Stress-strain relation

3-D Hooke's law

$$\epsilon_{11} = \frac{1}{E}\left(\sigma_{11} - \nu(\sigma_{22}+\sigma_{33})\right)$$

$\epsilon_{22} =$

$\epsilon_{33} =$

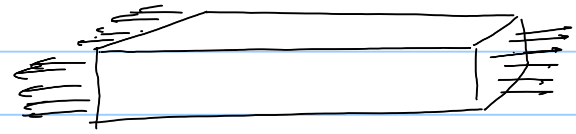
$\gamma_{12} = \tau_{12}/G \qquad G = \frac{E}{2(1+\nu)}$

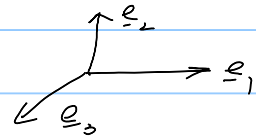
γ_{13}

γ_{23}

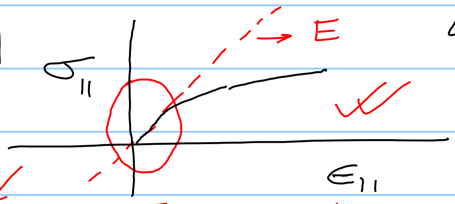
General loading!!

Uniaxial loading :





$\epsilon_{11} = \frac{\sigma_{11}}{E}$

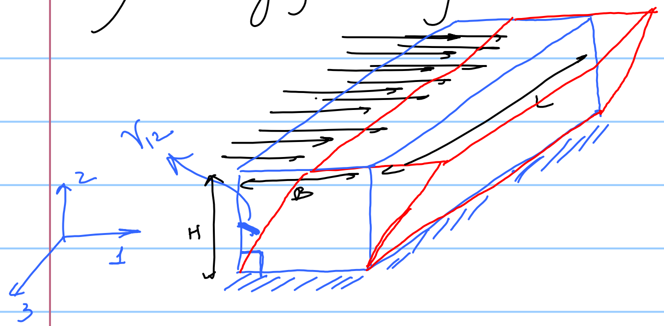


Graph for uniaxial tensile test

$$\left.\frac{d\sigma_{11}}{d\epsilon_{11}}\right|_{\epsilon_{11}=0} = E$$

$$\boxed{\nu = -\frac{\epsilon_{22}}{\epsilon_{11}}}$$

Physical significance of shear modulus :-



$$\underline{\sigma}\, \underline{e}_2 = \frac{F}{LB}\, \underline{e}_1$$

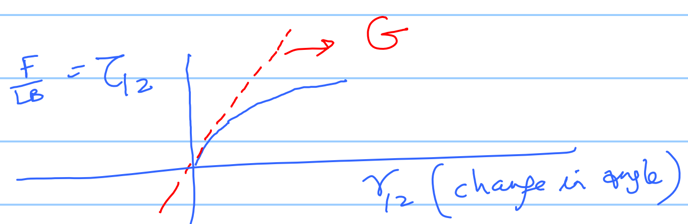
$\downarrow (\underline{e}_1, \underline{e}_2, \underline{e}_3)$

$$\begin{bmatrix}\tau_{12}\\ \sigma_{22}\\ \tau_{32}\end{bmatrix} = \begin{bmatrix}F/LB\\ 0\\ 0\end{bmatrix}$$

$$[\underline{\sigma}] = \begin{bmatrix}0 & \tau_{12} & 0\\ \tau_{12} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

for points in the bar near the top surface $\tau_{12} = F/LB$

$\sigma_{22} = \tau_{32} = 0$



$$\boxed{\left.\frac{d\tau_{12}}{d\gamma_{12}}\right|_{\gamma_{12}=0} = G}$$

Bulk modulus of elasticity

$$K = -\frac{\Delta P}{\Delta V/V}$$

(minus sign is kept here to obtain positive number)

K denotes compressibility / incompressibility —

$$\underline{\underline{\sigma}} = \underbrace{\frac{1}{3} I_1 \underline{\underline{I}}}_{\downarrow \; -p\underline{\underline{I}}} + \underline{\underline{\sigma}} - \frac{1}{3} I_1 \underline{\underline{I}}$$

$$\Downarrow \quad p_{eq} = -\frac{1}{3} I_1$$

$$K_{solids} = \frac{-p_{eq}}{\Delta V/V} = +\frac{1}{3}\frac{I_1}{J_1} = \frac{1}{3}\frac{(\sigma_{11}+\sigma_{22}+\sigma_{33})}{(\epsilon_{11}+\epsilon_{22}+\epsilon_{33})}$$

Adding first three stress-strain relation

$$\epsilon_{11}+\epsilon_{22}+\epsilon_{33} = \frac{1}{E}\left[(\sigma_{11}+\sigma_{22}+\sigma_{33}) - 2\nu(\sigma_{11}+\sigma_{22}+\sigma_{33})\right]$$

$$\Rightarrow J_1 = \frac{1-2\nu}{E} I_1$$

$$\Rightarrow \boxed{K_{solids} = \frac{E}{3(1-2\nu)}}$$

$$\Downarrow$$

$\nu = 1/2$ is the incompressible limit
(rubber, soft tissues, polymeric materials)

$\because K > 0 \Rightarrow \nu \le 1/2$

$$G = \frac{E}{2(1+\nu)}$$

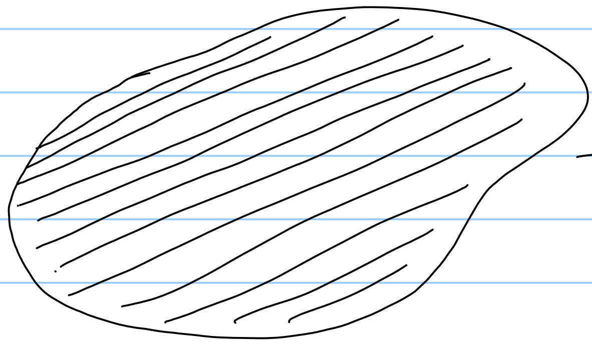
$$\Downarrow \quad G, E > 0$$

$$1+\nu > 0 \Rightarrow \nu > -1$$

Theoretical limit of Poisson's ratio for isotropic materials

$$\boxed{-1 < \nu \le 1/2}$$

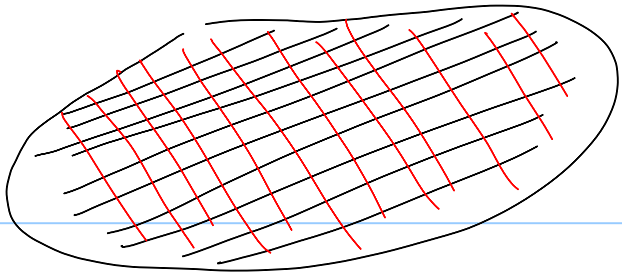
So, $\nu < 0$ is not very common but researchers are trying to develop such materials and they are called auxetic materials!



→ transversely isotropic materials

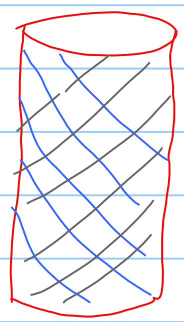
→ 5 independent material constants!

→ bullet-proof jackets!



→ Orthotropic materials

→ Wood, bones



→ plastic materials

→ do not come back to their original config on unloading!

for fluids:

$$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \overset{\text{viscosity}}{\mu}\underbrace{\left(\nabla \underline{v} + \nabla \underline{v}^T\right)}_{\text{strain rate}}$$

$\dot{\varepsilon}$

Viscoelastic materials

→ stress depends on both strain and strain rate!