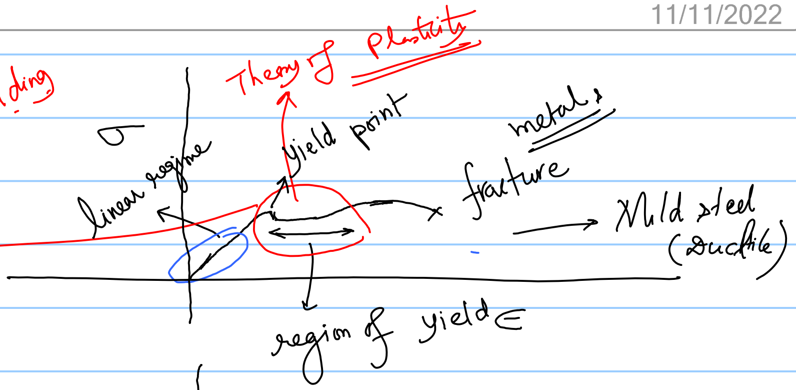
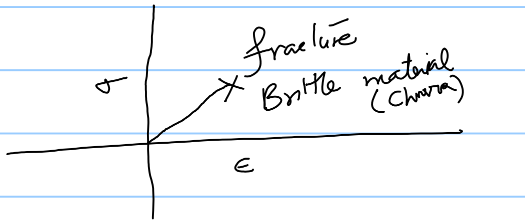
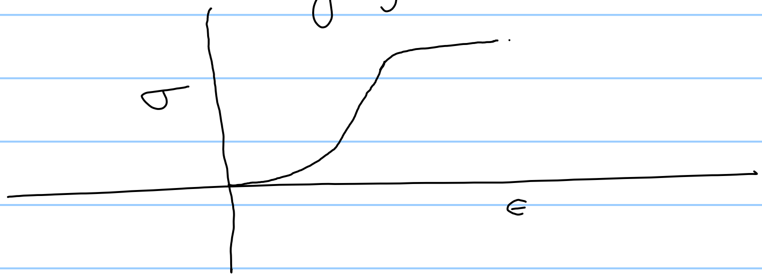
Lec 37 (Theories of Failure)

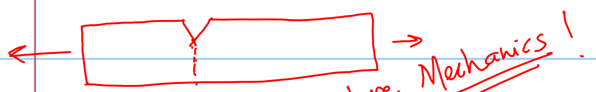
Note Title 11/11/2022



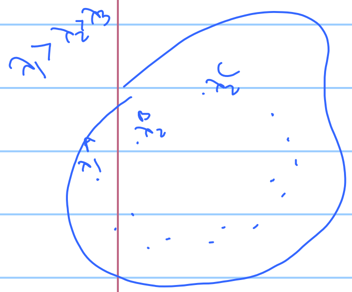
molecular origin of yielding (dislocation..)



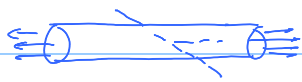


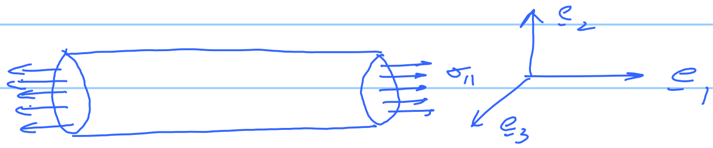


Fracture Mechanics!



