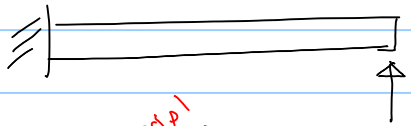
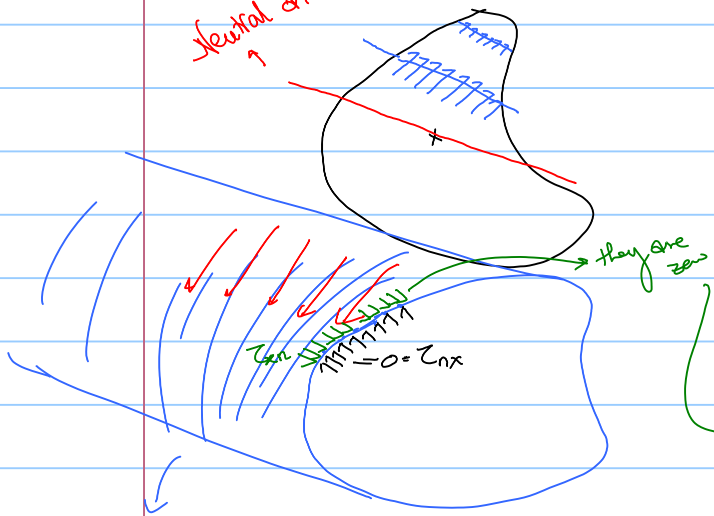
Lec 30 (Non-uniform bending of unsymmetrical beams)

Note Title 10/22/2022





$\sigma_{xx} = \,!$

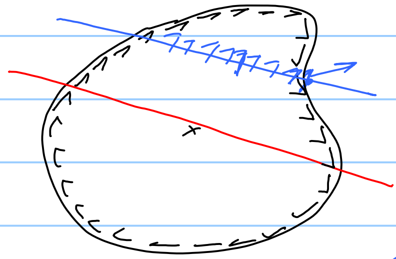
→ We want to also obtain shear stress distribution!

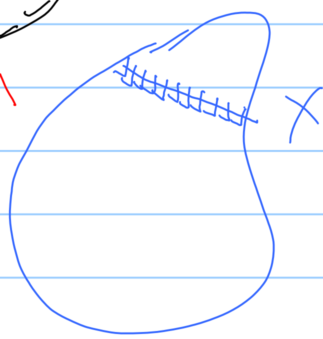
* Shear stress need not be the same on lines ∥ to neutral axis!

$\underline{\underline{\sigma}}\,\underline{n} = \underline{t}^{o} = \tau_{xn}$

$\underbrace{(\underline{\underline{\sigma}}\,\underline{n})\cdot\underline{e}_1}_{\underline{e}_1 \,\&\, \underline{n} \text{ are } \perp} = \underline{t}^{o}\cdot\underline{e}_1 = 0\,!$

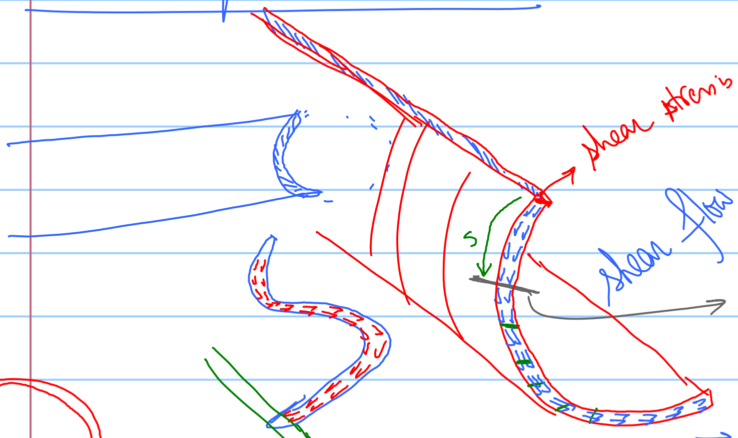
transverse load is applied on lateral surface having no component along axial direction!





