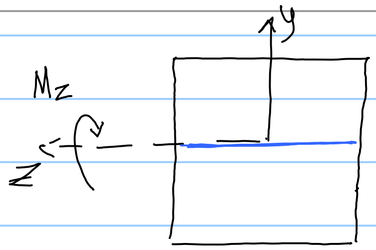
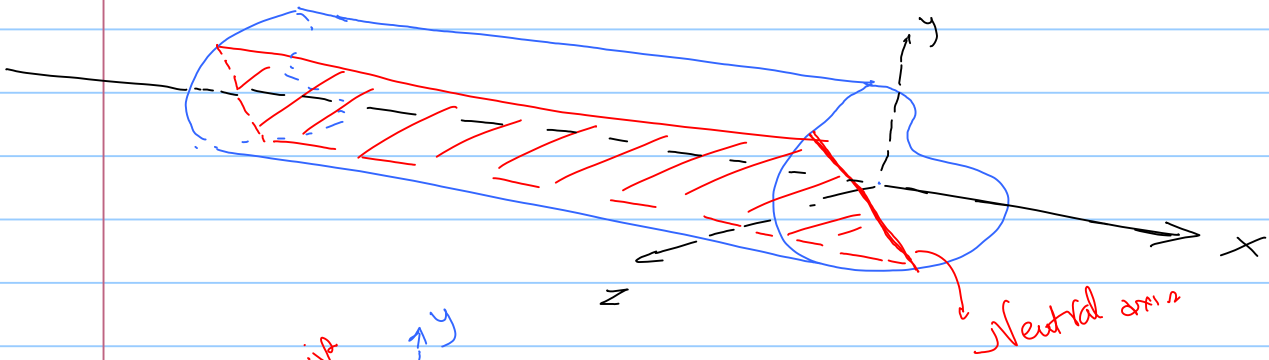
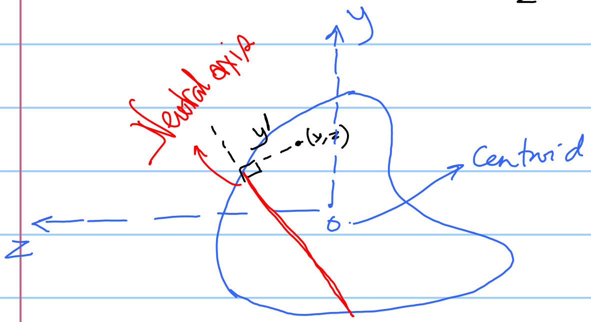
Lec 29 (Bending of unsymmetrical beams)

Note Title 10/21/2022



→ Neutral axis is $\parallel$ to applied moment direction!





$$\epsilon_{xx}(y,z) = -y'/R$$

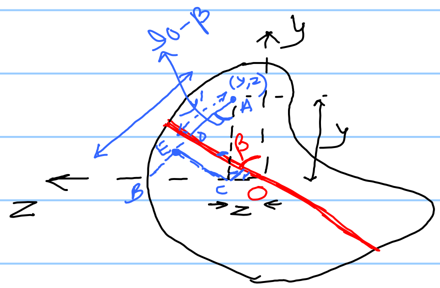
$$\sigma_{yy} = \sigma_{zz} = 0 \quad \text{(as earlier)}$$

$$\sigma_{xx} = E\,\epsilon_{xx} = -Ey'/R$$

$$F_x = \iint \sigma_{xx}\,dA = -E/R \iint y'\,dA = 0$$

$$-E/R \; y'_c\big|_{\text{Neutral axis}} A = 0$$

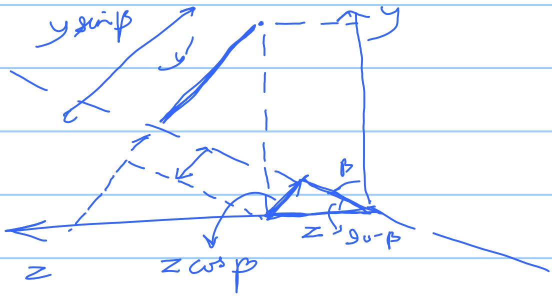
$$\Rightarrow y'_c\big|_{\text{Neutral axis}} = 0 \; !$$



