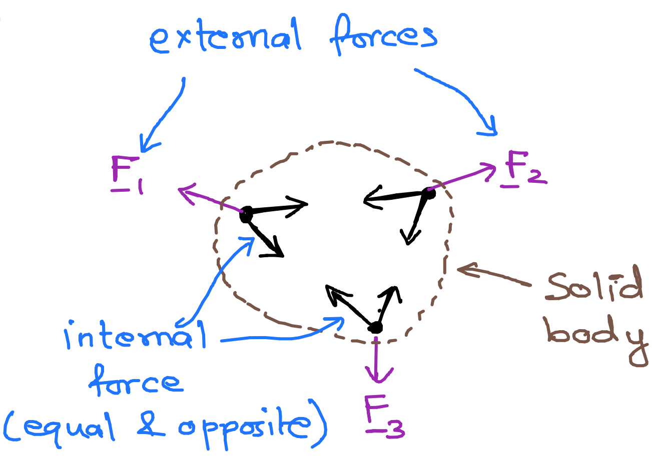
Necessary and sufficient condition for eq^m of a particle:



Resultant force = 0

$$\boxed{\underline{F}_R = \sum_{j=1}^{n} \underline{F}_j = \underline{0}}$$

- A solid body is composed of several particles
- A solid body is in equilibrium if all the particles comprising the body are in equilibrium
  - the internal resistive forces arise due to the application of external forces
  - if each particle is in eq^m, then the resultant force = 0
  - the sum of all forces together must also be zero
  - internal forces occur as self-cancelling pairs



Vector sum of all external forces must be zero

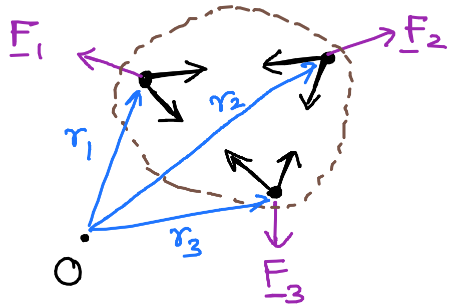
So what are the equations of equilibrium in this case?

- Resultant force at any point must be zero

$$\Rightarrow \boxed{\underline{F}_R = \sum_{j=1}^{n} \underline{F}_j = 0} \quad — \text{(A)}$$

- Resultant moment of ALL the forces about an arbitrary point must be zero

$$\Rightarrow \boxed{\underline{M}_O = \sum_{j=1}^{n} \underline{r}_j \times \underline{F}_j = \underline{0}}$$



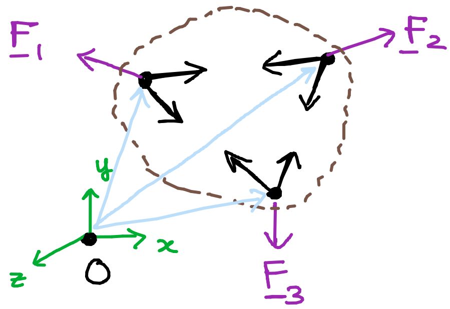
If there are couples C_j applied to the system, then

$$\boxed{\underline{M}_O = \sum_{j=1}^{n} \underline{r}_j \times \underline{F}_j + \sum_{j=1}^{n} \underline{C}_j = \underline{0}} \quad — \text{(B)}$$

If an (x, y, z) coordinate system is established with origin at O, we can resolve the force and moment vectors into components along each coordinate axis $\Rightarrow$ six scalar equilibrium equations

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$$

$$\sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$$



For cases where forces lie in the x-y plane (Coplanar forces)

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_z = 0$$

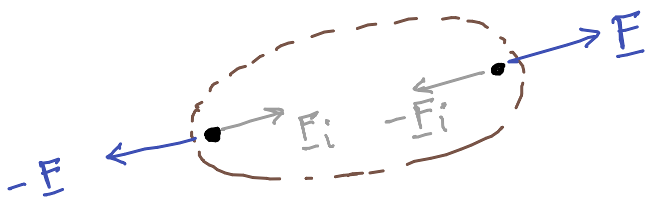
Eqns (A) and (B) are NECESSARY conditions for equilibrium

$\Rightarrow$ If the body is in equilibrium, then (A) & (B) must be satisfied

Usually if the system is at rest, we will know the system is in equilibrium and we will use (A) & (B) to obtain information about unknown forces (e.g. reactions)

However, if the external forces acting on a system of particles satisfy both (A) and (B), can you say that every constituting particle is in equilibrium? $\Longrightarrow$ NO for a deformable body

