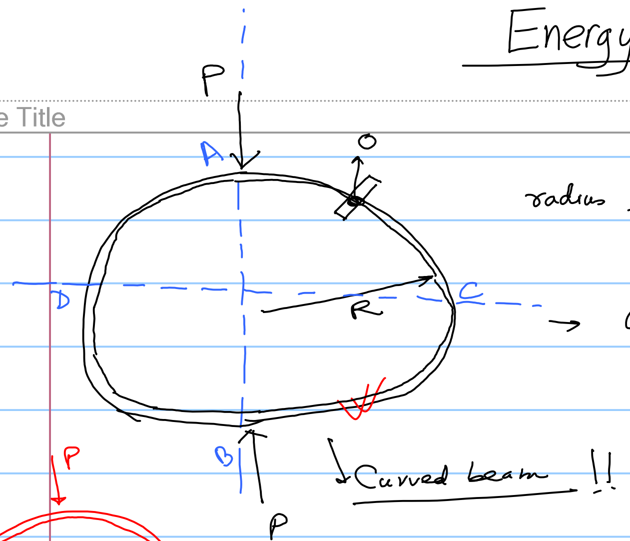
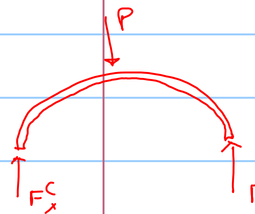
Energy methods

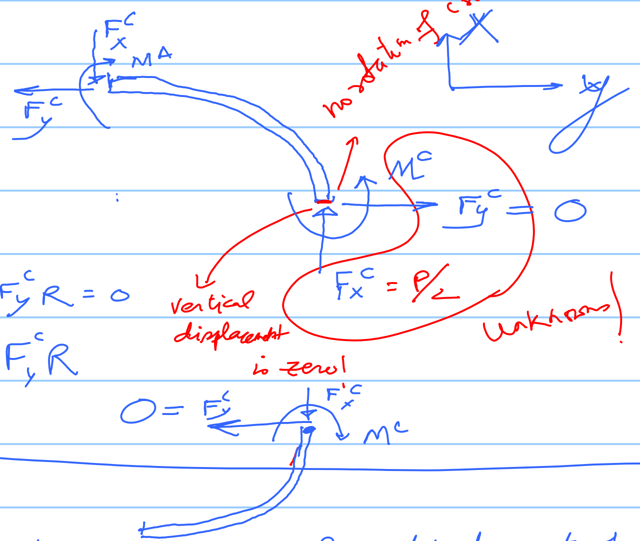
Note Title 11/9/2022



radius of cross-section: r

By how much do the pts where load is applied move inward?





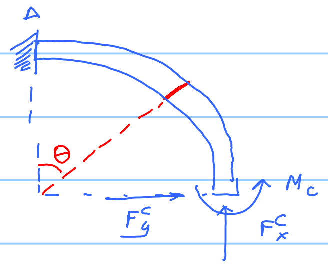
$F_x^C = P/2$ Moment balance about A:

$$-M^A + M^C + F_x^C R + F_y^C R = 0$$

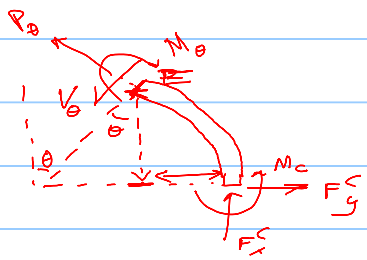
$$\Rightarrow M^A = M^C + F_x^C R + F_y^C R$$

* M^C is still an unknown but what we know is that the rotation of cross-section at C (corresponding displacement of M_C) is zero!

