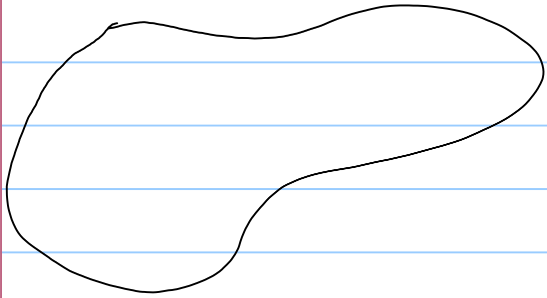
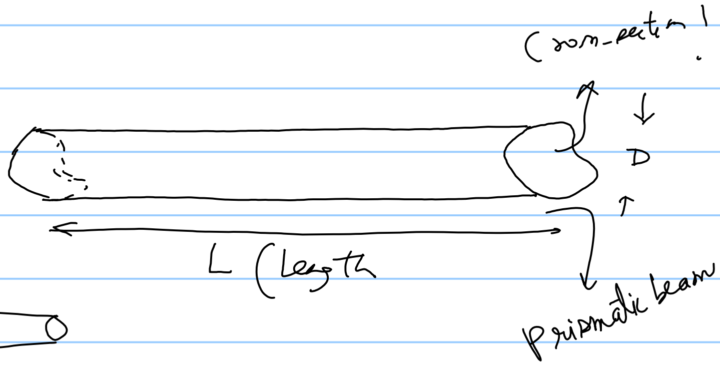
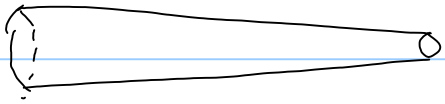
Lec 31 Theory of beams

Note Title 10/26/2022





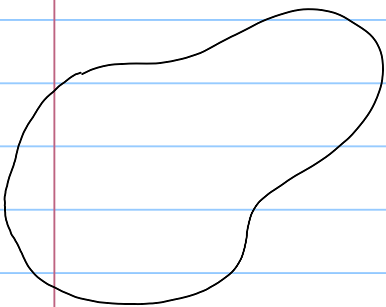


→ Length and cross-section!

$L >> D$

→ one of the dimension (length) is much much greater than other two (cross-section)

$$\underline{\underline{L/D > 10}}$$



$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial \tau_{13}}{\partial x_3} + b_1 = \rho a_1$$

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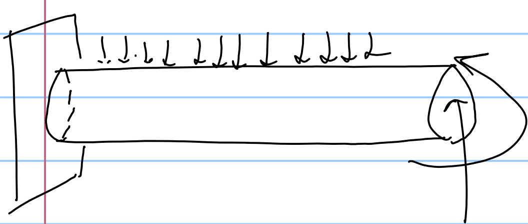
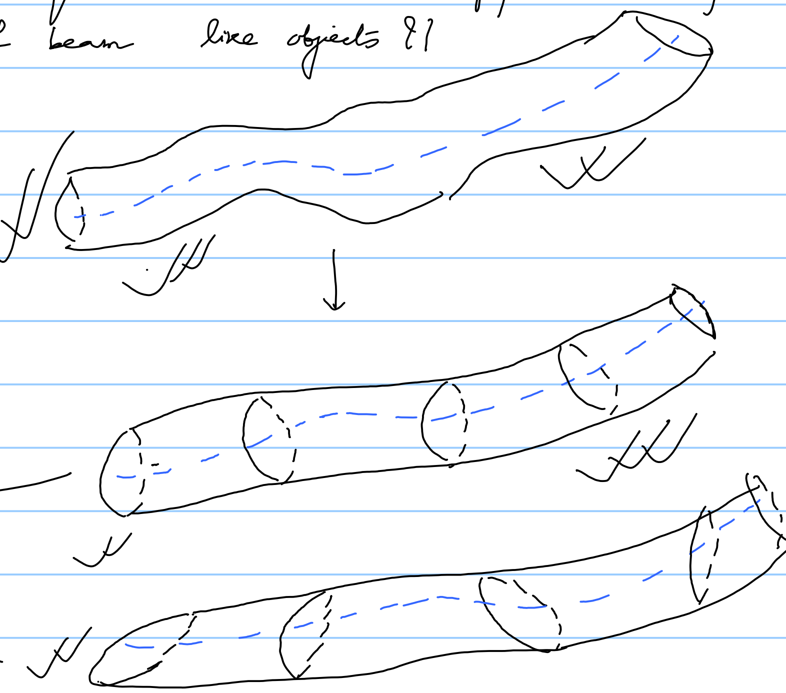
+ BC.

Can be applied to beams also!

* Can we derive simpler equations to obtain approximate deformation of beam like objects ??

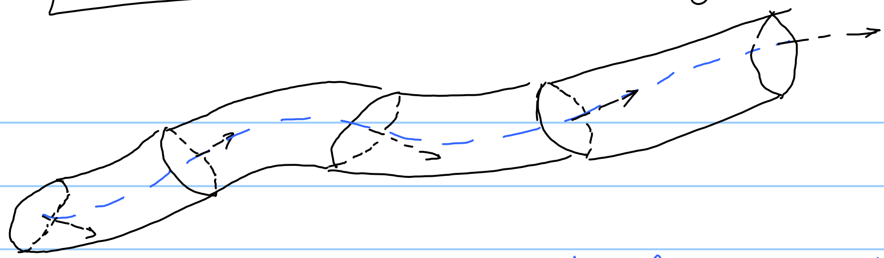




We want to obtain this def. where centerline deformation is as accurate as possible!

