Tutorial 9

APL 104 - 2022 (Solid Mechanics)

1. Suppose a solid disk of radius 'R' and a hollow disk of inner radius R^1 and outer radius R^2 are shrunk fit together (assume R > R^1). Further assume that no external pressure is applied on the outer hollow disk and that no axial displacement is allowed in the two disks. Let the two disks be made of same material. Only radial displacement u_r generates in this case.

   (a) Write down all the boundary conditions/ interface conditions required to obtain variation in radial stress σ_{rr} in the two disks.

   (b) Solve the governing equations and obtain expression for σ_{rr} in the two disks.

   (c) Obtain expression for circumferential/hoop stress too.

   (d) Draw plots for variation of both radial and circumferential stress.

2. Think of an isotropic solid cylinder which is glued to a rigid plate at both its ends. The two rigid plates are then pulled apart along the axis of the cylinder such the normals of the rigid plates are aligned with the axis of the cylinder. It turns out that u_θ is zero in this case but u_r and u_z do arise. Furthermore, (u_r, u_z) are not functions of θ coordinate. Assume that the deformation of the cylinder is such that every planar cross-section of the cylinder (z-plane or axial planes) remains planar even after deformation but it does change its radius.

   (a) What can you say about the dependence of u_r and u_z on radial and axial coordinates (r, z).

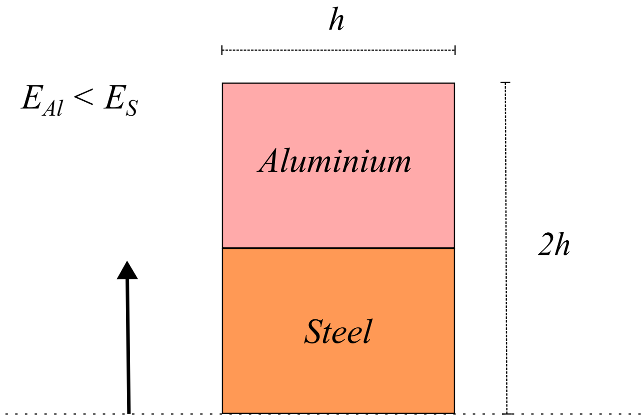
   (b) Obtain the strain matrix and stress matrix for the above problem.

   (c) Show that the θ-component of stress equilibrium equation is automatically satisfied.

   (d) What boundary condition will be used in order to solve the above deformation problem?

3. In class, we worked out distribution of the radial stress, the hoop stress as well as the axial stress for thick tubes. Work out their thin tube approximations, which is useful for pressure vessels. Can you obtain them directly from free body diagram of various sections of pressure vessels and doing their force balance?

4. Think of a composite beam having rectangular cross-section such that one half of the cross-section (having square shape) is aluminium while the other half is steel. When such a beam is bent, where will the neutral axis lie in the cross-section (calculated from the bottom line of cross-section)?

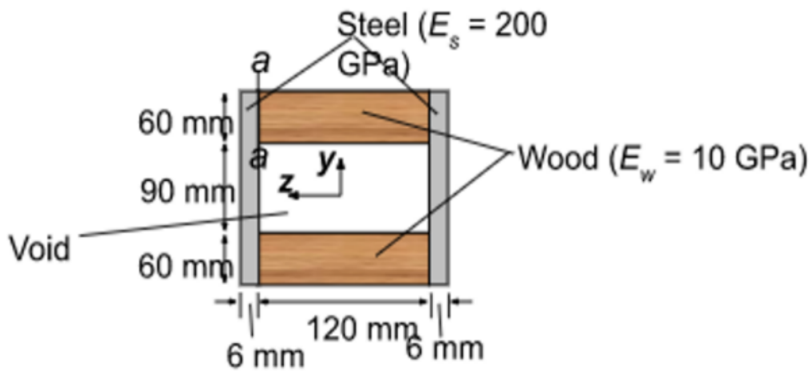


5. A beam of composite cross-section is subjected to bending moment $M_z = 30$kN. Find:

   (a) The curvature induced in the beam

   (b) Maximum bending stress in wood

   (c) Maximum bending stress in steel



6. Think of a composite circular beam formed by gluing together semi circular aluminium and steel beams such that top half is aluminium while the bottom half is steel. Assume the Young's moduli of the Aluminium and steel to be E_a and E_s respectively. Suppose the composite beam is bent into an arc of a circle in the vertical plane such that the radius of its neutral line is R

   (a) How would the normal strain ϵ_{xx} and normal stress ϵ_{xx} vary in the tube's cross-section? Draw two separate graphs for their variations - only qualitatively.

   (b) Obtain the location of the neutral axis in the beam.

   (c) Obtain expression for the bending moment required to bend the beam.

(d) Suppose it is subjected to bending moment such that it bends in horizontal pane. Obtain the expression for variation in bending stress σ_{xx} within cross-section?

7. A flat steel bar, 2.5cm wide by 0.6cm thick and 1m long, is bent by couples applied at the ends so that the midpoint deflection is 2.5ıcm. Compute the stress in the bar and the magnitude of the couples. Use $E = 200$GPa.

8. In a laboratory test of a beam loaded by end couples, the fibers at layer AB in Fig are found to increase 60×10^{-3}mm whereas those at CD decrease $100 \times 10^{-3} mm$ in the 200mm-gauge length. Using $E = 70GPa$, determine the flexural stress in the top and bottom fibers.