Ex Consider stretching of a rubber band



The ends of the rubber band accelerate away from each other

- The resultant of all external forces $\underline{F}$ and $-\underline{F}$ is zero
- The internal forces occur in self-cancelling pair
- But the particles will be in eq^m only when $\underline{F} = \underline{F}_i$

Necessary and sufficient conditions for equilibrium

Perfectly rigid body	Deformable body
Sum of all EXTERNAL forces and REACTIONS $= 0$ Sum of all EXTERNAL moments and REACTION moments about any point + EXTERNAL couples $= 0$	External and internal forces on every possible subsystem isolated out of the original system should satisfy (A) & (B)

In our analysis of mechanical systems, we will first treat engineering structural members to be relatively rigid and in equilibrium such that Eqns (A) and (B) are valid.

Later, we will derive equations of equilibrium for infinitesimal subsystems of deformable bodies, which will result in differential equations

The two vector equations (A) and (B) are equivalent to SIX scalar equations, so we can solve for SIX scalar unknowns in each set of external forces.

$$\sum F_{R_x} = 0 \qquad \sum M_{R_x} = 0$$

$$\sum F_{R_y} = 0 \qquad \sum M_{R_y} = 0$$

$$\sum F_{R_z} = 0 \qquad \sum M_{R_z} = 0$$

Coplanar Case

These conditions of equilibrium must be satisfied by the forces acting on an isolated system in equilibrium

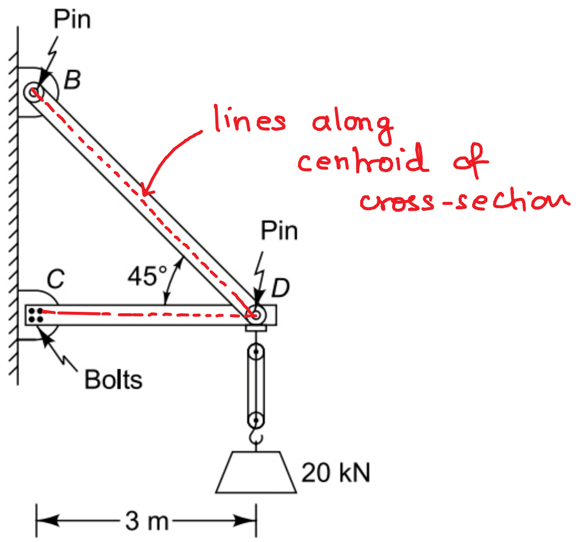
To show all forces & moments acting on an isolated subsystem

$\Rightarrow$ Draw the system's free-body diagram (FBD)

Despite drawing FBD, there are some systems for which the forces / moment reactions cannot be determined from equilibrium eqns alone $\Longrightarrow$ Such systems are called STATICALLY INDETERMINATE systems

Example 1

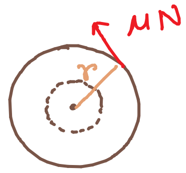
- We want to analyze this triangular frame
  - Study the forces and eq^m conditions
    - → Find out member forces
    - → Get support reactions



Simplifications / Idealizations

- Friction at the pin has been neglected, which could have resisted some moment at the pin support

$$M_{friction} = (\mu N)\, r$$



- The bolted joint C is assumed to have perfectly fitted bolts which can resist both translation and rotation

