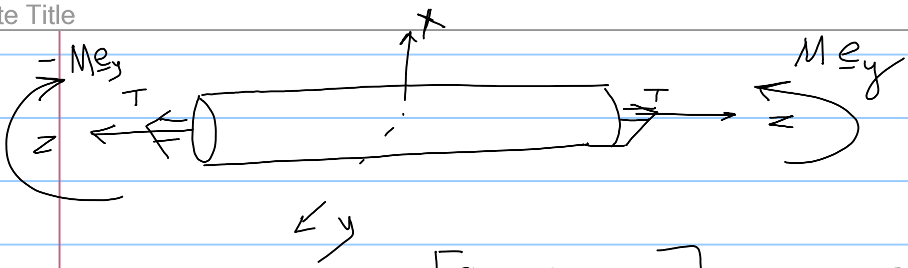
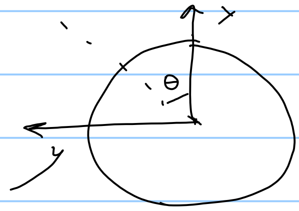
Lec 38

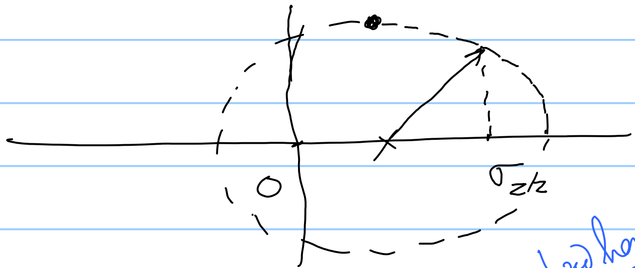
Note Title 11/15/2022



$$[\underline{\underline{\sigma}}]_{\substack{BM \\ (x,y,z)}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{Mx}{I_{yy}} \end{bmatrix} \qquad [\underline{\underline{\sigma}}]_{\substack{T \\ (r,\theta,z)}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{Tr}{J} \\ 0 & \frac{Tr}{J} & 0 \end{bmatrix}$$

$$[\underline{\underline{\sigma}}]_{(r,\theta,z)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & Tr/J \\ 0 & Tr/J & \frac{M r\cos\theta}{I_{yy}} \end{bmatrix}$$



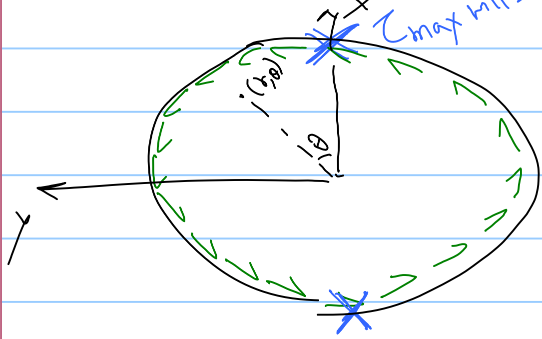


τ_{max} = radius of Mohr Circle

$$= \sqrt{\left(\frac{\sigma_{zz}}{2}\right)^2 + \tau_{\theta z}^2}$$

$$= \sqrt{\frac{M^2 r^2 \cos^2\theta}{I^2} + \frac{T^2 r^2}{4 I^2}}$$

τ_{max} will be realized here



$$\max_{(r,\theta)} \tau_{max} < \tau_y$$

$$\Rightarrow \boxed{\sqrt{\frac{M^2R^2}{I^2} + \frac{T^2R^2}{4I^2}} < \tau_y}$$

Factor of safety (N)

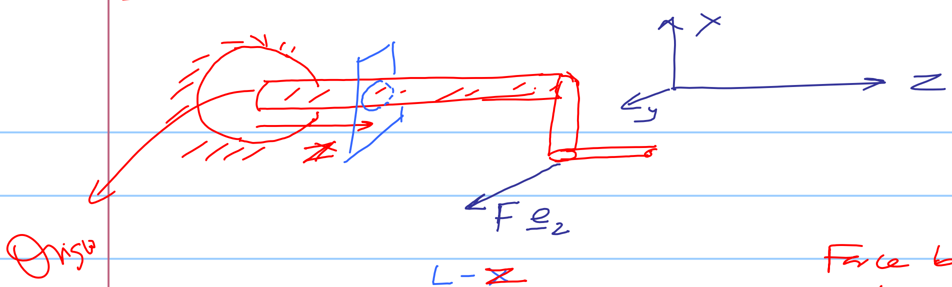
Let M & T be operational bending moment & Torque

