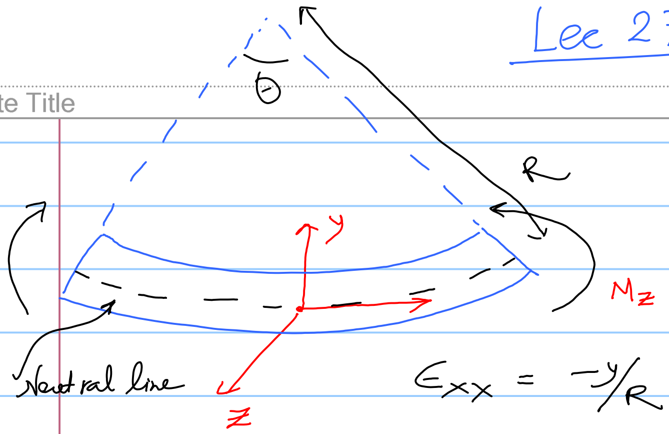
Note Title 10/18/2022

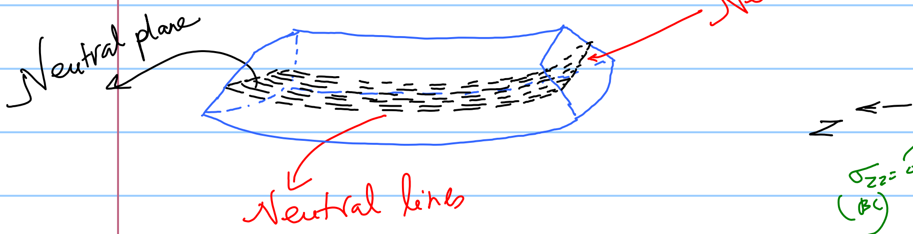
Lec 27 (pure bending of beams)

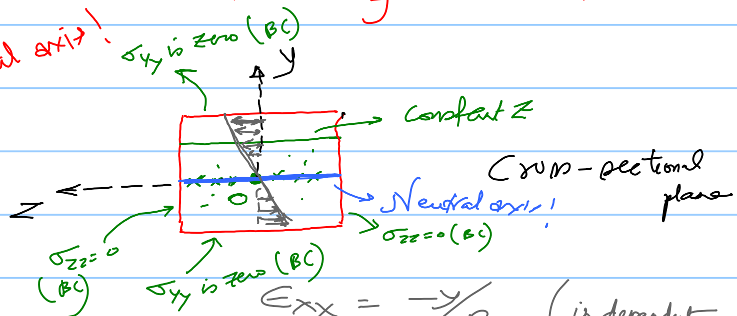
* Only bending moment is present in any internal section (no force!)



$\epsilon_{xx} = -y/R$

* unknowns are R and also the location of neutral line





$\epsilon_{xx} = -y/R$ (independent of z)

What is σ_{xx}?

$$\epsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})\right)$$

Approximation: $\sigma_{yy} = \sigma_{zz} = 0$ everywhere in the cross-section!

$$\sigma_{xx} = E\,\epsilon_{xx} = \underline{\underline{-E\,y/R}}$$

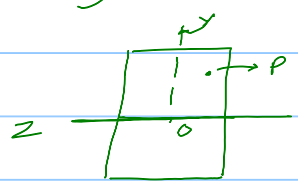
$$F_x = \iint_{\text{cross-sectional plane}} \sigma_{xx}\,dA = -\frac{E}{R}\iint_{\text{cross-secth}} y\,dA = -\frac{E}{R}\,y_c\,A = 0$$

$$\Rightarrow y_c = 0!$$

$\Rightarrow$ Neutral axis passes through the centroid of the cross-section!

$$\vec{M}_{/O} = \iint_{\text{cross-s}} \vec{r}_{P/O} \times \vec{E}(P)\,dA$$

(O: centre of cross-section)



$$= \iint (y\hat{j}+z\hat{k}) \times (\sigma_{xx}\hat{i} + \tau_{yx}\hat{j} + \tau_{zx}\hat{k})\,dA$$

$$M_z = \vec{M}_{/O}\cdot\hat{k} = \left[\iint (y\hat{j}+z\hat{k}) \times (\sigma_{xx}\hat{i} + \tau_{yx}\hat{j} + \tau_{zx}\hat{k})\,dA\right]\cdot\hat{k}$$

do not contribute to final result

$$= \left[\iint \left[y\hat{j} \times (\sigma_{xx}\hat{i} + \tau_{yx}\hat{j})\right]\right] \cdot \hat{k}\, dA$$

