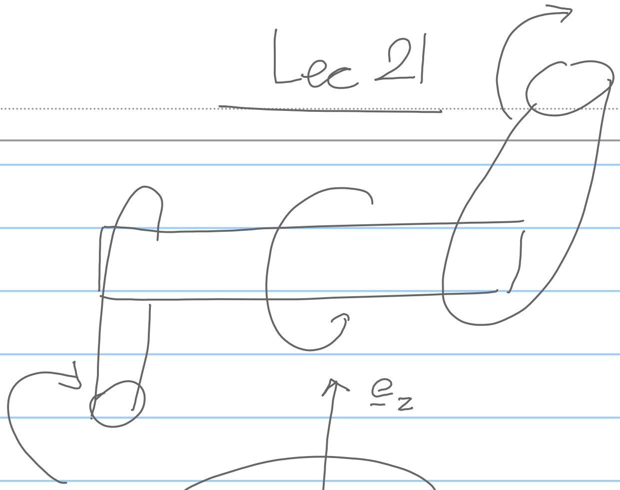
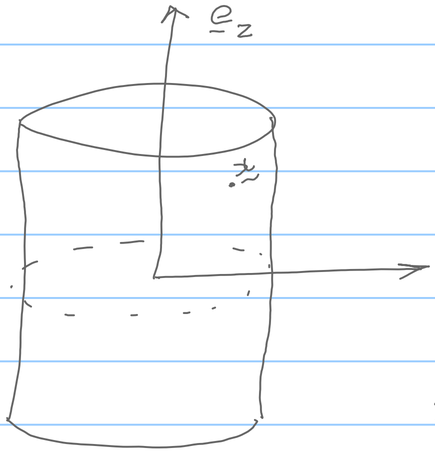
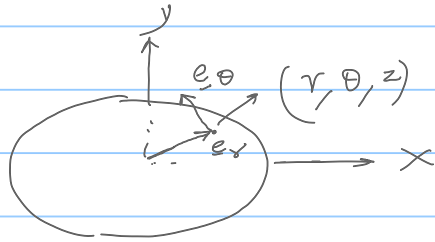
Note Title 9/21/2022

Lec 21



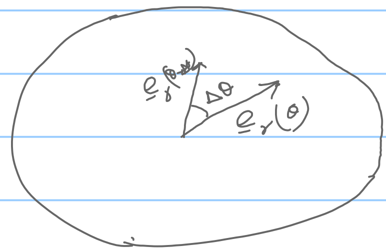




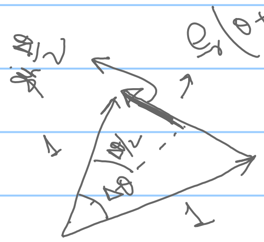
$$\underset{\sim}{x} = x_1 e_1 + x_2 e_2 + x_3 e_3$$
$$= r e_r + z e_z$$

→ e_r, e_θ change..

$e_r(\theta)$, $e_\theta(\theta)$



$$\frac{de_r}{d\theta} = e_\theta \qquad e_r(\theta+\Delta\theta) - e_r(\theta) = 2\sin\frac{\Delta\theta}{2}\, e_t$$



$$\frac{de_r}{d\theta} = \lim_{\Delta\theta\to 0} \frac{e_r(\theta+\Delta\theta) - e_r(\theta)}{\Delta\theta}$$
$$= \lim_{\Delta\theta\to 0} \frac{2\sin\frac{\Delta\theta}{2}}{\Delta\theta}\, e_t$$
$$= e_\theta$$

