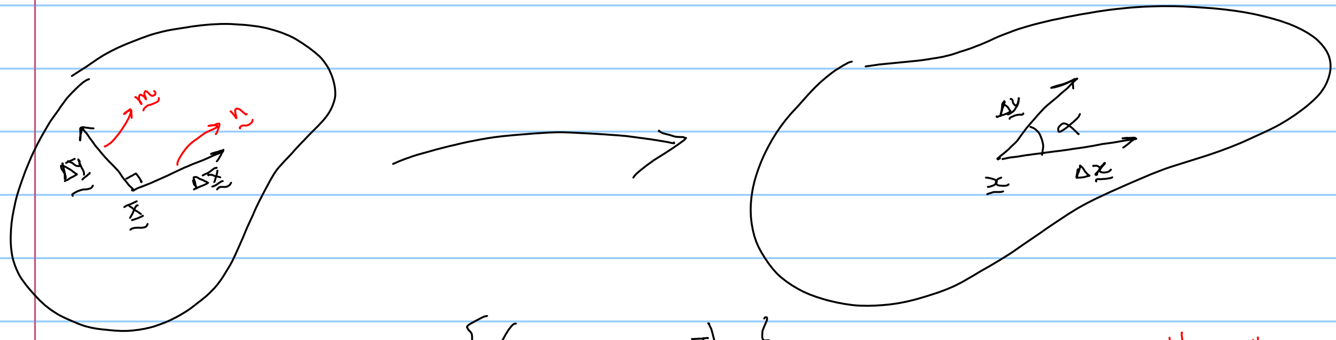
Lec 16 Shear strain & Vol. strain

Note Title 9/9/2022



$$\cos\alpha = \underbrace{\left\{\left(\nabla u + \nabla u^T\right) n\right\}\cdot m}$$

$$\|\nabla u\| << 1$$

$$\cos(90-\beta) = \sin\beta \simeq \beta$$

$$\Rightarrow \boxed{\beta = \left\{\left(\nabla u + \nabla u^T\right) n\right\}\cdot m}$$

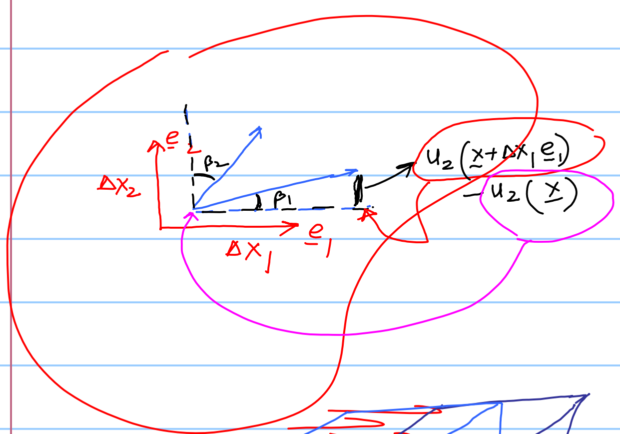
$$2\left\{\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \text{''} \\ \text{''} & \frac{\partial u_2}{\partial x_2} & \text{''} \\ \text{''} & \text{''} & \frac{\partial u_3}{\partial x_3} \end{bmatrix}\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}\right\}\cdot\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

$n = e_1$, $m = e_2$

$$\beta_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}$$

$$\beta_{23} = \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}$$

β_{13}



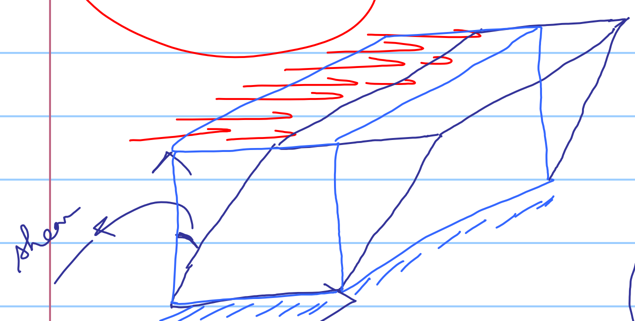
$$\beta_{12} = \beta_1 + \beta_2 \rightarrow \frac{\partial u_1}{\partial x_2}$$

