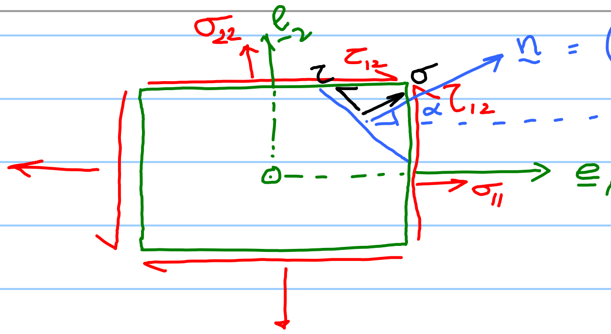
Lecture 12 (Mohr's Circle)

Note Title 8/31/2022



$(\underline{e}_1, \underline{e}_2, \underline{n}_3)$

$$\begin{bmatrix} \sigma_{11} & \tau_{12} & 0 \\ \tau_{12} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

$$[\underline{n}] = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix}$$

$$\sigma = (\underline{\underline{\sigma}}\,\underline{n}) \cdot \underline{n}$$

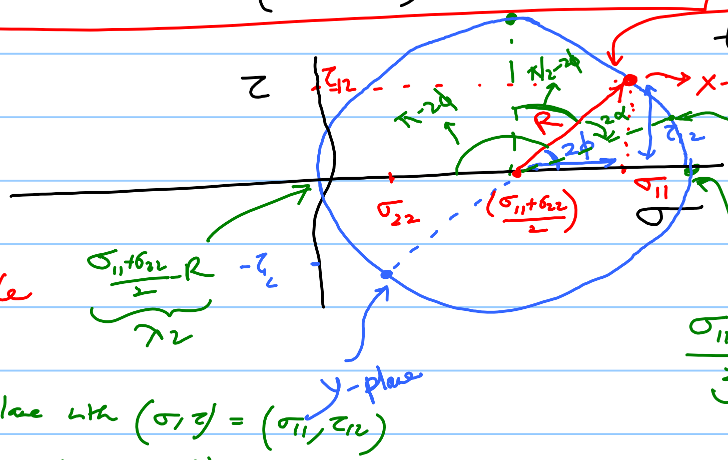
$$\tau = (\underline{\underline{\sigma}}\,\underline{n}) \cdot \underline{n}^{\perp}$$

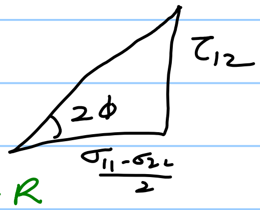
$$\boxed{\begin{aligned} \sigma &= \frac{\sigma_{11}+\sigma_{22}}{2} + R\cos(2\phi - 2\alpha) \\ \tau &= R\sin(2\phi - 2\alpha) \end{aligned}}$$

$$R = \sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2 + \tau_{12}^2}$$

$$\tan 2\phi = \frac{\tau_{12}}{\frac{\sigma_{11}-\sigma_{22}}{2}}$$

$$\boxed{\tau_{max} = R = \frac{\lambda_1 - \lambda_2}{2}}$$





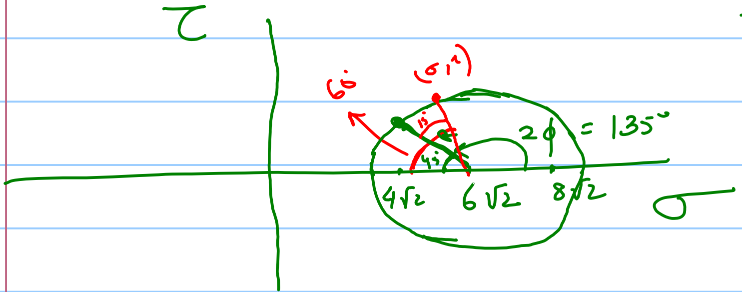
1. Draw the centre of circle $\left(\frac{\sigma_{11}+\sigma_{22}}{2}, 0\right)$
2. Draw the pt. on X-plane with $(\sigma, \tau) = (\sigma_{11}, \tau_{12})$
3. joint centre to X-plane pt. to obtain radius
4. With center and radius known, draw the circle
5. For point corresponding to α-plane, traverse 2α in opposite direction (clockwise) from X-plane

An example,

$$[\underline{\underline{\sigma}}]_{(\underline{e}_1 \cdot \underline{e}_2 \cdot \underline{e}_3)} = \begin{bmatrix} 4\sqrt{2} & 2\sqrt{2} & 0 \\ 2\sqrt{2} & 8\sqrt{2} & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$\underline{e}_3$ is a principle axis!

So, we can draw Mohr's Circle for all those planes having $\underline{n} \perp \underline{e}_3$



$$R = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$$

$$\lambda_i : \begin{array}{l} 6\sqrt{2}+4 \\ 6\sqrt{2}-4 \\ 10 \end{array}$$

$\tau_{max} = 4$

$2\phi = 135^\circ$

Q. What is the value of (σ, τ) on a plane which is at $+7.5^\circ$ from x-plane

$\sigma = 6\sqrt{2} - R\cos 60^\circ$

$\tau = R\sin 60^\circ$

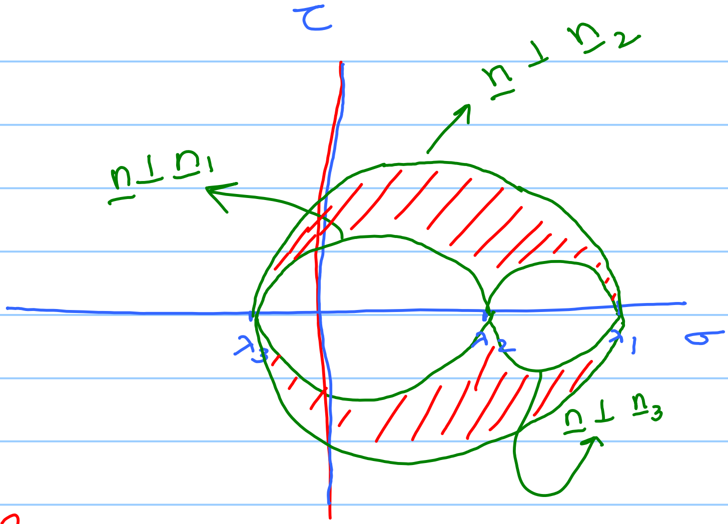
$\sigma = 6\sqrt{2} - 4/2 = 6\sqrt{2} - 2$

$\tau = 4\sqrt{3}/2 = 2\sqrt{3}$

if $\underline{e}_1$ is principal normal

$$[\underline{\sigma}] = \begin{bmatrix} X & 0 & 0 \\ 0 & \sigma_{22} & \sigma_{23} \\ 0 & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

$$[\underline{\sigma}]_{(\underline{n}_1, \underline{n}_2, \underline{n}_3)} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$



$\lambda_2 \longrightarrow \lambda_1 \Rightarrow$??