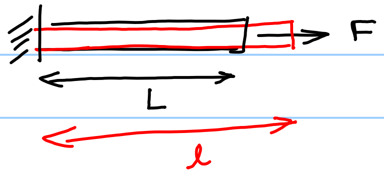
Lecture 14 Strain

Note Title 9/6/2022

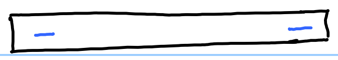


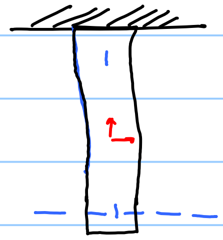
Longitudinal strain: $\dfrac{\text{change in length}}{\text{length}} = \dfrac{\Delta \ell}{\ell}$

Original length or deformed length??

$$\frac{\Delta \ell}{\ell} \approx \frac{\Delta \ell}{L} \quad (\Delta \ell << L)$$

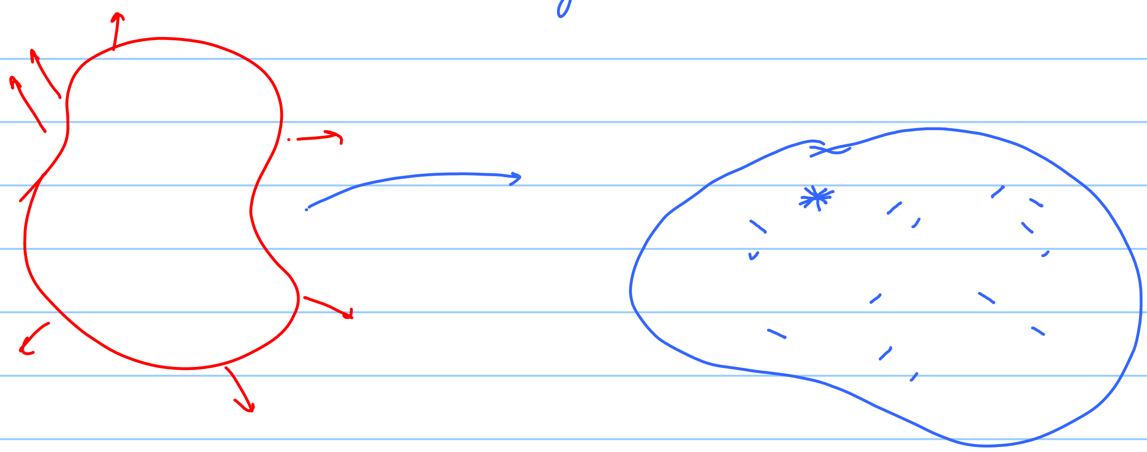
one single value for entire bar (global strain)





longitudinal strain varies in the bar!

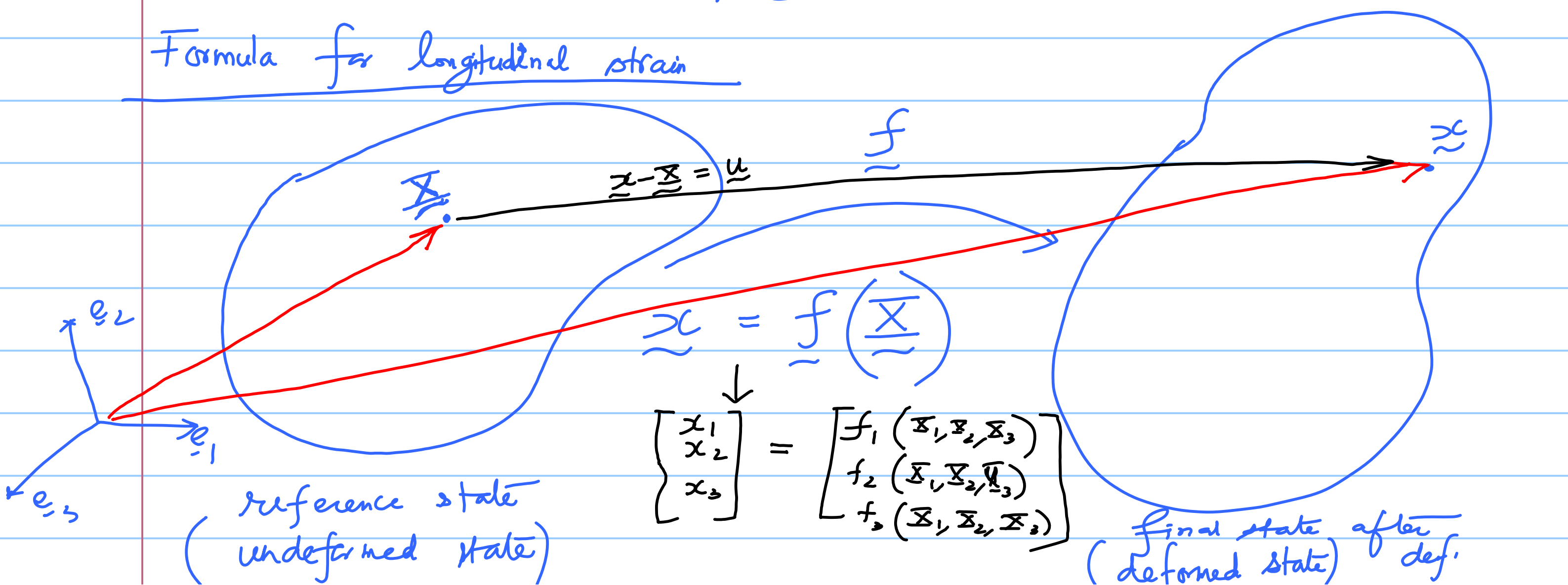
→ local strain and global strain are not the same!