0

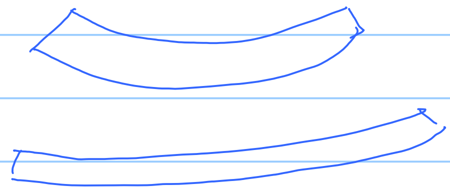
$$= -\iint y\,\sigma_{xx}\, dA$$

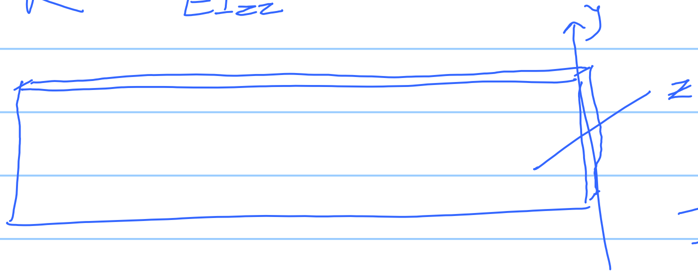
$$M_z = \frac{E}{R}\iint_{\text{cross-sect}} y^2 dA = \frac{E I_{zz}}{R}$$

$$\Rightarrow \boxed{R = \frac{E I_{zz}}{M_z}}$$

bending curvature

$$K = \frac{1}{R} = \frac{M_z}{E I_{zz}}$$





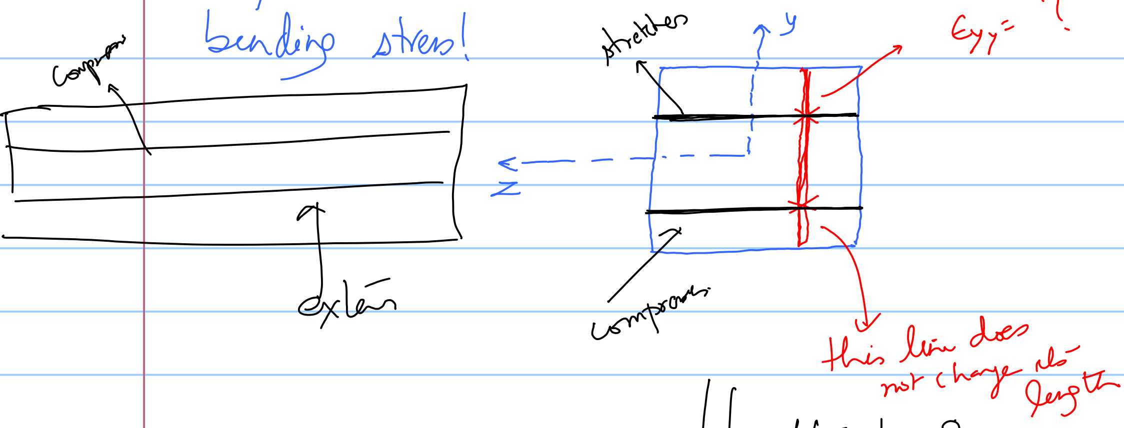
$$R = \frac{E I_{yy}}{M_y}$$

$$I_{zz} = \iint y^2 dA$$

$$I_{yy} = \iint z^2 dA$$

$$\sigma_{xx} = \frac{-Ey}{R} = -\frac{M_z\, y}{I_{zz}}$$

bending stress!



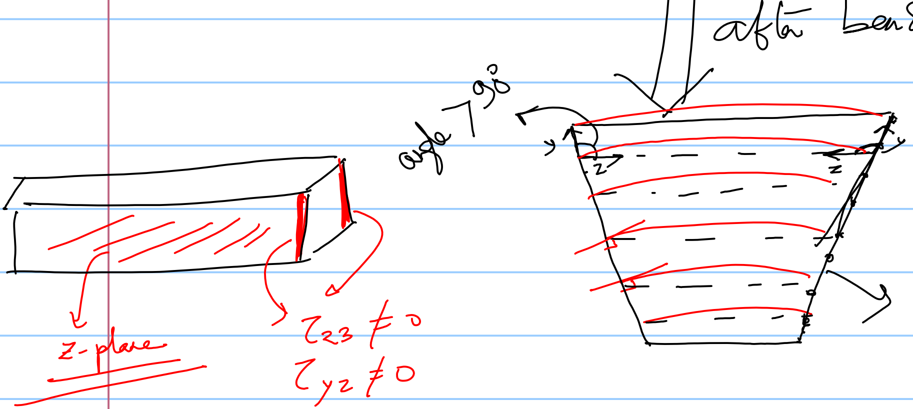
$\epsilon_{yy} = ?$

$$\epsilon_{yy} = \frac{1}{E}\left(\cancel{\sigma_{yy}} - \nu(\sigma_{xx} + \cancel{\sigma_{zz}})\right)$$

$$= -\nu\frac{\sigma_{xx}}{E} = -\nu\,\epsilon_{xx}$$

$$\epsilon_{yy} = -\nu\,\epsilon_{xx} = \nu y/R$$

$$\epsilon_{zz} = -\nu\,\epsilon_{xx} = \nu y/R$$



$\gamma_{23} < 0$

$\gamma_{23} < 0 \Rightarrow \tau_{23} < 0$

$\tau_{yz} < 0$