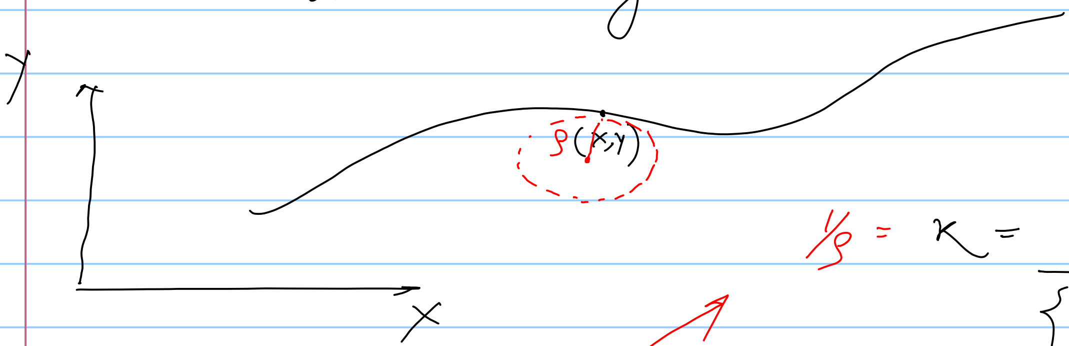
→ beam theory is about centerline and cross-section orientation!

Euler-Bernoulli beam theory



① → Cross-section normal & centerline tangent are aligned!

→ Centerline is the only unknown!



$$1/\rho = \kappa = \frac{\frac{d^2y}{dx^2}}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}} = \frac{M}{EI}$$

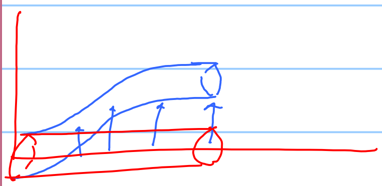
Bending of beam

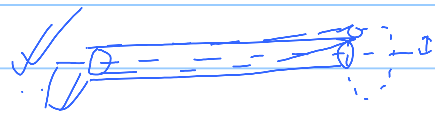
$M = EI\kappa$

$$\Rightarrow EI \frac{\frac{d^2y}{dx^2}}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}} = M$$

② $x \approx X$

$$\Rightarrow EI \frac{\frac{d^2y}{dX^2}}{\left\{1+\left(\frac{dy}{dX}\right)^2\right\}^{3/2}} = M(X)$$





③ $\left|\frac{dy}{dX}\right| \ll 1$

$1+\left(\frac{dy}{dX}\right)^2 \approx 1$

eq becomes linear in y

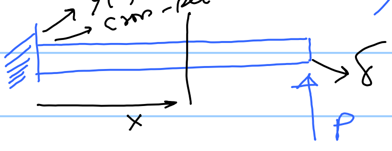
$$\boxed{EI \frac{d^2y}{dX^2} = M(X)}$$

linear & 2nd order

⇓

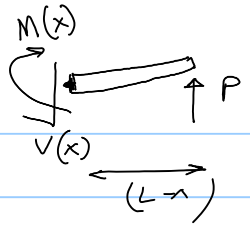
2 BCs + additional BC for unknown parameter in M(x)

A Cantilever



cross-section normal is along x ⇒ centerline tangent is along x ⇒ $\frac{dy}{dx}(0)=0$

→ First, we obtain bending moment profile



$$\Rightarrow -M(x) + P(L-x) = 0$$

$$\Rightarrow M(x) = P(L-x)$$

$$EI\frac{d^2y}{dx^2} = P(L-x) \Rightarrow \underline{\underline{2\ \text{B.C.s}}} \qquad y(0) = 0, \quad \frac{dy}{dx}(0) = 0$$

$$\Downarrow$$

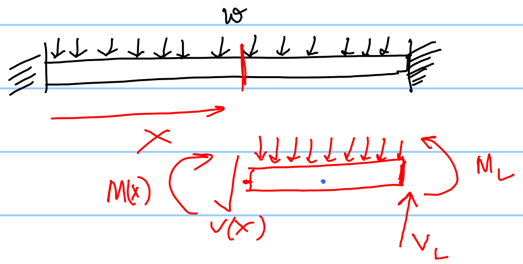
$$EI\frac{dy}{dx} = PLx - \frac{Px^2}{2} + C \Rightarrow C = 0$$

$$\Downarrow$$

$$EIy = PL\frac{x^2}{2} - P\frac{x^3}{6} + D \Rightarrow D = 0$$

$$\Rightarrow y = \frac{PL^3}{6EI}\left(3\frac{x^2}{L^2} - \frac{x^3}{L^3}\right)$$

$$\Rightarrow \boxed{\delta = y(L) = \frac{PL^3}{3EI}}$$



$$EI\frac{d^2y}{dx^2} = M(x)$$

$$\Rightarrow -M(x) + M_L + V_L(L-x) - \frac{w(L-x)^2}{2} = 0$$

$$\Rightarrow M(x) = M_L + V_L(L-x) - \frac{w(L-x)^2}{2}$$

$$EI\frac{d^2y}{dx^2} = M_L + V_L(L-x) - \frac{w(L-x)^2}{2}$$

2

4 BCs required $\Rightarrow$ $y(0) = y(L) = 0$, $\frac{dy}{dx}(0) = \frac{dy}{dx}(L) = 0$