In ΔABC, $AB = \dfrac{AC}{\cos(90-\beta)} = \dfrac{y}{\sin\beta}$

In ΔACE, $AE = y \sin\beta$

$$y' = y\sin\beta - z\cos\beta$$



$$\vec{M}_O = \iint (y\hat{j} + z\hat{k}) \times (\sigma_{xx}\,dA)\,\hat{i}$$

$$= -\frac{E}{R}\left[\iint y y'\,dA\,(-\hat{k}) + \iint y' z\,dA\,\hat{j}\right]$$

$$\Rightarrow \vec{M}_{/0} = \frac{E}{R}\left[\iint \left(y^2 \sin\beta - yz\cos\beta\right) dA\ \hat{k} - \iint \left(yz\sin\beta - z^2\cos\beta\right) dA\ \hat{j}\right]$$

$$= \frac{E}{R}\left[\left(I_{zz}\sin\beta - I_{yz}\cos\beta\right)\hat{k} - \left(I_{yz}\sin\beta - I_{yy}\cos\beta\right)\hat{j}\right]$$

$$\Rightarrow M_z = \frac{E}{R}\left(I_{zz}\sin\beta - I_{yz}\cos\beta\right) \Rightarrow$$

$$M_y = \frac{E}{R}\left(I_{yy}\cos\beta - I_{yz}\sin\beta\right) \Rightarrow K = \frac{M_y}{E\left(I_{yy}\cos\beta - I_{yz}\sin\beta\right)}$$

$$\Rightarrow \frac{M_z}{M_y} = \frac{I_{zz}\sin\beta - I_{yz}\cos\beta}{I_{yy}\cos\beta - I_{yz}\sin\beta} \Rightarrow \boxed{\frac{M_z}{M_y} = \frac{I_{zz}\tan\beta - I_{yz}}{I_{yy} - I_{yz}\tan\beta}}$$

> If $I_{yz} = 0$

$$\Rightarrow \frac{M_z}{M_y} = \frac{I_{zz}}{I_{yy}}\tan\beta$$

$$\Rightarrow \text{If } M_y = 0 \Rightarrow \infty = \tan\beta \Rightarrow \beta = 90^\circ$$

$$\Rightarrow \text{If } M_z = 0 \Rightarrow 0 = \tan\beta \Rightarrow \beta = 0$$

$$\text{If } M_y = 0 \Rightarrow \infty = \frac{I_{zz}\tan\beta - I_{yz}}{I_{yy} - I_{yz}\tan\beta} \Rightarrow \frac{I_{yy}}{I_{yz}} = \tan\beta$$

$$K = \frac{1}{R} = \quad , \quad \sigma_{xx} = -\frac{E}{R}y' = -EK\left(y\sin\beta - z\cos\beta\right)$$

$$= -\frac{M_y\left(y\sin\beta - z\cos\beta\right)}{\left(I_{yy}\cos\beta - I_{yz}\sin\beta\right)}$$

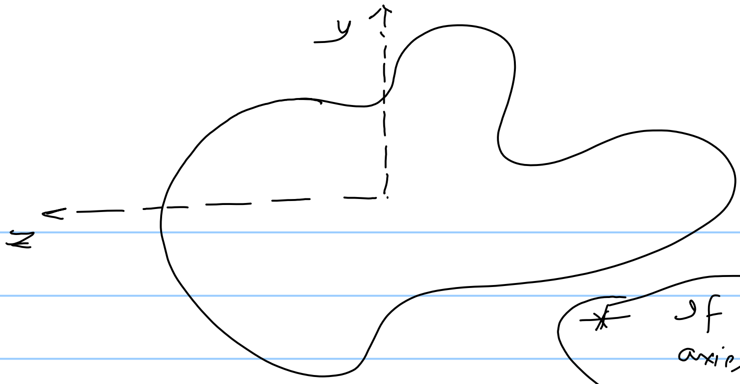
$$= -\frac{M_y\left(y\tan\beta - z\right)}{\left(I_{yy} - I_{yz}\tan\beta\right)}$$

$$\Rightarrow \sigma_{xx} = \frac{M_z\left(yI_{yy} - zI_{yz}\right) + M_y\left(yI_{yz} - zI_{zz}\right)}{I_{yz}^2 - I_{yy}I_{zz}} \qquad \left(-\frac{M_{zz}y}{I_{zz}}\right)$$

$M_y = 0$, $I_{yz} = 0$: $\quad -\dfrac{M_z y}{I_{zz}}$

$M_z = 0$, $I_{yz} = 0$: $\quad \dfrac{M_y z}{I_{yy}}$

$I_{yz} = 0$: $\quad -\dfrac{M_z y}{I_{zz}} + \dfrac{M_y z}{I_{yy}}$



* If Coordinate system is aligned with principal axis, then the beam acts like a symmetrical beam

* If you apply moment abt principal axis, then neutral axis is aligned with same principal axis!