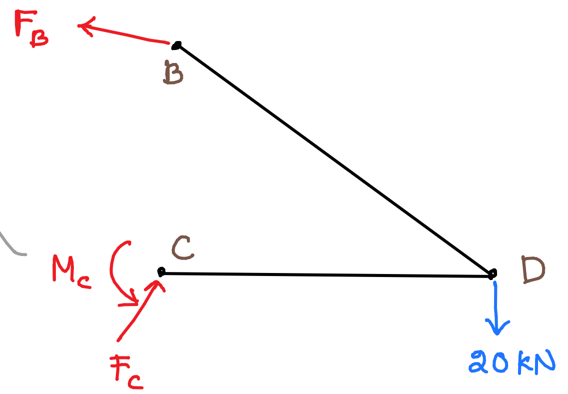
  ⇒ Acting like FIXED support at C

  If the bolts are loosely fitted, the moment resisted would be much less

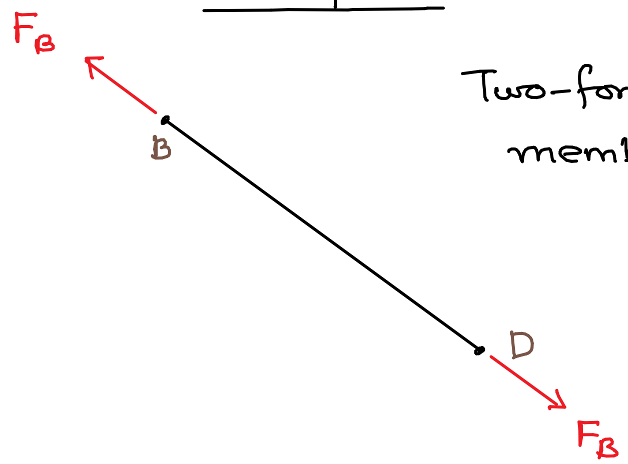
- We could neglect the weight of the frame

The directions of the reactions can just be assumed

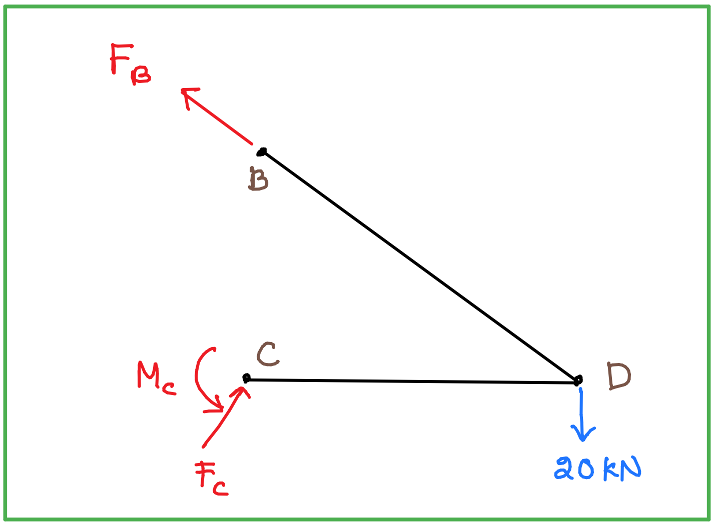
FBD of entire structure



FBD of BD



Two-force member



– Unknowns

F_B (magnitude)

F_c (magnitude & direction)

M_c (magnitude)

4 unknowns

– Knowns

Coplanar force system

$\Sigma F_{R_x} = 0, \quad \Sigma F_{R_y} = 0, \quad \Sigma M_R = 0$

3 equations

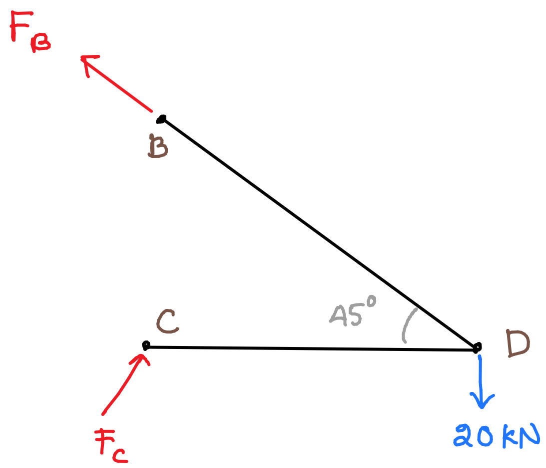
$\Downarrow$

Statically Indeterminate

(Cannot solve using just eq^m eqns)

If we approximate the bolted joint by a pin support then the system becomes statically determinate

FBD of entire structure



Apply eq^m equations:

$$\circlearrowleft + \sum M_B = 0$$

$$\Rightarrow \quad F_{Cx}(3) - 20(3) = 0$$

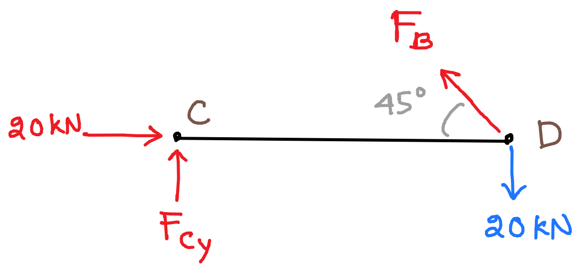
$$\Rightarrow \quad F_{Cx} = 20 \text{ kN}$$