→ Longitudinal strain varies from pt. to pt. in the body and even at a pt., it depends on the orientation of line element

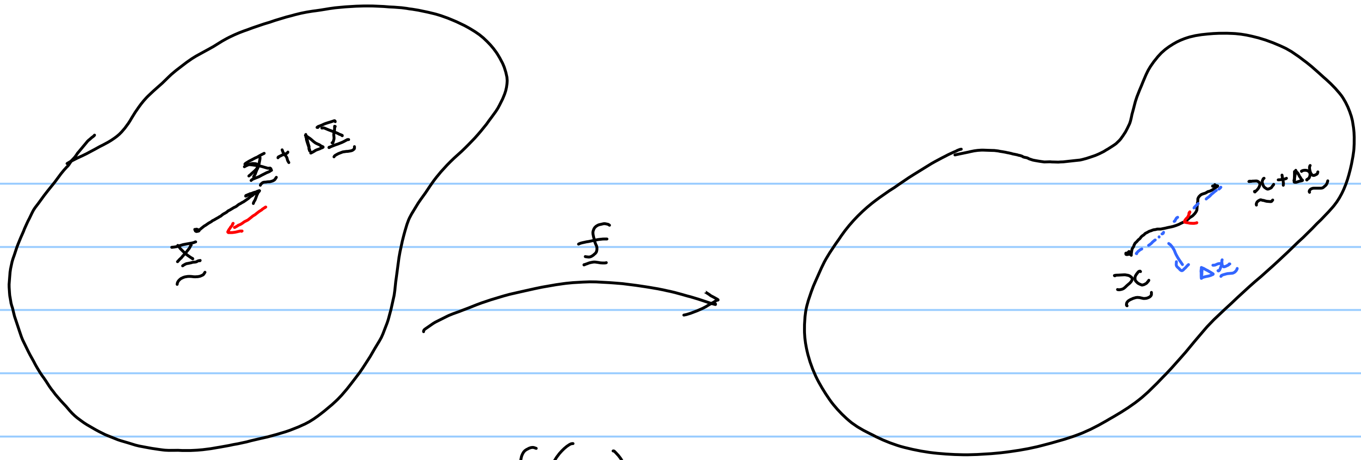
→ it is a scalar quantity!

Formula for longitudinal strain



$$\underset{\sim}{x} = \underset{\sim}{f}(\underset{\sim}{X})$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} f_1(X_1, X_2, X_3) \\ f_2(X_1, X_2, X_3) \\ f_3(X_1, X_2, X_3) \end{bmatrix}$$



$$\underset{\sim}{x} = \underset{\sim}{f}(\underset{\sim}{X})$$

$$\underset{\sim}{x} + \Delta \underset{\sim}{x} = \underset{\sim}{f}(\underset{\sim}{X} + \Delta \underset{\sim}{X})$$

As $\|\Delta \underset{\sim}{X}\| \to 0$, $\Delta \underset{\sim}{x}$ becomes the deformed line element!

$$\Downarrow$$

$$\Delta \underset{\sim}{x} = \underset{\sim}{f}(\underset{\sim}{X} + \Delta \underset{\sim}{X}) - \underset{\sim}{f}(\underset{\sim}{X})$$

$$\Delta x_1 = f_1(X_1 + \Delta X_1, X_2 + \Delta X_2, X_3 + \Delta X_3) - f_1(X_1, X_2, X_3)$$

$$= \cancel{f_1(X_1, X_2, X_3)} + \frac{\partial f_1}{\partial X_1}\Delta X_1 + \frac{\partial f_1}{\partial X_2}\Delta X_2 + \frac{\partial f_1}{\partial X_3}\Delta X_3 + \cdots\cdots - \cancel{f_1(X_1, X_2, X_3)}$$

$$\Delta x_1 = \sum_j \frac{\partial f_1}{\partial X_j}\Delta X_j + O(\|\Delta \underset{\sim}{X}\|^2)$$

$$\Rightarrow \quad \Delta x_i = \sum_j \frac{\partial f_i}{\partial X_j}\Delta X_j + O(\|\Delta \underset{\sim}{X}\|^2)$$

$$\Rightarrow \quad \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_i}{\partial X_j} \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \end{bmatrix} + O(\|\Delta \underset{\sim}{X}\|^2)$$

$$\boxed{\Delta \underset{\sim}{x} = \underset{=}{F}\, \Delta \underset{\sim}{X} + O(\|\Delta \underset{\sim}{X}\|^2)} \checkmark$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \frac{\partial f_1}{\partial X_2} & \frac{\partial f_1}{\partial X_3} \\ \frac{\partial f_2}{\partial X_1} & \frac{\partial f_2}{\partial X_2} & \frac{\partial f_2}{\partial X_3} \\ \frac{\partial f_3}{\partial X_1} & \frac{\partial f_3}{\partial X_2} & \frac{\partial f_3}{\partial X_3} \end{bmatrix}$$

$$\underset{=}{F} = \sum_i \sum_j \frac{\partial f_i}{\partial X_j}\, \underset{\sim}{e}_i \otimes \underset{\sim}{e}_j \qquad \underset{=}{F} \equiv$$

$$= \sum_j \frac{\partial \underset{\sim}{f}}{\partial X_j} \otimes \underset{\sim}{e}_j = \underset{\sim}{\nabla}\, \underset{\sim}{f}$$

$$\underset{\sim}{\nabla}(\cdot) = \sum_j \frac{\partial}{\partial X_j}(\;) \otimes \underset{\sim}{e}_j$$