Tutorial 4: Stress equilibrium equations and Principal Stresses

APL 104 - 2022 (Solid Mechanics)

1. The sectional view of a dam is shown in Fig.1.

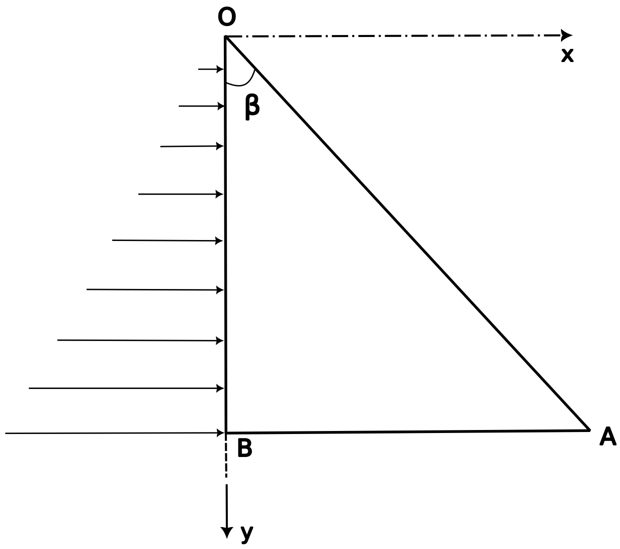

Figure 1

The pressure of water on vertical face (denoted by line OB) is also shown. With the axes Ox and Oy, as shown in Fig.1, the stress components at any point (x, y) are given by (γ = specific weight of water and ρ = specific weight of dam material)

$$\sigma_{xx} = -\gamma \; y$$

$$\sigma_{yy} = \left(\frac{\rho}{\tan\beta} - \frac{2\gamma}{\tan^3\beta}\right)x + \left(\frac{\gamma}{\tan^2\beta} - \rho\right)y$$

$$\tau_{xy} = \tau_{yx} = -\frac{\gamma}{\tan^2\beta}x$$

$$\tau_{yz} = 0, \; \tau_{zx} = 0, \; \sigma_{zz} = 0$$

Check if these stress components satisfy the differential equations of equilibrium. Also, verify if the boundary conditions are satisfied on the vertical face OB.

2. Consider the rectangular beam shown in Fig.2. According to the elementary theory of bending, the 'fiber stress' within the elastic range due to bending is given by

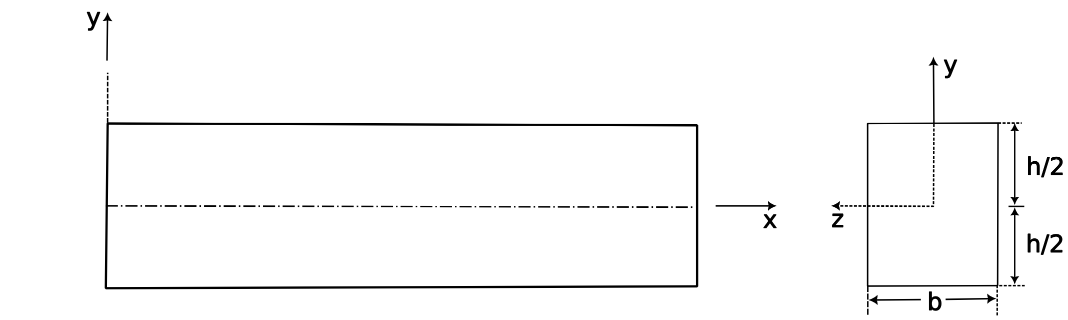

Figure 2

$$\sigma_{xx} = -\frac{My}{I} = -\frac{12My}{bh^3}$$

where M is the bending moment in the beam's cross-section and is a function of x. Assume that $\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0$ everywhere. Furthermore $\tau_{xy} = 0$ on the top and bottom face and $\sigma_{yy} = 0$ on the bottom face. Using the differential equations of equilibrium, determine τ_{xy} and σ_{yy}. Compare these with the values given in the elementary strength of materials.

3. A cylindrical rod (Fig.3) is subjected to a torque T. At any point P of the cross-section LN, the following stress components exist:

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = \tau_{yx} = 0,\ \tau_{xz} = \tau_{zx} = -G\theta y,\ \tau_{yz} = \tau_{zy} = G\theta x$$