$$\xrightarrow{+} \sum F_x = 0$$

$$\Rightarrow \quad F_{Cx} - F_B \cos 45^\circ = 0$$

$$\Rightarrow \quad F_B = 28.28 \text{ kN}$$

FBD of member CD

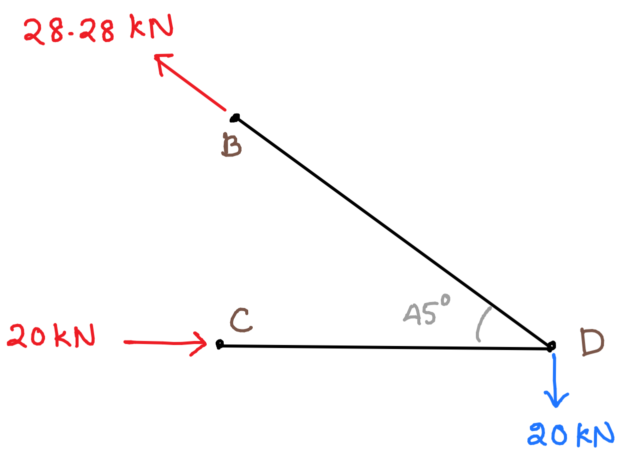


Apply eq^m eqns:

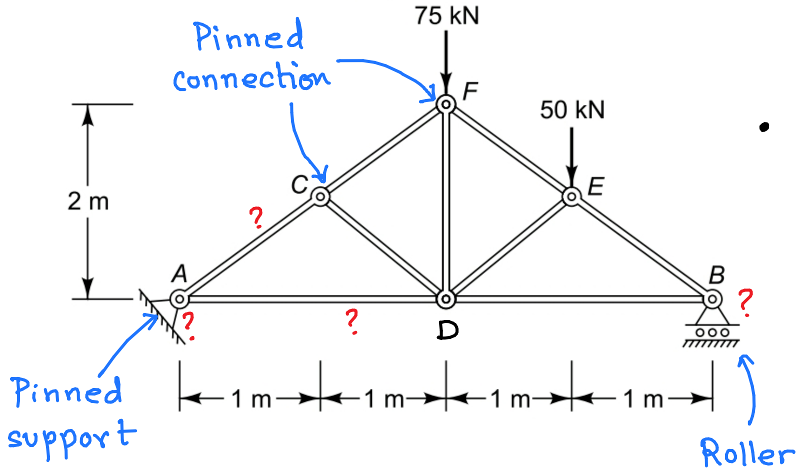
$$\circlearrowleft + \sum M_D = 0$$

$$\Rightarrow \quad F_{Cy}(3) = 0$$

$$\Rightarrow \quad F_{Cy} = 0$$



Example 2 :



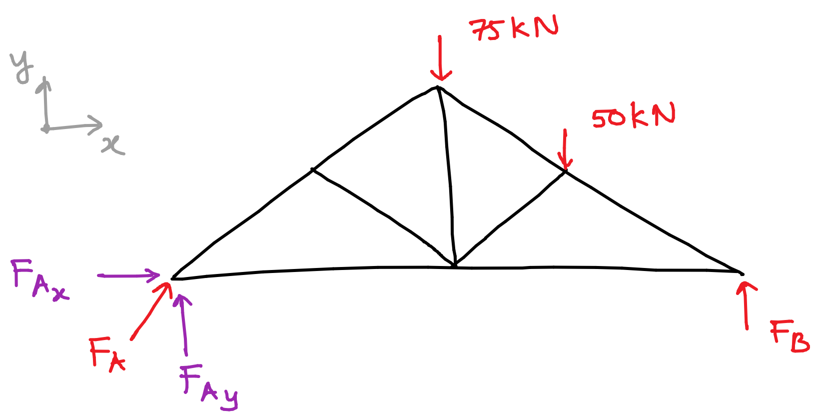
- Determine reactions at A and B
- Determine member forces AC and AD

The pinned truss is shown in equilibrium condition post application of the load

All joints are pinned connections

Solution :

Draw FBD of the entire truss



Replace the supports by appropriate forces

Apply eq^m equations:

