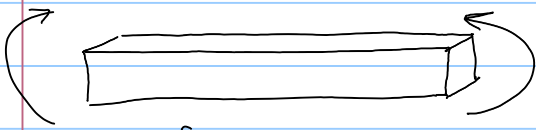
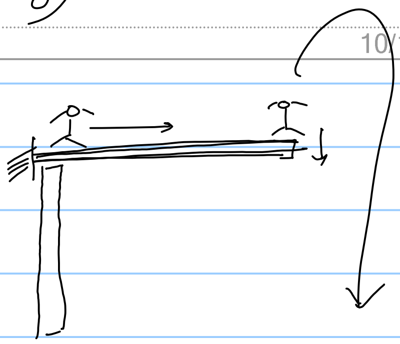
Lec 28 (Non-uniform bending)

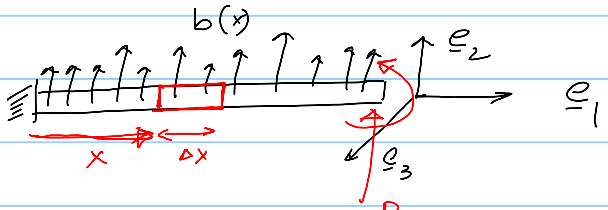
Note Title 10/19/2022

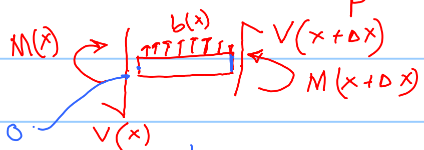


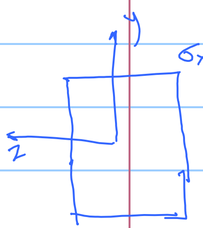


Uniform bending.

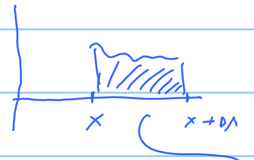
$$\sigma_{xx} = -\frac{M_z\, y}{I_{zz}} = -E\,\frac{y}{R}$$







σ_{xx}, τ_{yx}, τ_{zx}



Force balance:

$$\lim_{\Delta x\to 0}\frac{V(x+\Delta x) - V(x) + \int_x^{x+\Delta x} b(\xi)\,d\xi}{\Delta x}$$

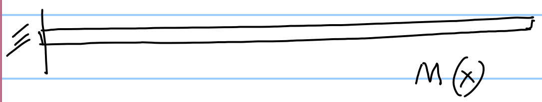
$$\Rightarrow \quad V' + b(x) = 0$$

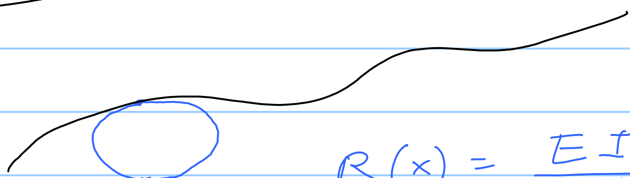
$$\overrightarrow{M}/_O = \lim_{\Delta x\to 0}\frac{M(x+\Delta x) - M(x) + V(x+\Delta x)\,\Delta x + \int_x^{x+\Delta x}(\xi - x)\, b(\xi)\,d\xi}{\Delta x} = 0$$

$$\Rightarrow \quad M' + V(x) + \cancel{\lim_{\Delta x\to 0}\frac{\int_x^{x+\Delta x}(\xi-x)\, b(\xi)\,d\xi}{\Delta x}}^{\,0}$$

$$\Rightarrow \quad \frac{dM}{dx} + V(x) = 0 \quad !$$

$$R = \frac{E I_{zz}}{M_z}$$

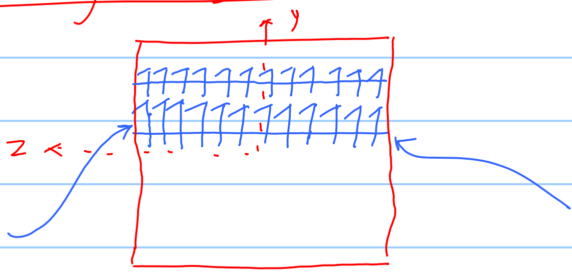




$$R(x) = \frac{E I_{zz}}{M_z(x)}$$

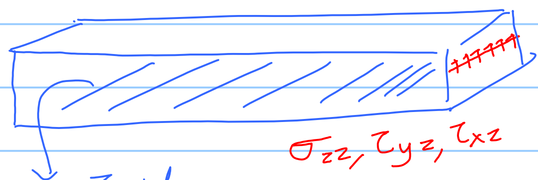
$$\sigma_{xx}(x,y,z) = -\frac{M_z(x)\, y}{I_{zz}}$$