1) Maximum principal stress theory σ_y
2) Maximum shear stress theory: τ_y
3) Maximum normal strain theory ϵ_y
4) Maximum shear strain theory γ_y
5) Distortional energy theory
6) Octahedral shear stress theory





$$[\underline{\underline{\sigma}}] = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [\underline{\underline{\epsilon}}] = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$$

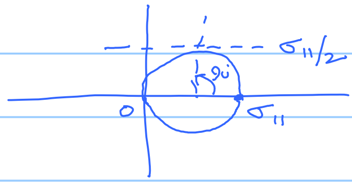
(1) σ_{11}^*: the load value at which the body failed

$\sigma_{11}^* = \sigma_y$ ↖ yield stress!

$$\boxed{\max_{\underline{x}} \lambda_1(\underline{x}) \le \sigma_y}$$

$$[\underline{\underline{\sigma}}] = \begin{bmatrix} \times & \times & \lambda \\ \times & \times & \lambda \\ \times & \times & \times \end{bmatrix}$$

(2)



$$\frac{\sigma_{11}^*}{2} = \tau_y$$

$$\max_{\underline{x}} \frac{\lambda_1(\underline{x}) - \lambda_3(\underline{x})}{2} \le \tau_y$$

(5) Distortional energy theory..

$$E = \sum_i \sum_j \frac{1}{2} \sigma_{ij} \epsilon_{ij}$$

In the coordinate system of principal direction

$$[\underline{\underline{\sigma}}] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}, \quad [\underline{\underline{\epsilon}}] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

$$E = \frac{1}{2} \sum_i \sigma_i \epsilon_i \qquad \epsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right]$$

ϵ_2 -

ϵ_3 -

$$= \frac{1}{2E}\left[\sigma_1\left(\sigma_1 - \nu(\sigma_2+\sigma_3)\right) + \sigma_2\left(\sigma_2 - \nu(\sigma_1+\sigma_3)\right) + \sigma_3\left(\sigma_3 - \nu(\sigma_1+\sigma_2)\right)\right]$$

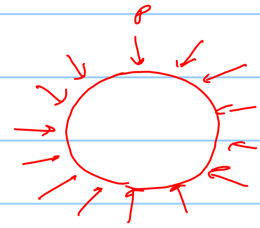
$$= \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Vol. strain energy density $= \frac{1}{2} \sigma^P . \epsilon_{vol}$

$$= \frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)(\epsilon_1+\epsilon_2+\epsilon_3)$$

$$= \frac{(1-2\nu)}{6E}(\sigma_1+\sigma_2+\sigma_3)^2$$

p



$\underline{\underline{\sigma}} = -p\,\underline{\underline{I}}$

→ only volume change is happening without any distortion (shear is absent..)

$$\underline{\underline{\sigma}} = p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon & & \\ & \epsilon & \\ & & \epsilon \end{bmatrix}$$

$$\sum\sum \frac{1}{2}\sigma_{ij}\epsilon_{ij} = \frac{1}{2} p \cdot 3\epsilon$$
$$= \frac{1}{2} p\, \epsilon_{vol}$$

$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2+\sigma_3)\right]$

ϵ_2

ϵ_3

$$\Rightarrow \epsilon_1 + \epsilon_2 + \epsilon_3 = \frac{1}{E}\left[(\sigma_1+\sigma_2+\sigma_3) - 2\nu(\sigma_1+\sigma_2+\sigma_3)\right]$$

$$= \frac{1-2\nu}{E}(\sigma_1+\sigma_2+\sigma_3)$$

Distortional energy density $= \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\nu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1) - \frac{(1-2\nu)}{3}(\sigma_1+\sigma_2+\sigma_3)^2\right]$

$$= \frac{1+\nu}{3E}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right)$$

for isotropic materials

simple tension test

$$\frac{1+\nu}{3E}\sigma_1^{*2} = \boxed{\frac{1+\nu}{3E}\sigma_y^2}$$

$$\max_{\underline{x}} \frac{1+\nu}{3E}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right) < \frac{1+\nu}{3E}\sigma_y^2$$

$$\max_{\underline{x}} \underbrace{\left(\sigma_1^2+\sigma_2^2+\sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right)}_{K\,\tau_{oct}^2} < \sigma_y^2$$

, therefore, we also call it octahedral shear stress theory!

Last class on Tuesday

Quiz-2

4:5:30

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