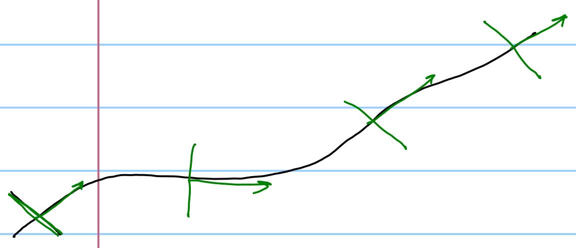
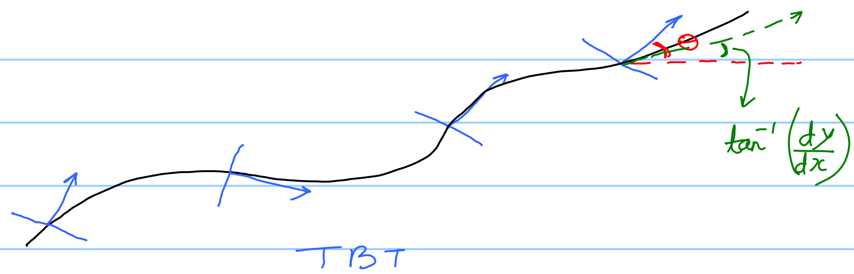
Lec 32   Timoshenko beam theory

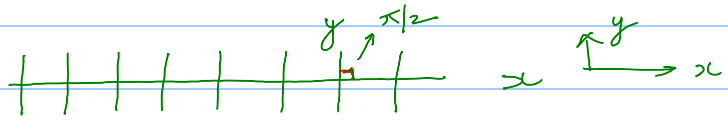
Note Title   10/28/2022

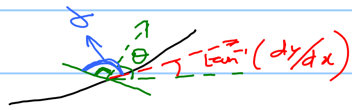




Single Variable: y

2 Variables: y, θ





$$\alpha = \pi/2 + \theta - \tan^{-1}(dy/dx)$$

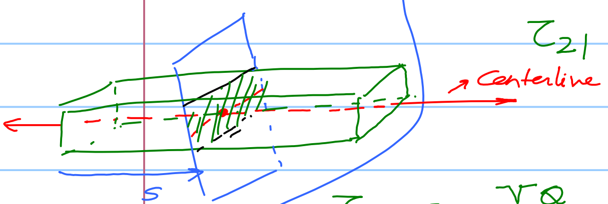
$$\gamma_{12} = \pi/2 - \alpha = \pi/2 - \left(\pi/2 + \theta - \tan^{-1}(dy/dx)\right)$$

$$= \tan^{-1}\frac{dy}{dx} - \theta$$

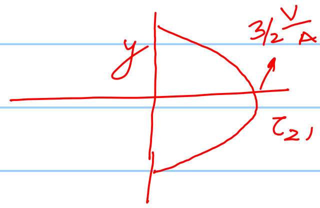
$$\approx \frac{dy}{dx} - \theta \qquad \left(\text{assuming } \left|\frac{dy}{dx}\right| << 1\right)$$

$$\approx \frac{dy}{dX} - \theta \qquad (\text{assuming } x \approx X)$$

$$\tau_{21} = \tau_{12} = G\gamma_{12} = G\left(\frac{dy}{dX} - \theta\right)$$



$$\tau_{21} = \frac{VQ}{It} \Rightarrow \tau_{21}(y=0) = \frac{3}{2}\frac{V}{A}$$



$$\Rightarrow \gamma_{21}(y=0) = \frac{\tau_{21}(y=0)}{G} = \frac{3}{2}\frac{V}{GA} = \frac{V}{\frac{2}{3}GA} = \frac{V}{\kappa GA}$$

→ shear correction factor!

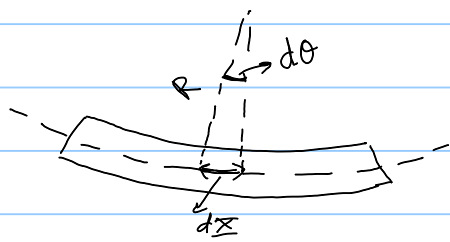
$$\Rightarrow \boxed{\frac{dy}{dx} - \theta = \frac{V}{\kappa GA}}$$

relates shear force to y & θ

In EBT

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$EI\,\kappa(x) = M(x)$$



$$R\,d\theta = dX$$

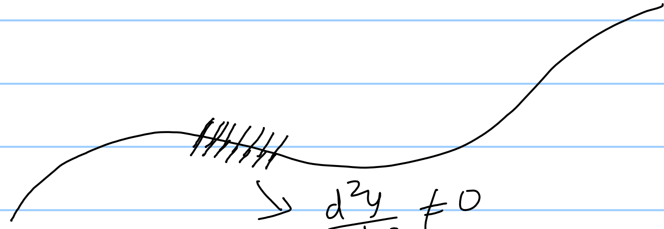
$$\Rightarrow \frac{d\theta}{dX} = \frac{1}{R} = \kappa$$

In TBT
$\theta' = \kappa$
$\nparallel$
$\frac{d^2y}{dx^2}$

$\theta \neq \frac{dy}{dx}$

For EBT, $\theta = \frac{dy}{dx} \Rightarrow \theta' = \frac{d^2y}{dx^2} \Rightarrow \kappa = \frac{d^2y}{dx^2}$

$$\boxed{EI\,\theta' = M(x)} \rightarrow \text{a more general equation!}$$



$\Rightarrow \frac{d^2y}{dx^2} \neq 0$

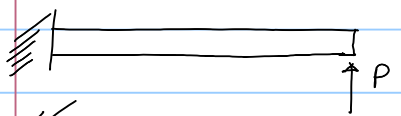
$\theta' = 0$!