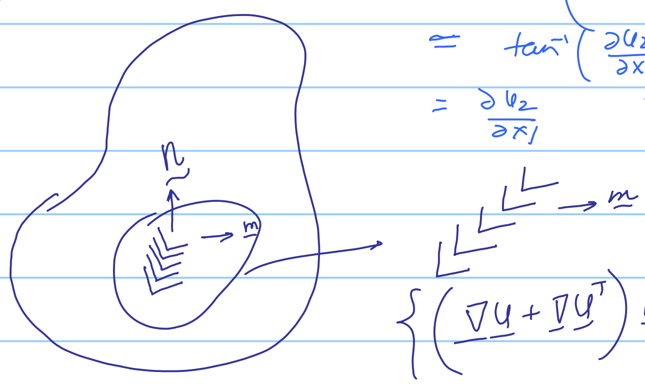
$$\beta_1 \simeq \tan^{-1}\left\{\frac{u_2(x+\Delta x_1 e_1) - u_2(x)}{\Delta x_1}\right\}$$

$$\beta_1 \simeq \tan^{-1}\left(\lim_{\Delta x_1\to 0}\frac{u_2(x+\Delta x_1 e_1) - u_2(x)}{\Delta x_1}\right)$$

$$\simeq \tan^{-1}\left(\frac{\partial u_2}{\partial x_1}\right)$$

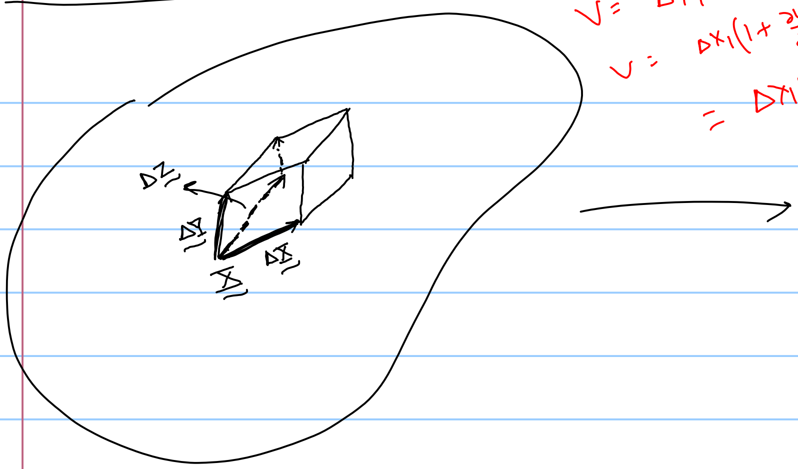
$$= \frac{\partial u_2}{\partial x_1}$$





$$\left\{\left(\nabla u + \nabla u^T\right) n\right\}\cdot m$$

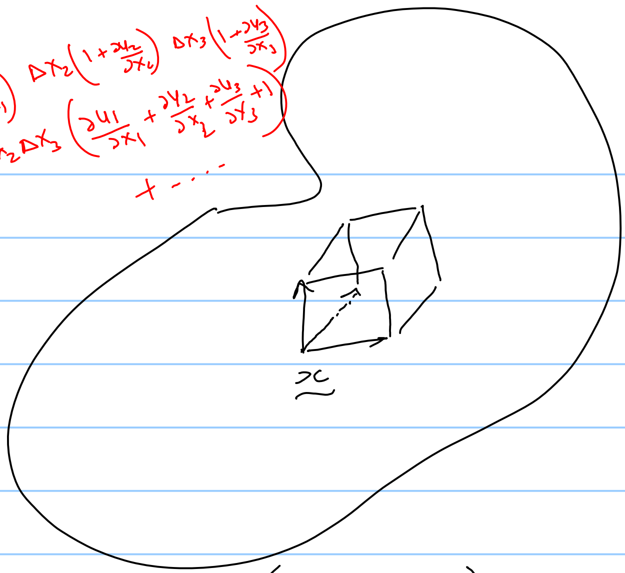
Volumetric strain :-



$V = \Delta X_1 \Delta X_2 \Delta X_3$

$v = \Delta X_1\left(1+\frac{\partial u_1}{\partial X_1}\right) \Delta X_2\left(1+\frac{\partial u_2}{\partial X_2}\right) \Delta X_3\left(1+\frac{\partial u_3}{\partial X_3}\right)$

$= \Delta X_1 \Delta X_2 \Delta X_3 \left(\frac{\partial u_1}{\partial X_1} + \frac{\partial u_2}{\partial X_2} + \frac{\partial u_3}{\partial X_3} + 1\right) + \cdots$



$$V = \left(\Delta \underset{\sim}{X} \times \Delta \underset{\sim}{Y}\right) \cdot \Delta \underset{\sim}{Z} \qquad v = \left(\Delta \underset{\sim}{x} \times \Delta \underset{\sim}{y}\right) \cdot \Delta \underset{\sim}{z}$$

$$\epsilon_V = \lim_{V \to 0} \frac{v}{V} - 1$$

$$V \quad \det \begin{bmatrix} [\Delta \underset{\sim}{X}] & [\Delta \underset{\sim}{Y}] & [\Delta \underset{\sim}{Z}] \end{bmatrix} = \left(\Delta \underset{\sim}{X} \times \Delta \underset{\sim}{Y}\right) \cdot \Delta \underset{\sim}{Z}$$

