Lec 26 (Extension-torsion-inflation)

Note Title 10/12/2022

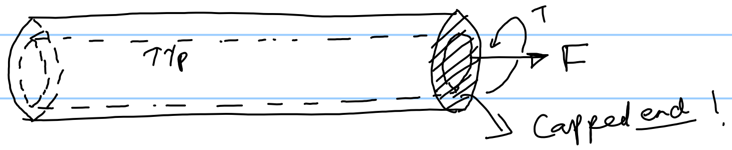
Torque $\longleftrightarrow$ end-to-end rotation

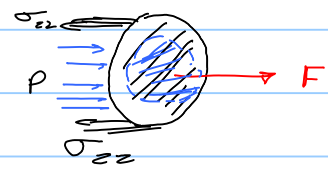
T $\qquad\qquad$ Ω

$$T = \underbrace{GJ}_{\text{Torsional stiffness}} \underbrace{\kappa}_{\text{twist}} = GJ\frac{\Omega}{L} \Rightarrow \boxed{\Omega = \frac{TL}{GJ}}$$

Relating Axial force to axial strain

F $\qquad\qquad$ ϵ





Net force balance in z-direction

$$F + \pi r_1^2 p - \iint_{\text{cross-section of cylinder}} \sigma_{zz}\, dA = 0$$

$$\sigma_{zz} = \lambda\left(u_r' + \frac{u_r}{r} + \epsilon\right) + 2\mu\epsilon$$

$$= \lambda c + (\lambda + 2\mu)\epsilon$$

$$\Rightarrow F = -\pi r_1^2 p - \{\lambda c + (\lambda+2\mu)\epsilon\} A$$

$$c = -\frac{\lambda}{\lambda+\mu}\epsilon + \frac{p}{(\lambda+\mu)}\frac{r_1^2}{r_2^2 - r_1^2}$$

$$= -\pi r_1^2 p + \underbrace{\left[\frac{\lambda^2}{\lambda+\mu} - (\lambda+2\mu)\right]}_{E}\epsilon A$$

$$- \frac{\lambda}{(\lambda+\mu)}\frac{p r_1^2}{(r_2^2 - r_1^2)} A$$

$$\boxed{F = EA\epsilon - \pi r_1^2 p\,[1 + 2\nu]}$$

$$\Rightarrow \epsilon = \frac{F + \pi r_1^2 p(1+2\nu)}{EA}$$

When $p = 0$ $\Rightarrow$ $F = EA\epsilon = EA\frac{\Delta l}{l}$

$$\Rightarrow \Delta l = \frac{Fl}{EA}$$

$$\Omega = \frac{Tl}{GJ}$$

$$\text{inflation} = \left.\frac{\Delta r}{r}\right|_{r=r_1} = \frac{u_r(r_1)}{r_1} = \frac{\left[-\nu\epsilon + \frac{P}{2(\lambda+\mu)}\frac{r_1^2}{r_2^2-r_1^2}\right]r_1 + \frac{P}{2\mu r_1}\frac{r_1^2 r_2^2}{r_2^2-r_1^2}}{r_1}$$

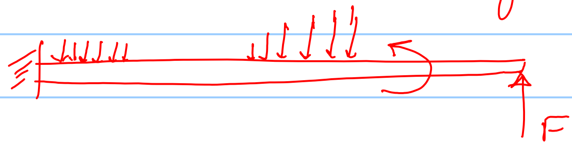
$$= -\nu\epsilon + \frac{P r_1^2}{2(\lambda+\mu)\,(r_2^2-r_1^2)} + \frac{P r_2^2}{2\mu\,(r_2^2-r_1^2)}$$

$$\sigma_{rr} = A/2 + B/r_2^2$$
$$\sigma_{\theta\theta} = A/2 - B/r^2$$

* What happens to all the formulas when $r_2 \simeq r_1$ $\quad \boxed{r_2 - r_1 = t}$