$E(M^C, P)$ $\qquad \frac{\partial E}{\partial M^C} = 0$



$E(F_x^C, F_y^C, M^C)$



Moment balance about E:

$$\Rightarrow -M_\theta + M_c + F_y^c R\cos\theta + F_x^c R(1-\sin\theta) = 0$$

$$\Rightarrow M_\theta = M_c + F_y^c R\cos\theta + F_x^c R(1-\sin\theta)$$

$$E(M_c, F_c^x, F_c^y) = \int_0^{\pi/2} \left(\frac{V_\theta^2}{2kGA} + \frac{P_\theta^2}{2EA} + \frac{M_\theta^2}{2EI}\right) R\,d\theta$$

Let's neglect energy due to shear force & bending moment!

$$\Rightarrow E(M_c, F_c^x, F_c^y) = \int_0^{\pi/2} \frac{\{M_c + F_c^y R\cos\theta + F_c^x R(1-\sin\theta)\}^2}{2EI} R\,d\theta$$

$$\delta_c^x = \frac{\partial E}{\partial F_c^x}$$

$$\delta_c^y = \frac{\partial E}{\partial F_c^y} = \int_0^{\pi/2} \frac{2\left[\overset{=??}{M_c} + \overset{\to 0}{F_c^y} R\cos\theta + \overset{=P/2}{F_c^x} R(1-\sin\theta)\right] R\cos\theta}{2EI} R\,d\theta$$

$$\theta_c = \frac{\partial E}{\partial M_c} = 0$$

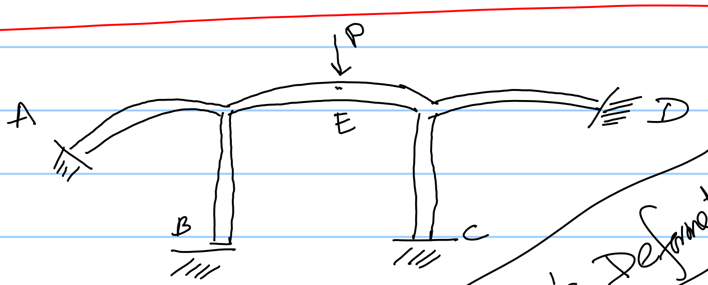
$$\Rightarrow \int_0^{\pi/2} \frac{2\left(M_c + \overset{0}{F_c^y} R\cos\theta + \overset{P/2}{F_c^x} R(1-\sin\theta)\right)}{2EI} R\,d\theta$$

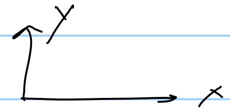
$$\Rightarrow \int_0^{\pi/2} \frac{M_c + P/2\,R(1-\sin\theta)}{EI} R\,d\theta = 0$$

$$\Rightarrow \frac{M_c R}{EI}\frac{\pi}{2} + \frac{P/2\,R^2}{EI}\int_0^{\pi/2}(1-\sin\theta)\,d\theta = 0 \quad \to \pi/2 - 1$$

$$\Rightarrow M_c \frac{\pi}{2} + \frac{P}{2} R\left(\frac{\pi}{2} - 1\right) = 0$$

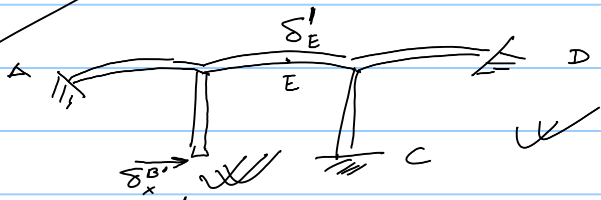
$$\Rightarrow M_c = -\frac{PR(\pi/2 - 1)}{\pi}$$





BETTI's Deformation

Reciprocal theorem!



$\delta_y^{B'} = 0$

$\theta^{B'} = 0$

$B_x,\ B_y,\ M_B$

Force & moment at A, B, C, D, P → δ_E

(0,0,0) (0,0,0) (0,0,0) (0,0,0),

A_x	A_y	M_A	B_x	B_y	M_B	C_x	C_y	M_C	D_x	D_y	M_D	P
0	0	0	$\delta_x^{B'}$	0	0	0	0	0	0	0	0	δ_E'

A_x'	A_y'	M_A'	B_x'	B_y'	M_B'	C_x'	C_y'	M_C'	D_x'	D_y'	M_D'	$P' = 0$
0	0	0	0	0	0	0	0	0	0	0	0	δ_E

$$B_x \delta_x^{B'} + P\delta_E' =$$

$$\Rightarrow B_x = -\frac{P\delta_E'}{\delta_x^{B'}}$$

$$B_y = -\frac{P\delta_E'}{\delta_y^{B'}}$$

$$M_B = \frac{-P\delta_E'}{\theta^{B'}}$$