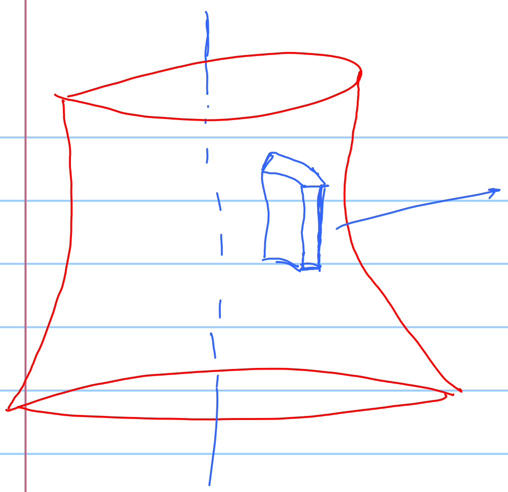
$$\frac{de_\theta}{d\theta} = -e_r$$

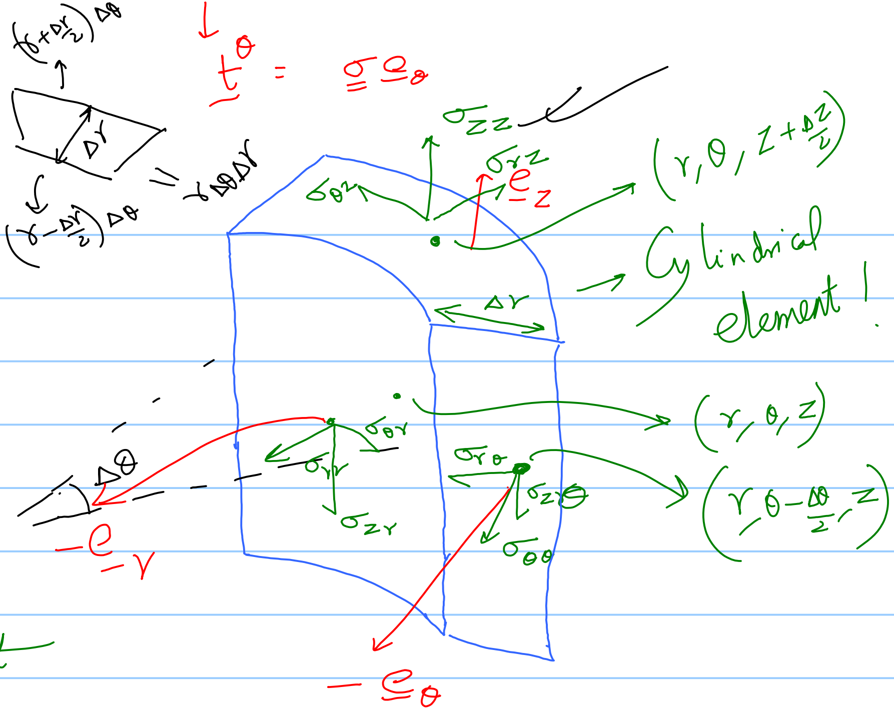
$$\underset{=}{\sigma} = \sum_i \sum_j \sigma_{ij}\, e_i \otimes e_j$$

$$= \sigma_{rr}\, e_r \otimes e_r + \sigma_{r\theta}\, e_r \otimes e_\theta + \sigma_{rz}\, e_r \otimes e_z \dots$$

$$[\underset{=}{\sigma}]_{(e_r, e_\theta, e_z)} = \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{\theta r} & \sigma_{\theta\theta} & \sigma_{\theta z} \\ \sigma_{zr} & \sigma_{z\theta} & \sigma_{zz} \end{bmatrix} \qquad \underset{=}{\sigma}\, e_z$$

$$[t^r] = [\underset{=}{\sigma}\, e_r]$$





LMB for Cylindrical element

$\sum \underline{F}^{ext}$

Start with traction force..

$$\underline{t}^{+z} = \sigma_{zz}\left(r,\theta,z+\frac{\Delta z}{2}\right)\underline{e}_z + \sigma_{\theta z}\left(r,\theta,z+\frac{\Delta z}{2}\right)\underline{e}_\theta(\theta) + \sigma_{rz}\left(r,\theta,z+\frac{\Delta z}{2}\right)\underline{e}_r(\theta)$$

$$\underline{t}^{-z} = -\sigma_{zz}\left(r,\theta,z-\frac{\Delta z}{2}\right)\underline{e}_z - \sigma_{\theta z}\left(r,\theta,z-\frac{\Delta z}{2}\right)\underline{e}_\theta(\theta) - \sigma_{rz}\left(r,\theta,z-\frac{\Delta z}{2}\right)\underline{e}_r(\theta)$$