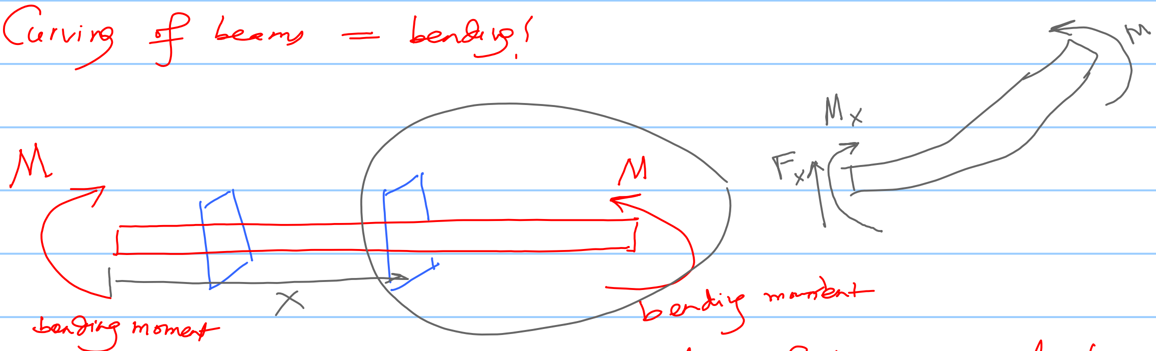
Thin tube approximation $\left(\frac{t}{r_1} \ll 1\right)$

Bending of beams



Shear force diagram
bending moment diagram

→ Curving of beams = bending!



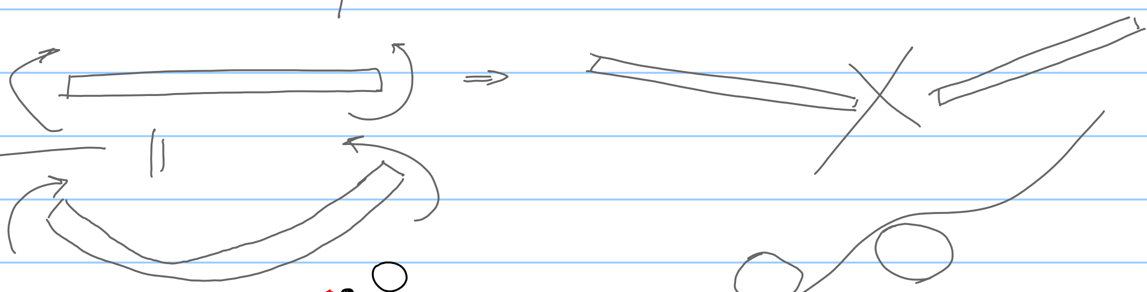
Torque → acts along length of beam → twist

Bending moment → acts ⊥ to axis of beam → bending

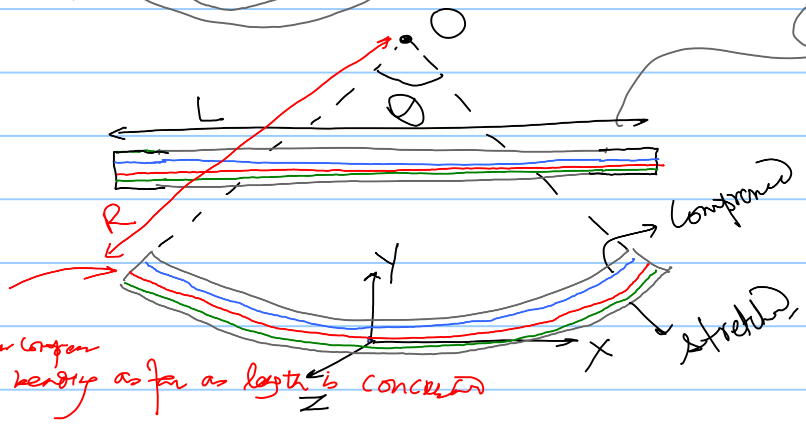
→ ~~Same~~ Same moment everywhere!

→ No internal force in the cross-section!

Pure bending!



it has to bend into a part of a perfect circle



length of fiber: $R\Theta$

neither stretch/nor compress
neutral to bending as far as length is concerned

$$\epsilon_b = \frac{l_b - L}{L} = \frac{(R-y)\theta - R\theta}{R\theta} = -y/R$$

* Radius is unknown!
* location of red fiber!