Distribution of τ_{yx}



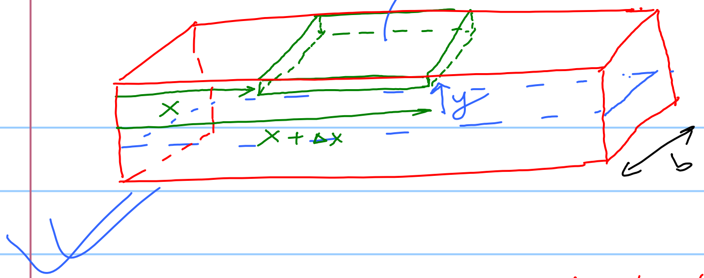
$\tau_{yx}(x,y)$

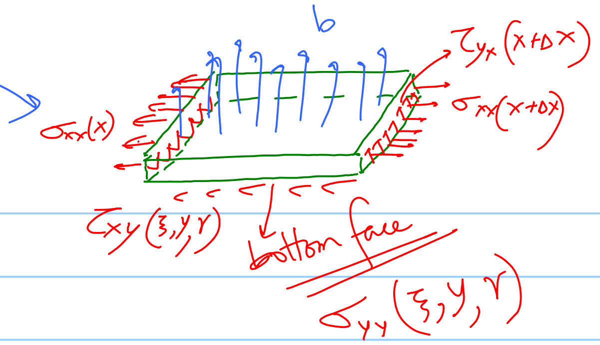
τ_{yx} is independent of z !



σ_{zz}, τ_{yz}, τ_{xz}

Z-plane





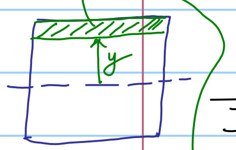
Force balance in x-direction!

$$\iint_{x\text{-face}} \left[\sigma_{xx}(x+\Delta x,\eta,\gamma) - \sigma_{xx}(x,\eta,\gamma)\right] dA + \iint_{-y\text{ face}} -\tau_{xy}(\xi,y,\gamma)\, dA$$

$$\iint_{x\text{-face}} \left[\frac{-M_z(x+\Delta x)\eta + M_z(x)\eta}{I_{zz}}\right] d\eta\, d\gamma - \iint_{-y\text{ face}} \tau_{xy}(\xi,y)\, d\xi\, d\gamma$$

$$- \int_{x}^{x+\Delta x} \tau_{xy}(\xi,y)\, d\xi \int_{-b/2}^{b/2} d\gamma$$

Divide by Δx and let $\Delta x \to 0$

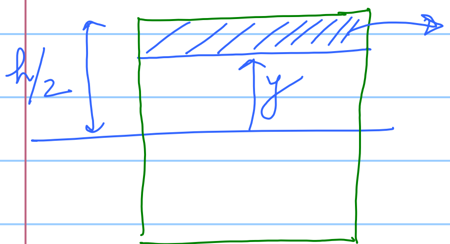


$$\frac{V(x)}{I_{zz}} \iint_{x\text{-face}} \eta\, dA - \lim_{\Delta x \to 0} \frac{b}{\Delta x} \int_{x}^{x+\Delta x} \tau_{xy}(\xi,y)\, d\xi$$

$$\Rightarrow \frac{V(x)}{I_{zz}} \iint \eta\, dA - b\, \tau_{xy}(x,y) = 0$$

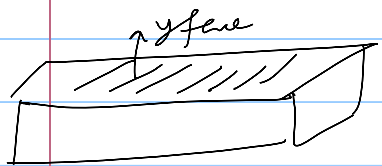
1st moment of shaded area!

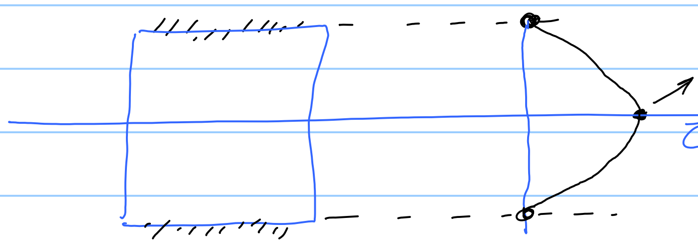
$$\Rightarrow \boxed{\tau_{xy}(x,y) = \frac{V(x)\, Q_y(y)}{I_{zz}\, b}}$$



$$\left[y + \tfrac{1}{2}\left(\tfrac{h}{2} - y\right)\right] \cdot b \left(\tfrac{h}{2} - y\right) = Q_y$$

$$I_{zz} = \tfrac{1}{12} b h^3$$





$$\tau_{xy}(y=0) = \frac{3}{2}\frac{V}{bh}$$

$$\tau_{xy} = \tau_{yx}$$