$\Downarrow$

$M(x) = 0$

Two eqs of TBT

$$\boxed{\begin{aligned} EI\,\theta' &= M(x) \\ \frac{dy}{dx} - \theta &= \frac{V}{KGA} \end{aligned}}$$



$y(0) = 0$

$\theta(0) = 0$, $\cancel{\frac{dy}{dx}(0) = 0}$

$$V(x) = P$$
$$M(x) = P(L-x)$$
$$EI\,\theta' = P(L-x)$$
$$\frac{dy}{dx} - \theta = \frac{P}{KGA}$$
$$\theta = \frac{P}{EI}\left(Lx - \frac{x^2}{2}\right) + C$$
$$\Downarrow C = 0$$
$$\Rightarrow \theta(x) = \frac{P}{EI}\left(Lx - x^2/2\right)$$

$$\frac{dy}{dx} = \frac{P}{EI}\left(Lx - \frac{x^2}{2}\right) + \frac{P}{KGA}$$
$$\Rightarrow y = \frac{P}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + \frac{Px}{KGA} + D$$
$$\downarrow D = 0$$
$$\Rightarrow y(x) = \frac{PL^3}{6EI}\left(3\frac{x^2}{L^2} - \frac{x^3}{L^3}\right) + \frac{Px}{KGA}$$

$$\epsilon_\delta = \frac{\delta_{EBT} - \delta_{TBT}}{\delta_{TBT}} = \frac{\cancel{\frac{PL^3}{3EI}} - \cancel{\frac{PL^3}{3EI}} - \frac{PL}{KGA}}{PL^3/3EI + \frac{PL}{KGA}}$$

$$= -\frac{1}{1 + \frac{L^2 KGA}{3EI}}$$

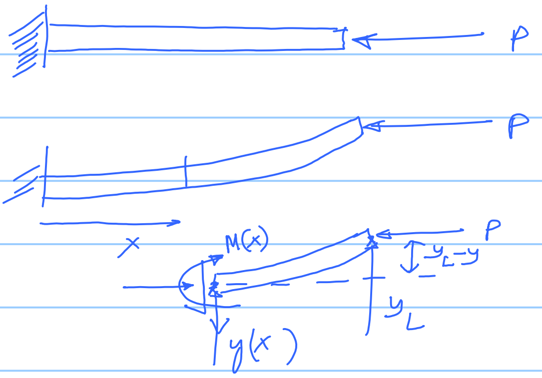
For rectangular cross-section!

$$\Rightarrow \frac{L^2 KGA}{3EI} = \frac{L^2\, \frac{2}{3}\, G\, bh}{3E\, \frac{1}{12} bh^3} = \frac{L^2}{h^2}\,\frac{8}{3}\,\frac{G}{E}$$

$$\epsilon_\delta = -\frac{1}{1+\frac{L^2}{h^2}\frac{8}{3}\frac{G}{E}} = -\frac{1}{1+\frac{L^2}{h^2}\frac{8}{3}\frac{1}{2(1+\nu)}}$$

$\Rightarrow$ ϵ_δ would be very small if $\underline{\underline{L/h > 10}}$

Buckling in beams (using EBT)



$$\Rightarrow -M(x) + P(y_L - y) = 0$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = P(y_L - y)$$

$$\Rightarrow EI\frac{d^2y}{dx^2} + Py - Py_L = 0$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} + \frac{P}{EI}y - \frac{P}{EI}y_L = 0}$$

Non-homogeneous 2nd order ODE

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

$\downarrow y = e^{\lambda x}$

$$\Rightarrow \lambda^2 + \frac{P}{EI} = 0 \Rightarrow \lambda = \pm\sqrt{-P/EI} = \pm i\sqrt{P/EI}$$

$$y = C_1\cos\left(\sqrt{P/EI}\,x\right) + C_2\sin\left(\sqrt{\frac{P}{EI}}\,x\right) + y_L$$

B.C. $\Rightarrow$ $y(0) = 0 \Rightarrow C_1 + y_L = 0 \Rightarrow C_1 = -y_L$

$\frac{dy}{dx}(0) = 0 \Rightarrow C_2 = 0$

$$\Rightarrow y = y_L\left(1 - \cos\left(\sqrt{\frac{P}{EI}}\,x\right)\right)$$

$\Downarrow$ $x = L$, $y = y_L$

$$\Rightarrow y_L = y_L(1 - \cos)$$

$$\Rightarrow 1 - \cos = 1$$

$$\Rightarrow \cos\left(\sqrt{\frac{P}{EI}}\,L\right) = 0$$

$$\Rightarrow \sqrt{\frac{P}{EI}}\,L = \frac{2n+1}{2}\pi$$

$$\Rightarrow \frac{P}{EI} L^2 = \frac{\pi^2}{4} \quad n=0$$

$$\Rightarrow \boxed{P = \frac{\pi^2}{4} \frac{EI}{L^2}}$$

Friday 6-7

~~Quiz~~