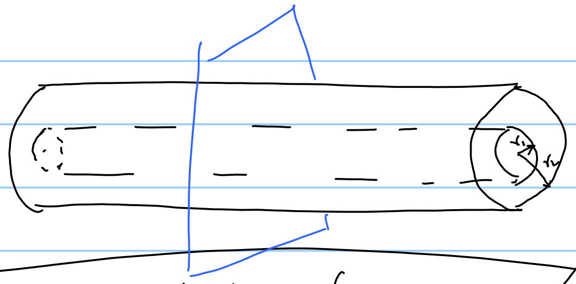
Note Title 10/11/2022

Lec 25

Extension-torsion-inflation in cylinders.

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} = 0 \quad \Rightarrow \quad \boxed{u_r' + \frac{u}{r} = C} \quad \text{or} \quad \boxed{\epsilon_{rr} + \epsilon_{\theta\theta} = C}$$

$$\Rightarrow u_r = \frac{Cr}{2} + \frac{D}{r}$$



$$\sigma n = 0 \quad (r = r_2), \qquad n = e_r$$

$$\begin{bmatrix} \sigma_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{r\theta} & \sigma_{\theta\theta} & \tau_{\theta z} \\ \tau_{rz} & \tau_{\theta z} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\sigma_{rr}(r_2) = 0}$$

$$\tau_{r\theta}(r_2) = 0$$

$$\tau_{rz}(r_2) = 0$$

On inner lateral surface

$$\sigma(-e_r) = p e_r$$

$$\begin{bmatrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{\sigma_{rr}(r_1) = -p}$$

$$\tau_{r\theta}(r_1) = 0$$

$$\tau_{rz}(r_1) = 0$$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} = 0,$$

$$\sigma_{rr} = (\lambda + 2\mu) u_r' + \lambda\left(\frac{u_r}{r} + \epsilon\right)$$

$$\sigma_{\theta\theta} = (\lambda + 2\mu)\frac{u_r}{r} + \lambda(u_r' + \epsilon)$$

$$\sigma_{rr} + \sigma_{\theta\theta} = 2(\lambda+\mu)\left(u_r' + \frac{u_r}{r}\right) + 2\lambda\epsilon$$

$$\boxed{= 2(\lambda+\mu)C + 2\lambda\epsilon = A}$$

$$\frac{d\sigma_{rr}}{dr} + 2\frac{\sigma_{rr}}{r} - \frac{(\sigma_{rr}+\sigma_{\theta\theta})}{r} = 0$$

$$\Rightarrow \frac{d\sigma_{rr}}{dr} + 2\frac{\sigma_{rr}}{r} - \frac{A}{r} = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\sigma_{rr}\right) = \frac{A}{r}$$

$$\Rightarrow \frac{d}{dr}\left(r^2\sigma_{rr}\right) = Ar$$

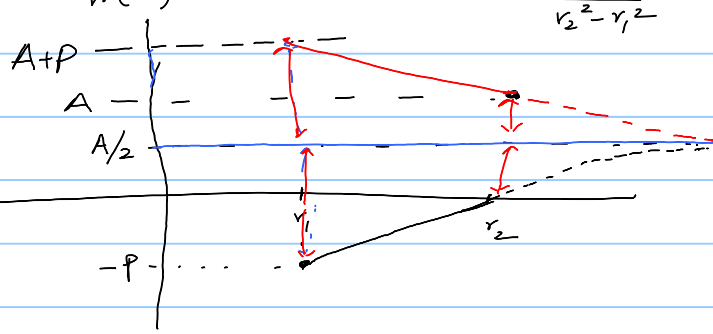
$$\Rightarrow r^2\sigma_{rr} = \frac{Ar^2}{2} + B$$

$$\Rightarrow \sigma_{rr} = \frac{A}{2} + \frac{B}{r^2}$$

$A,\ r \to \infty$, $\sigma_{rr} \to A/2$

$$\sigma_{\theta\theta} = \frac{A}{2} - \frac{B}{r^2}$$

$$\sigma_{rr}(r_1) = -p, \quad \sigma_{rr}(r_2) = 0 \quad \Rightarrow \quad A = 2p\frac{r_1^2}{r_2^2 - r_1^2}, \quad B = -p\frac{r_1^2 r_2^2}{r_2^2 - r_1^2}$$