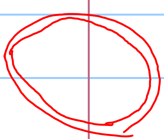
Special case

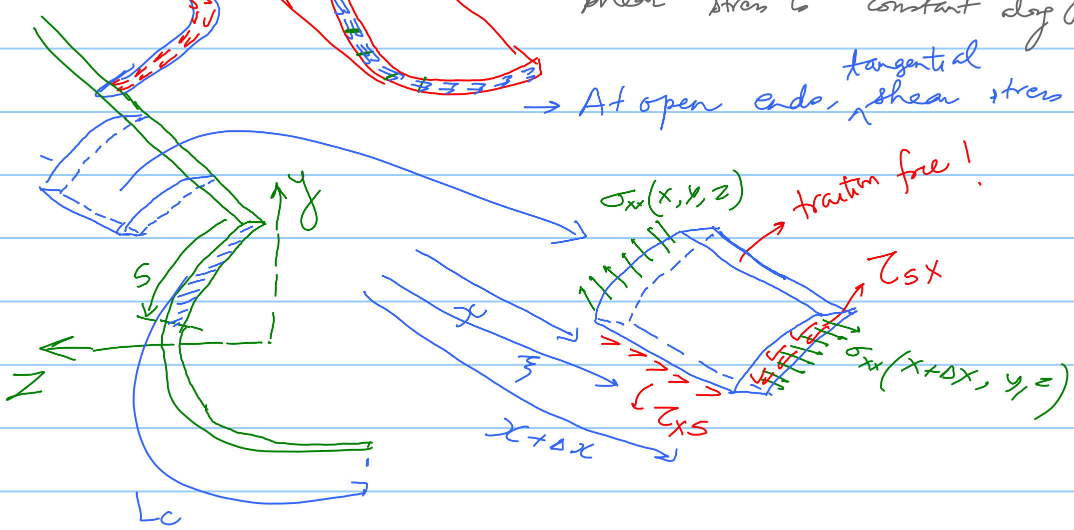
Thin & open cross-sections



Due to thinness, we can say that shear stress is constant along thickness direction!

→ At open ends, tangential shear stress vanishes!





$$F_x = \iint\limits_{\substack{x\text{-plane}\\ \text{shaded area}\\ \text{from } 0 \to s}} \left[\sigma_{xx}(x+\Delta x, y, z) - \sigma_{xx}(x,y,z)\right] dA + \iint\limits_{s\text{-plane}} \tau_{xs}(\xi, s)\, dA = 0$$

$$\iint\limits_{s\text{-plane}} \tau_{xs}(\xi,s)\,dA = t\int_{x}^{x+\Delta x} \tau_{xs}(\xi,s)\,d\xi$$

thickness

→ divide the above value by Δx & take the limit $\Delta x \to 0$

$$\iint\limits_{\substack{x\text{-plane}\\ (\text{shaded area}\\ 0 \to s)}} \frac{d\sigma_{xx}}{dx}(x,y,z)\, dA + t\,\tau_{xs}(x,s) = 0$$

$$\sigma_{xx}(x,y,z) = \frac{M_z(y I_{yy} - z I_{yz}) + M_y(y I_{yz} - z I_{zz})}{I_{yz}^2 - I_{yy} I_{zz}}$$

$$\iint\limits_{x\text{-plane}} \frac{\underbrace{M_z'}_{-V_y}(y I_{yy} - z I_{yz}) + \underbrace{M_y'}_{V_z}(y I_{yz} - z I_{zz})}{I_{yz}^2 - I_{yy} I_{zz}} + t\,\tau_{xs}(x,s) = 0$$

$$\Rightarrow \tau_{xs}(x,s) = \frac{1}{t} \iint\limits_{\substack{x\text{-plane}\\ (0\to s)}} \frac{V_y(y I_{yy} - z I_{yz}) - V_z(y I_{yz} - z I_{zz})}{I_{yz}^2 - I_{yy} I_{zz}}\, dA$$

$$\tau_{xs}(x,s) = \frac{1}{t}\, \frac{V_y(Q_y^s I_{yy} - Q_z^s I_{yz}) - V_z(Q_y^s I_{yz} - Q_z^s I_{zz})}{I_{yz}^2 - I_{yy} I_{zz}}$$

Special case: $I_{yz} = 0$, $V_z = 0$

$$\Rightarrow \boxed{\tau_{xs}(x,s) = -\frac{1}{t}\,\frac{V_y Q_y^s}{I_{zz}}}$$