Cylindrical coordinate system

Note Title 10/7/2022

Strain matrix :

radial strain

ϵ_r

$$[\epsilon]_{(r,\theta,z)} = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{2}\left[\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) + \frac{\partial u_\theta}{\partial r}\right] & \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \\ & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) & \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \\ & & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

shear strains.

normal strains

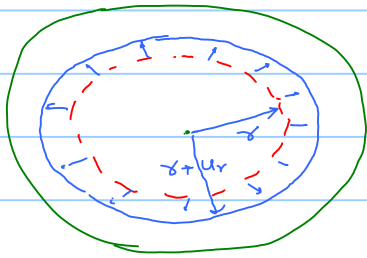
Circumferential strain

hoop strain $\epsilon_{\theta\theta}$

axial strain

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right)$$

u_r
$u_\theta = 0$
$u_z = 0$



$L = 2\pi r$

$l = 2\pi(r + u_r)$

$\frac{l-L}{L} = \frac{u_r}{r}$

$$\gamma_{r\theta} = 2\epsilon_{r\theta} = \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) + \frac{\partial u_\theta}{\partial r}$$

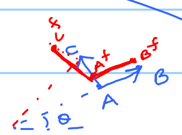
$u_r = 0$

$u_\theta \neq 0 = \text{constant}$

$u_z = 0$

$\underline{u} = u_\theta \underline{e}_\theta$ (constant over u_θ)

(not a rigid displacement because $\underline{e}_\theta$ is changing)



$$\gamma_{r\theta} = -\frac{u_\theta}{r}$$

$\underline{\sigma} = \underline{\underline{C}}\,\underline{\epsilon}$

$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl}$

$\hat{\sigma}_{ij} = \hat{C}_{ijkl}\,\hat{\epsilon}_{kl}$ for isotropic material

$C_{ijkl} = \hat{C}_{ijkl}$

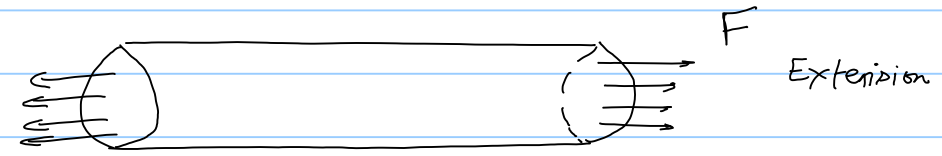
$$\epsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right)$$