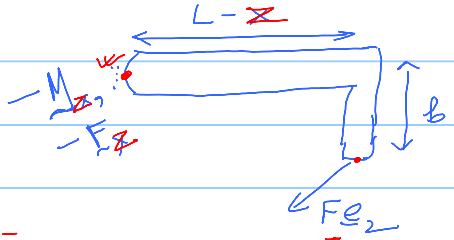
$\Rightarrow$ We have to design for $M^* = NM$ & $T^* = NT$

$$\Rightarrow \sqrt{\frac{M^{*2}R^2}{I^2} + \frac{T^{*2}R^2}{4I^2}} < \tau_y$$

$$\Rightarrow \sqrt{\frac{M^2R^2}{I^2} + \frac{T^2R^2}{4I^2}} < \tau_y/N$$

Lever of a Machine





Force balance

$$\Rightarrow \; -\underset{\sim}{F}_z + F\underset{\sim}{e}_2 = \underset{\sim}{0}$$

$$\Rightarrow \; \underset{\sim}{F}_z = F\underset{\sim}{e}_2$$

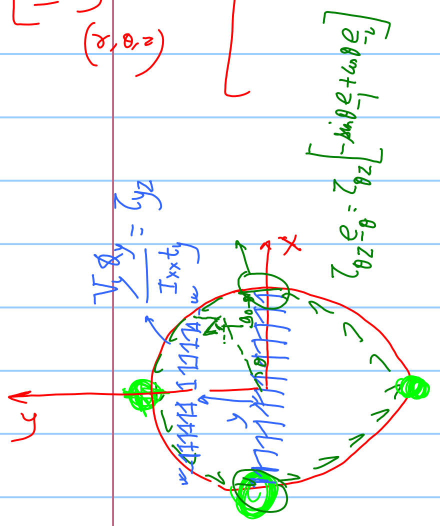
Moment balance

$$-\underset{\sim}{M}_z + \underbrace{\vec{r}}_{(L-z)\underset{\sim}{e}_3 - b\underset{\sim}{e}_1} \times F\underset{\sim}{e}_2 = \underset{\sim}{0}$$

$$\Rightarrow -\underset{\sim}{M}_z - (L-z)F\underset{\sim}{e}_1 - Fb\,\underset{\sim}{e}_3 = \underset{\sim}{0}$$

$$\Rightarrow \underset{\sim}{M}_z = \underbrace{-(L-z)F\underset{\sim}{e}_1}_{\text{bending moment}} \underbrace{- Fb\,\underset{\sim}{e}_3}_{\text{Torque}}$$

$$[\underset{=}{\sigma}]_{(r,\theta,z)} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$



Due to shear force $F\underset{\sim}{e}_2$

$$\tau_{yz} = \frac{V_y Q_y}{I_{xx} t_y} = \frac{F Q_y}{I t_y}$$

Due to bending moment,

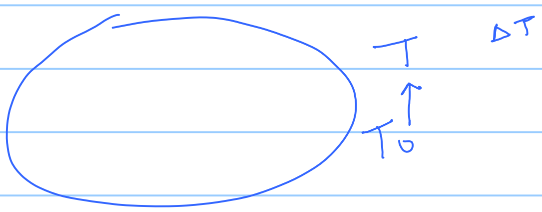
$$\sigma_{zz} = \frac{M_x y}{I_{xx}} = \frac{-(L-z)F\,y}{I}$$

Due to torque

$$\tau_{\theta z} = \frac{Tr}{2I} = \frac{-Fbr}{2I}$$

$$[\underset{=}{\sigma}] = \begin{bmatrix} 0 & 0 & -\tau_{\theta z}\sin\theta \\ 0 & 0 & \tau_{yz} + \tau_{\theta z}\cos\theta \\ -\tau_{\theta z}\sin\theta & \tau_{yz} + \tau_{\theta z}\cos\theta & \sigma_{zz} \end{bmatrix}_{(x,y,z)}$$

Thermoelasticity



* Thermal part of strain does not lead to stress generation

Due to Temp. alone

$$\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = \alpha\Delta T$$

Due to stress alone

$$\epsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})\right]$$

$\vdots$

elastic strain

$$\epsilon_{xx} - \epsilon_{xx}^T = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})\right]$$

$$\epsilon_{yy} - \epsilon_{yy}^T = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx}+\sigma_{zz})\right]$$

$$\epsilon_{zz} - \epsilon_{zz}^T = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx}+\sigma_{yy})\right]$$

$$\gamma_{xy} = \tau_{xy}/G$$

$\vdots$

* Thermoelasticity —
  * Design
  * Beam Theory

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