$$A^{+z} = A^{-z} = r\Delta\theta\Delta r$$

$$\underline{F}^{+z} = r\Delta\theta\Delta r\left[\sigma_{zz}(r,\theta,z)\underline{e}_z + \frac{\partial\sigma_{zz}}{\partial z}\frac{\Delta z}{2}\underline{e}_z + \cdots\right]$$

$$\underline{F}^{-z} = r\Delta\theta\Delta r\left[-\sigma_{zz}(r,\theta,z)\underline{e}_z + \frac{\partial\sigma_{zz}}{\partial z}\frac{\Delta z}{2}\underline{e}_z + \cdots\right]$$

$$\underline{F}^{+z} + \underline{F}^{-z} = \underbrace{r\Delta\theta\Delta r\Delta z}_{\Delta V}\left[\frac{\partial\sigma_{zz}}{\partial z}\underline{e}_z + \frac{\partial\sigma_{rz}}{\partial z}\underline{e}_r + \frac{\partial\sigma_{\theta z}}{\partial z}\underline{e}_\theta\right]$$

$$\underline{t}^{+\theta} = \underbrace{\sigma_{r\theta}\left(r,\theta+\frac{\Delta\theta}{2},z\right)\underline{e}_r\left(\theta+\frac{\Delta\theta}{2}\right)} + \sigma_{\theta\theta}\left(r,\theta+\frac{\Delta\theta}{2},z\right)\underline{e}_\theta\left(\theta+\frac{\Delta\theta}{2}\right) + \sigma_{z\theta}\left(r,\theta+\frac{\Delta\theta}{2},z\right)\underline{e}_z$$

$$\underline{t}^{-\theta} = -\sigma_{r\theta}\left(r,\theta-\frac{\Delta\theta}{2},z\right)\underline{e}_r\left(\theta-\frac{\Delta\theta}{2}\right) \cdots\cdots$$

$$A^{+\theta} = A^{-\theta} = \Delta r\Delta z$$

$$\underset{\sim}{F}^{+\theta} = \Delta r \Delta z \left[\sigma_{r\theta}\, \underline{e}_r + \frac{\partial}{\partial\theta}\left(\sigma_{r\theta}\, \underline{e}_r\right)\frac{\Delta\theta}{2} + \sigma_{\theta\theta}\, \underline{e}_\theta + \frac{\partial}{\partial\theta}\left(\sigma_{\theta\theta}\, \underline{e}_\theta\right)\frac{\Delta\theta}{2} + \sigma_{z\theta}\, \underline{e}_z + \frac{\partial}{\partial\theta}\sigma_{z\theta}\, \underline{e}_z \right.$$

$$= \Delta r \Delta z \left[\sigma_{r\theta}\, \underline{e}_r + \sigma_{\theta\theta}\, \underline{e}_\theta + \sigma_{z\theta}\, \underline{e}_z + \left\{ \frac{\partial \sigma_{r\theta}}{\partial\theta}\, \underline{e}_r + \sigma_{r\theta}\, \underline{e}_\theta + \frac{\partial \sigma_{\theta\theta}}{\partial\theta}\, \underline{e}_\theta - \sigma_{\theta\theta}\, \underline{e}_r + \frac{\partial \sigma_{z\theta}}{\partial\theta}\, \underline{e}_z \right\} \frac{\Delta\theta}{2} \right]$$

$$\underset{\sim}{F}^{-\theta} = \Delta r \Delta z \left[\quad - \quad - \quad - \quad - \quad - \quad \right]$$

$$\underset{\sim}{F}^{+\theta} + \underset{\sim}{F}^{-\theta} = \underbrace{r \Delta r \Delta z \Delta\theta}_{\Delta V} \left[\frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial\theta}\, \underline{e}_r + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial\theta}\, \underline{e}_\theta + \frac{1}{r}\frac{\partial \sigma_{z\theta}}{\partial\theta}\, \underline{e}_z + \frac{\sigma_{r\theta}}{r}\, \underline{e}_\theta - \frac{\sigma_{\theta\theta}}{r}\, \underline{e}_r \right]$$