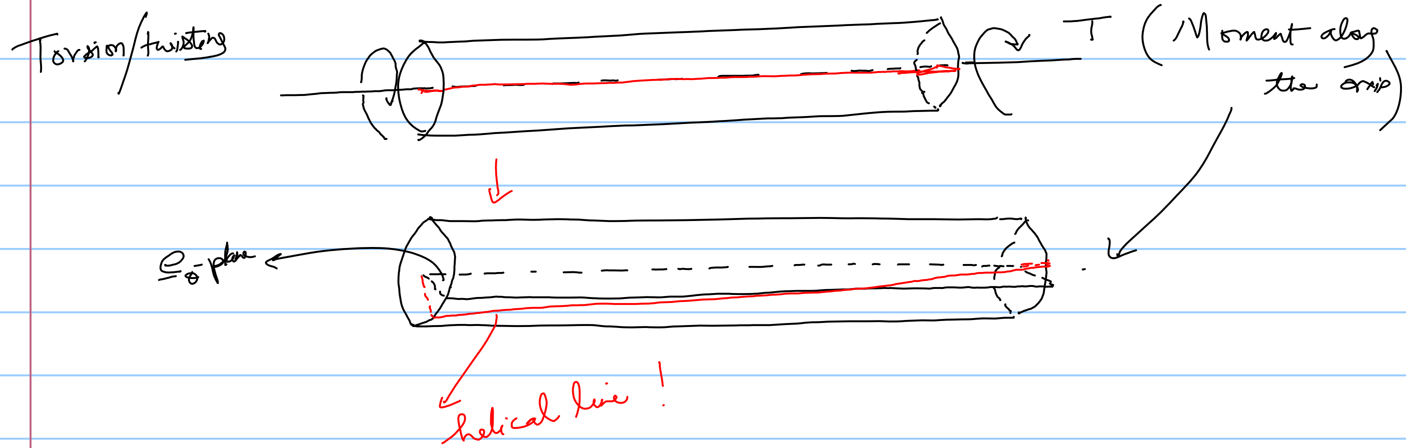
$$\epsilon_{\theta\theta} =$$

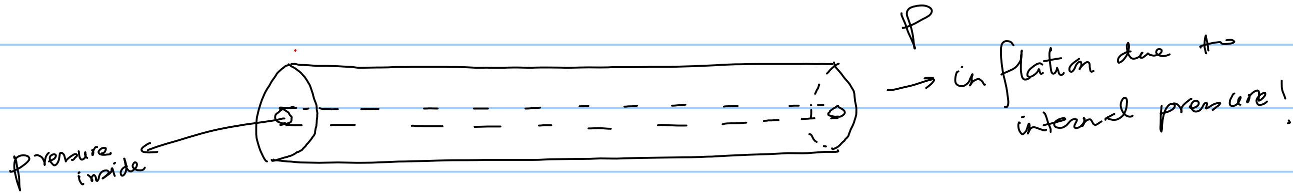
$$\epsilon_{zz} =$$

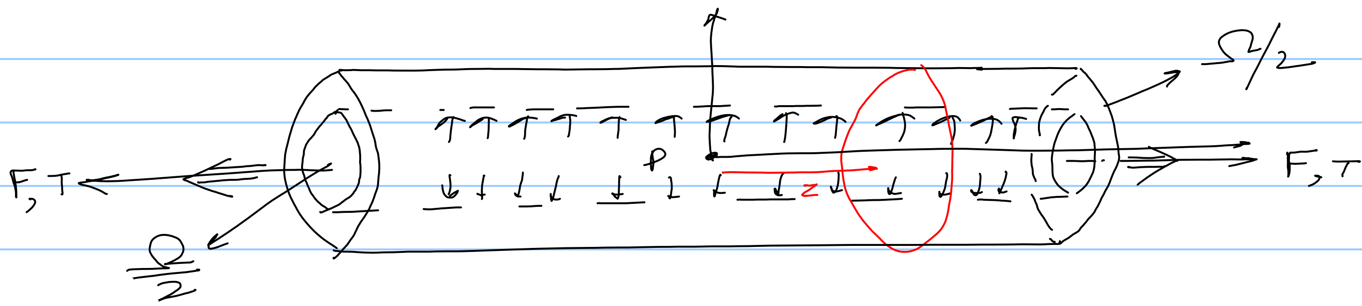
$$\gamma_{r\theta} = \frac{\tau_{r\theta}}{G}, \quad - \; - \; - \; . \; .$$

Extension torsion inflation in cylinders









Simplifications:-

Axisym.

$u_r(r,\theta,z) \Rightarrow u_r(r,z)$

$u_\theta(r,\theta,z) \Rightarrow u_\theta(r,z)$

$u_z(r,\theta,z) \Rightarrow u_z(r,z)$

Axisymmetry

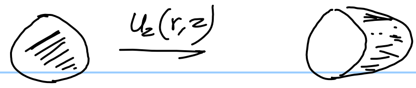
— no θ dependence!

→ No warping of cross-section!

→ plane section remaining plane

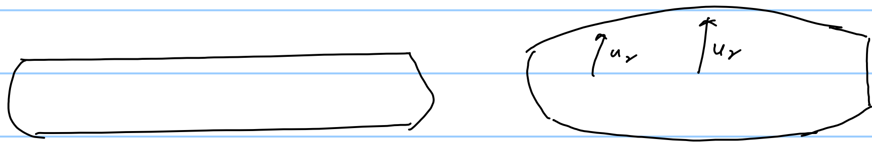
$\Downarrow$

$u_z(z)$

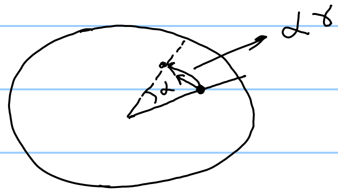


Axial homogeneity:

$u_r(r,z) \Rightarrow u_r(r)$



u_θ arises only due to torsion:



end-to-end rotation!

$$u_\theta = \underbrace{\alpha}_{= \frac{\Omega}{L} z} r$$

$$\boxed{\begin{aligned} u_\theta &= \frac{\Omega}{L} r z \\ u_r &= u_r(r) \\ u_z &= u_z(z) \end{aligned}}$$