$$v = \det \begin{bmatrix} \Delta \underset{\sim}{x} & \Delta \underset{\sim}{y} & \Delta \underset{\sim}{z} \end{bmatrix} = \det \begin{bmatrix} \underset{\approx}{F} \Delta \underset{\sim}{X} + O(\|\Delta \underset{\sim}{X}\|^2) & \underset{\approx}{F} \Delta \underset{\sim}{Y} + O(\|\Delta \underset{\sim}{Y}\|^2) & \underset{\approx}{F} \Delta \underset{\sim}{Z} + O(\|\Delta \underset{\sim}{Z}\|^2) \end{bmatrix}$$

$$\begin{bmatrix} \underset{\approx}{A} \underset{\sim}{a} & \underset{\approx}{A} \underset{\sim}{b} & \underset{\approx}{A} \underset{\sim}{c} \end{bmatrix} = [\underset{\approx}{A}] \begin{bmatrix} \underset{\sim}{a} & \underset{\sim}{b} & \underset{\sim}{c} \end{bmatrix}$$

$$[\underset{\approx}{A}][\underset{\approx}{B}] = [\underset{\approx}{C}]$$

1st col.

$$v = \det\left([\underset{\approx}{F}] \begin{bmatrix} \Delta \underset{\sim}{X} & \Delta \underset{\sim}{Y} & \Delta \underset{\sim}{Z} \end{bmatrix}\right)$$

$$= \det \underset{\approx}{F} \det\left(\begin{bmatrix} \Delta \underset{\sim}{X} & \Delta \underset{\sim}{Y} & \Delta \underset{\sim}{Z} \end{bmatrix}\right) = \det \underset{\approx}{F}\, V$$

$$\Rightarrow \lim_{V \to 0} \frac{v}{V} = \det \underset{\approx}{F} \quad \Rightarrow \epsilon_V = \det(\underset{\approx}{F}) - 1$$

$$\underline{\underline{F}} = \underline{\underline{I}} + \nabla \underline{u}$$

$$\det\left(\begin{bmatrix} 1+\frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & 1+\frac{\partial u_2}{\partial x_2} & \partial u_2/\partial x_3 \\ - & - & - \end{bmatrix}\right) - 1$$

$$= 1 + \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right) + \cancel{O\left(\|\nabla u\|^2\right)} \rightarrow 0 \quad -1$$

$$\epsilon_V = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = I_1\left(\nabla \underline{u}\right) = I_1\left(\frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right)\right)$$

$$\epsilon_{nn} = \left\{\frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right)\underline{n}\right\} \cdot \underline{n}$$

$$\gamma_{nm} = 2\left\{\frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right)\underline{n}\right\} \cdot \underline{m}$$

Infinitesimal strain tensor!!

$$\sigma_{nn} = \left(\underline{\underline{\sigma}}\,\underline{n}\right) \cdot \underline{n}$$

$$\tau_{nm} = \left(\underline{\underline{\sigma}}\,\underline{n}\right) \cdot \underline{m}$$

$$\epsilon_V = tr\left(\frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right)\right)$$

unique at a pt.!!

Principal strain direction: direction of line element along which normal strain is getting max. or minimized!

Principal strains: corresponding normal strain values!

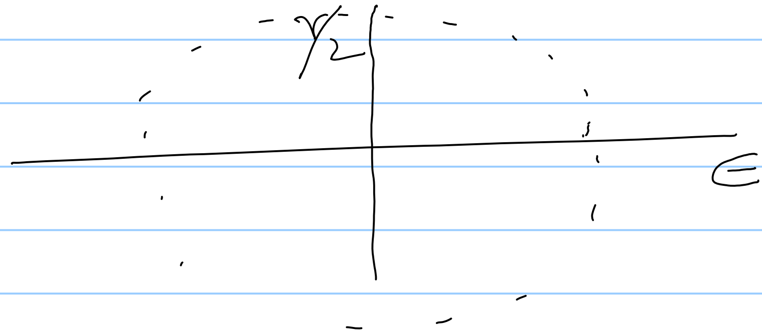
$$\underline{\underline{\epsilon}} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right)$$

$$\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ & \epsilon_{yy} & \epsilon_{yz} \\ & & \epsilon_{zz} \end{bmatrix}$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) = \gamma_{xy}/2$$

$$[\underline{\underline{\epsilon}}] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

→ third direction is principal strain direct..!



stress invariants : I_1, I_2, I_3

strain " : J_1, J_2, J_3

strain tensor = hydrostatic strain tensor + strain deviator

$$\underline{\underline{\epsilon}} = \frac{1}{3} J_1 \underline{\underline{I}} + \left(\underline{\underline{\epsilon}} - \frac{1}{3} J_1 \underline{\underline{I}}\right)$$

$$\underline{\underline{F}} = \underline{\underline{I}} + \underline{\nabla u}$$

$$= \underline{\underline{I}} + \underbrace{\frac{1}{2}\left(\underline{\nabla u} + \underline{\nabla u}^T\right)}_{\text{responsible for inducing strain}} + \frac{1}{2}\left(\underline{\nabla u} - \underline{\nabla u}^T\right)$$

?!..