Tutorial 1: Mathematical Preliminaries

APL 104 - 2022 (Solid Mechanics)

Q1. Show that $\underline{a} \cdot \left(\underline{\underline{A}}\, \underline{b}\right) = \left(\underline{\underline{A}}^T \underline{a}\right) \cdot \underline{b}$

Q2. There exists a tensor $\underline{\underline{A}}$ such that $\underline{\underline{A}} \cdot \underline{e}_1 = \underline{a}$, $\underline{\underline{A}} \cdot \underline{e}_2 = \underline{b}$, $\underline{\underline{A}} \cdot \underline{e}_3 = \underline{c}$. What will be the matrix form of $\underline{\underline{A}}$ in $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ coordinate system?

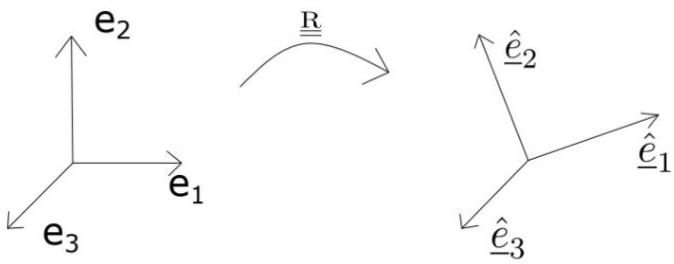
Q3. Show that

(a) $(\underline{a} \otimes \underline{b})\, \underline{\underline{C}} = \underline{a} \otimes \left(\underline{\underline{C}}^T \underline{b}\right)$

(b) $\underline{\underline{C}}\, (\underline{a} \otimes \underline{b}) = \left(\underline{\underline{C}}\, \underline{a}\right) \otimes \underline{b}$

Q4. Given an anti-symmetric tensor $\underline{\underline{A}}$, prove that $\left(\underline{\underline{A}}\, \underline{x}\right) \cdot \underline{x} = 0 \;\; \forall\, \underline{x}$

Q5. In class we learnt that a unique rotation tensor $\underline{\underline{R}}$ can be associated with transforming a set of orthonormal triad into anoother say $(\underline{e}_1, \underline{e}_2, \underline{e}_3) \rightarrow (\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$. In particular, we discussed a specific case where $(\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$ is obtained by rotation of $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ about $\underline{e}_3$ axis by angle θ. Find the matrix form of this rotation tensor $\underline{\underline{R}}$ in $(\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$ coordinate system.



Q6. **Extra questions for practice**:

- If $\underline{a}$ and $\underline{b}$ are two vectors, show that their dot product $\underline{a} \cdot \underline{b}$ remain the same in different coordinate system (say $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ and $(\hat{\underline{e}}_1, \hat{\underline{e}}_2, \hat{\underline{e}}_3)$)

- Prove that $\left[(\underline{u} \cdot \underline{a}) \left(\underline{\underline{A}}^T \underline{b}\right)\right] \cdot \underline{n} = \underline{a} \cdot \left[\left(\underline{u} \otimes \underline{\underline{A}}\underline{n}\right)\right] \underline{b}$

- If $\underline{\underline{A}}$ is a symmetric tensor and $\underline{\underline{B}}$ is an anti-symmetric tensor, then show that

$$\underline{\underline{A}} : \underline{\underline{B}} = 0$$