Tutorial 2: Traction vector

APL 104 - 2022 (Solid Mechanics)

Q1. Show that $\underline{t}^n = \sum_i \underline{t}^i\,(\underline{n} \cdot \underline{e}_i) = \sum_i \underline{t}^{\hat{i}}\,(\underline{n} \cdot \hat{\underline{e}}_i)$, i.e, the formula is independent of what three planes are chosen to determine $\underline{t}^n$!

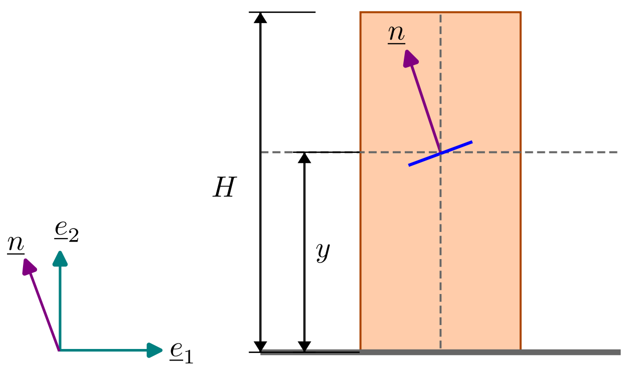
Q2. Suppose $[\underline{t}^1] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $[\underline{t}^2] = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$, $[\underline{t}^3] = \begin{bmatrix} 0 \\ 7 \\ 9 \end{bmatrix}$ in $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ coordinate system.
What will be the traction on a plane with normal $\underline{n} = \hat{\underline{e}}_1$ where $(\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$ is obtained from rotation of $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ about $\underline{e}_3$ by 45°? What are the normal and shear components of traction on this plane?

Q3. Show that the component of a traction vector on $\underline{n}$-plane in the direction $\underline{m}$ equals the component of the traction on $\underline{m}$-plane in the direction $\underline{n}$, i.e, $\underline{t}^n \cdot \underline{m} = \underline{t}^m \cdot \underline{n}$.

Q4. Consider a vertical bar having mass density ρ. Assume its length be to H and is subjected to uniform body force due to gravity. Find the traction vector on an infinitesimal internal section of the bar located at the center of its cross-section with outward normal

$$\underline{n} = -\sin\theta\ \underline{e}_1 + \cos\theta\ \underline{e}_2$$

and at a height of y from the base (see figure). Also find the normal and tangential components of the traction vector on this plane.



Q5. Think of a bar lying along $\underline{e}_1$ axis and loaded axially. We will learn later in the class that during the tensile loading of a bar, the traction on a section with normal along $\underline{e}_1$ has no shear components of traction. Also, as the bar is allowed to contract freely in the transverse direction, the traction on sections having normals perpendicular to $\underline{e}_1$ completely vanish. What will be the traction on the plane whose normal makes an angle θ from axial direction. What are the normal and shear components of traction on this plane?