* σ_{rr} & $\sigma_{\theta\theta}$ vanish if applied pressure is zero

$$u_r = \frac{Cr}{2} + \frac{D}{r}, \qquad 2(\lambda+\mu)C + 2\lambda\epsilon = A$$

$$\Rightarrow C = -\frac{\lambda}{(\lambda+\mu)}\epsilon + \frac{A}{2(\lambda+\mu)}$$

$$\sigma_{rr} = (\lambda+2\mu)\, u_r' + \lambda\left(\frac{u_r}{r} + \epsilon\right)$$

$$= (\lambda+2\mu)\left(\frac{C}{2} - \frac{D}{r^2}\right) + \lambda\left(\frac{C}{2} + \frac{D}{r^2} + \epsilon\right)$$

$$= (\lambda+\mu)\, C + \lambda\epsilon - 2\mu \frac{D}{r^2}$$

$$= \frac{A}{2} + \frac{B}{r^2}$$

$$\Rightarrow \quad B = -2\mu D \quad \text{or,} \quad D = \frac{-B}{2\mu}$$

$$u_r = \left[-\frac{\lambda}{2(\lambda+\mu)}\epsilon + \frac{P}{2(\lambda+\mu)} \frac{r_1^2}{r_2^2 - r_1^2}\right] r + \frac{P}{2\mu r} \frac{r_1^2 r_2^2}{r_2^2 - r_1^2}$$

if $P = 0$

$$\Rightarrow \quad u_r = -\frac{\lambda}{2(\lambda+\mu)}\epsilon\, r = -\nu\epsilon$$

$$\Rightarrow \quad \frac{u_r}{r} = u_r' = -\nu\epsilon$$

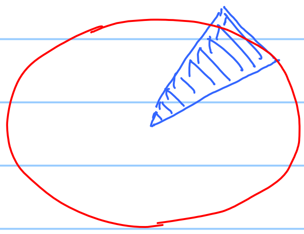
$$u_z = \epsilon z$$

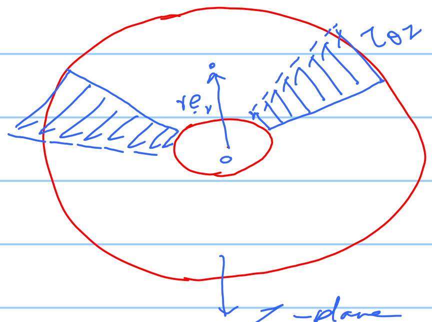
twist $= \Omega/L$

$$u_\theta = \frac{\Omega}{L} r z = \kappa r z$$

$$\gamma_{\theta z} = \frac{\partial u_\theta}{\partial z} + \cancel{\frac{1}{r}\frac{\partial u_z}{\partial \theta}}^{\,0} = \frac{\Omega}{L} r = \kappa r$$

$$\tau_{\theta z} = G\kappa r$$





$$\vec{M}/_o = \iint_{\text{Cross-Area}} r e_r \times (\sigma_{zz} e_z + \tau_{\theta z} e_\theta)\, r\, dr\, d\theta \qquad \tau_{\theta z},\ \sigma_{zz},\ \cancel{\tau_{rz}}^{\,0}$$

$$= -\iint r^2 \sigma_{zz} e_\theta\, dr\, d\theta + \left(\iint r^2 \tau_{\theta z}\, dr\, d\theta\right) e_z$$

$$= -\int_{r_1}^{r_2} r^2 \sigma_{zz}\, dr \cancel{\int_0^{2\pi} e_\theta\, d\theta}^{\,0} + \left(G\kappa \iint r^2 \underbrace{r\, dr\, d\theta}_{dA}\right) e_z$$

$$= GJ\kappa\, e_z$$

J: polar moment of area

$$\Rightarrow \quad \vec{M}/_o = GJ\kappa\, e_z$$

$$\Rightarrow \quad T e_z = GJ\kappa\, e_z$$

$$\boxed{T = GJ\kappa}$$

GJ: torsional stiffness!

$$T = GJ\frac{\Omega}{L} \quad \Rightarrow \quad \boxed{\Omega = \frac{TL}{GJ}}$$

$\boxed{F = \underbrace{EA}_{\text{Extensional stiffness / stretching stiffness!}} \in}$ when there is no pressure