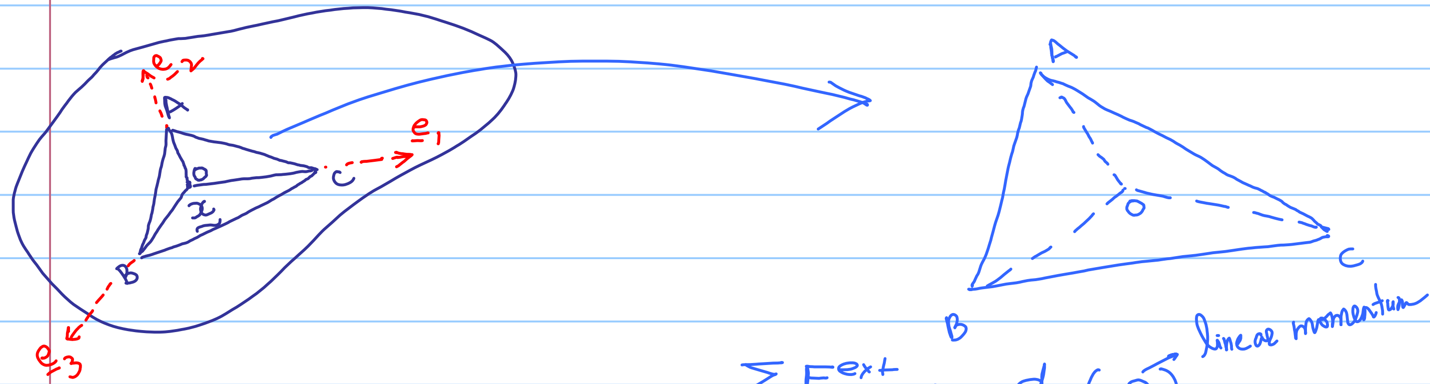
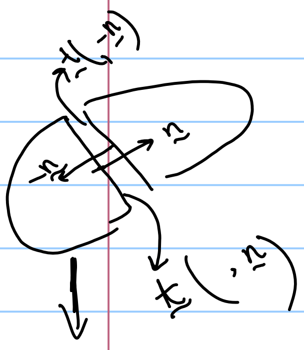
Lecture 5 (Traction vector)



$$\sum \underset{\sim}{F}^{ext} = \frac{d}{dt}(\underset{\sim}{p})$$ linear momentum



$\underset{\sim}{t}(\,,-\underset{\sim}{e}_1) \leftarrow$ OAB

$\downarrow$

$\underset{\sim}{t}^{-1}$

faces	outward normals
OAB :	$-\underset{\sim}{e}_1$
OAC :	$-\underset{\sim}{e}_3$
OBC :	$-\underset{\sim}{e}_2$
ABC :	$\underset{\sim}{n}$

* traction is applied by that part of the body into which the normal vector points to and it acts on that part of the body from which the normal vector originates!

eg body force (force/vol.) contact force :

surface force (force/area) non-contact force :

$$\sum \underset{\sim}{F}^{ext} = \underset{\text{traction force}}{\sum} + \underset{\text{body force}}{\sum}$$

$$\underset{\sim}{t}^{-1} A_{OAB} + \underset{\sim}{t}^{-2} A_{OBC} + \underset{\sim}{t}^{-3} A_{OAC} + \underset{\sim}{t}^{n} A_{ABC} + \rho V \underset{\sim}{g}$$

We have approximated the traction to be uniform over each faces

$$= \rho V \underset{\sim}{a}_{CM}$$

$$\underset{\sim}{t}^{-1} \cancel{A_{ABC}} (\underset{\sim}{n}\cdot\underset{\sim}{e}_1) + \underset{\sim}{t}^{-2} \cancel{A_{ABC}} (\underset{\sim}{n}\cdot\underset{\sim}{e}_2) + \underset{\sim}{t}^{-3} \cancel{A_{ABC}} (\underset{\sim}{n}\cdot\underset{\sim}{e}_3)$$

$$+ \underset{\sim}{t}^{n} \cancel{A_{ABC}} + \rho(\underset{\sim}{g} - \underset{\sim}{a}_{CM}) \tfrac{1}{3} h \cancel{A_{ABC}}$$

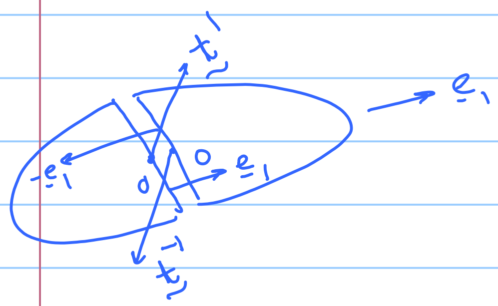
$$\Rightarrow \sum_{i=1}^{3} \underset{\sim}{t}^{-i} (\underset{\sim}{n}\cdot\underset{\sim}{e}_i) + \underset{\sim}{t}^{n} + \cancel{\rho(\underset{\sim}{g} - \underset{\sim}{a}_{CM}) \tfrac{1}{3} h} = \underset{\sim}{0}$$ $\to 0$

lt $h \to 0$ (shrink the tetrahedron to point "O")

$$\Rightarrow \quad \sum_{i=1}^{3} \underline{t}^{-i}\,(\underline{n}\cdot\underline{e}_i) + \underline{t}^{n} = \underline{0}$$

$$\Rightarrow \quad -\sum_{i=1}^{3} \underline{t}^{i}\,(\underline{n}\cdot\underline{e}_i) + \underline{t}^{n} = \underline{0}$$

$$\Rightarrow \quad \boxed{\underline{t}^{n}(\underline{x}) = \sum_{i=1}^{3} \underline{t}^{i}(\underline{x})\,(\underline{n}\cdot\underline{e}_i)}$$



in a coordinate system $(\underline{e}_1-\underline{e}_2-\underline{e}_3)$

normal components of traction

$$\begin{bmatrix} t^n_1 \\ t^n_2 \\ t^n_3 \end{bmatrix} = n_1 \begin{bmatrix} t^1_1 \\ t^1_2 \\ t^1_3 \end{bmatrix} + n_2 \begin{bmatrix} t^2_1 \\ t^2_2 \\ t^2_3 \end{bmatrix} + n_3 \begin{bmatrix} t^3_1 \\ t^3_2 \\ t^3_3 \end{bmatrix}$$

shear components of traction

+ve normal traction: tensile

−ve " " : compressive

$$\underline{t}^{n} = \sum \underline{t}^{i}\,(\underline{n}\cdot\underline{e}_i)$$

$$= \sum \underline{t}^{i}\,(\underline{e}_i\cdot\underline{n})$$

$$= \sum [\underline{t}^{i}]\left([\underline{e}_i]^T [\underline{n}]\right)$$

$$= \sum \left([\underline{t}^{i}][\underline{e}_i]^T\right)[\underline{n}]$$

$$= \sum \left(\underline{t}^{i}\otimes\underline{e}_i\right)\underline{n}$$

$$= \Big[\underbrace{\sum \underline{t}^{i}\otimes\underline{e}_i}_{\text{Stress tensor}}\Big]\underline{n}$$