$\circlearrowleft + \sum M_A = 0$

$\Rightarrow F_B(4) - 75(2) - 50(3) = 0$

$\Rightarrow F_B = 75 \text{ kN}$

$+\uparrow \sum F_y = 0$

$\Rightarrow F_{Ay} + F_B - 50 - 75 = 0$

$\Rightarrow F_{Ay} = 50 \text{ kN}$

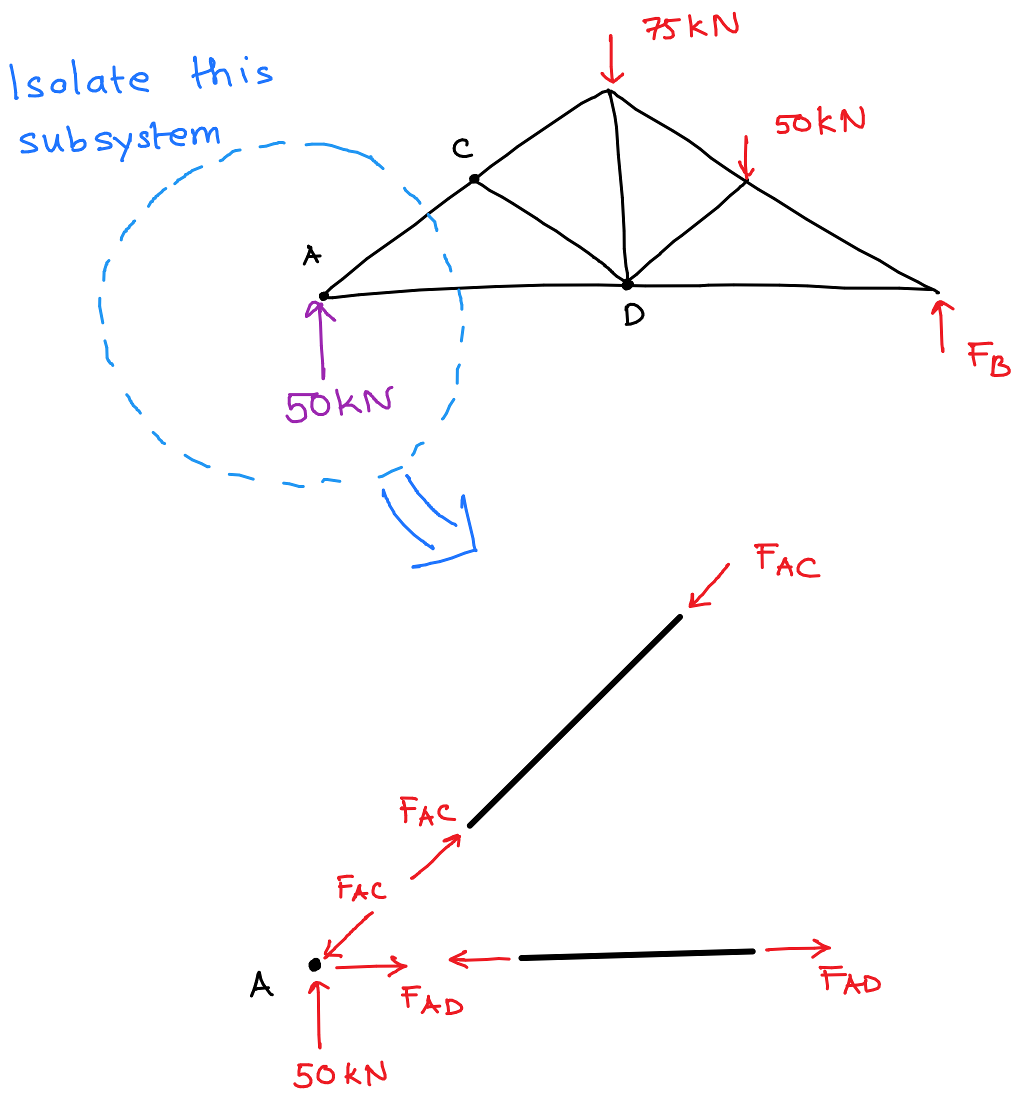
$\xrightarrow{+} \sum F_x = 0$

$\Rightarrow F_{Ax} = 0$

$$\boxed{\begin{aligned} F_B &= 75 \text{ kN} \\ F_A &= \sqrt{50^2 + 0^2} = 50 \text{ kN} \end{aligned}}$$

Note that the solution for the reactions did not use the design of the truss. However, for finding the member forces, you would need the design of the truss

Next, find member forces AC and AD. For that, draw FBDs of members AC and AD, and also draw FBD of joint A



Apply eq^m equation at joint A

This angle is obtained from geometry of the truss

$$\uparrow + \sum F_y = 0 \quad \Rightarrow \quad -F_{AC} \sin 45° + 50 = 0$$

$$\Rightarrow \quad F_{AC} = 70.71 \text{ kN}$$

$$\xrightarrow{+} \sum F_x = 0 \quad \Rightarrow \quad F_{AD} - F_{AC} \cos 45° = 0$$

$$\Rightarrow \quad F_{AD} = 50 \text{ kN}$$