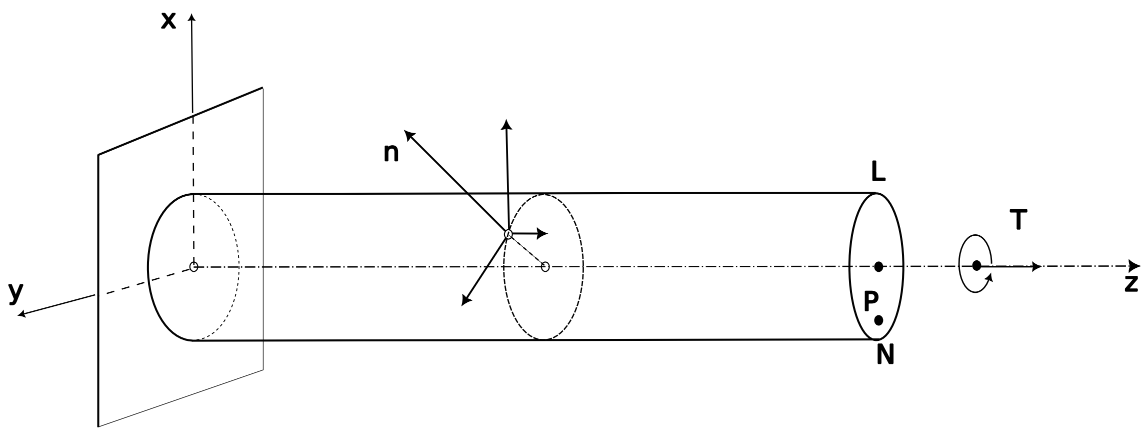

Figure 3

Check whether these satisfy the equations of equilibrium. Also show that the above distribution implies that the lateral surface should be free of external load.

4. For the state of stress given in Q3, determine the principal stress components, maximum shear stress values and the associated plane normals on which they are realized.

5. A cylindrical boiler, 180cm in diameter, is made of plates 1.8cm thick and is subjected to an internal pressure of 1400 kPa. Determine the maximum shearing stress in the plate at point P and the plane on which it acts.

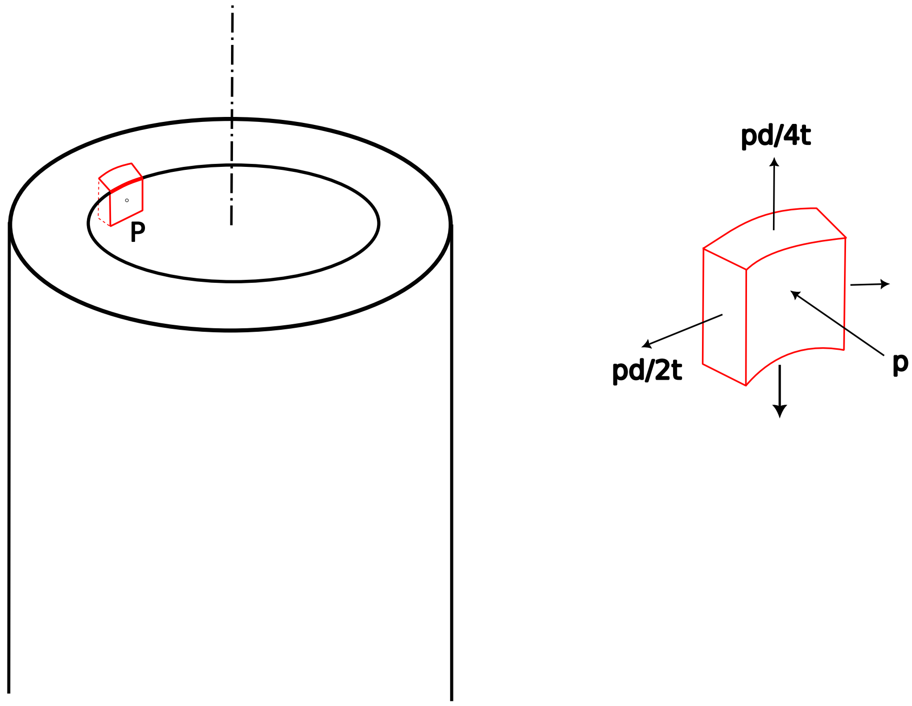

Figure 4

6. **Divergence operator**
Divergence of a tensor is defined as follows:

$$\underline{\nabla} \cdot (\circ) = \sum_i \frac{\partial}{\partial x_i} (\circ) \cdot \underline{e}_i$$

For example

$$\begin{aligned}\underline{\nabla} \cdot \underline{v} = \sum_i \frac{\partial}{\partial x_i} (\underline{v}) \cdot \underline{e}_i &= \sum_i \frac{\partial}{\partial x_i} \left(\sum_j v_j \underline{e}_j \right) \cdot \underline{e}_i \\ &= \sum_i \sum_j \frac{\partial v_j}{\partial x_i} \delta_{ij} = \sum_i \frac{\partial v_i}{\partial x_i}.\end{aligned}$$

Show that $\underline{\nabla} \cdot \underline{\underline{\sigma}} = \sum_i \sum_j \frac{\partial \sigma_{ji}}{\partial x_i} \underline{e}_j$