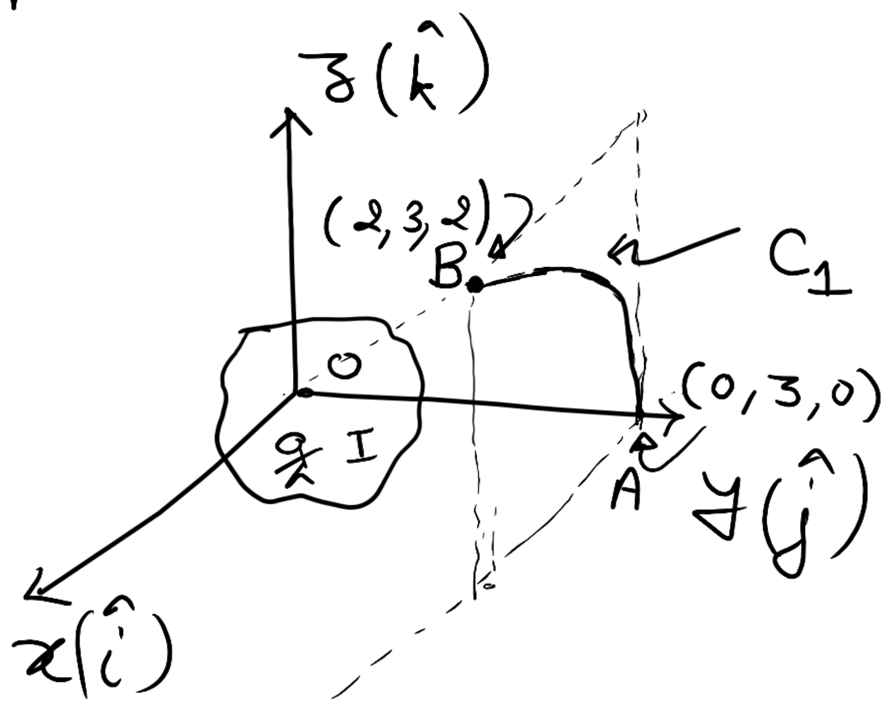
Set 9 A (four problems) & B (7 problems): To be discussed during the weeks of March 30 and April 6

Q 1) (a) Is the force $\underline{F} = (-2xy + yz)\hat{i} + (-x^2 + xz - z)\hat{j} + (xy - y)\hat{k}$ conservative?

(b) If it is conservative, find its potential function.

(c) Find the work done by this force while a particle (say P) moves along an open quarter circular path C_1. (Start at A, end at B)



$$\underline{\nabla} \times \underline{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2xy+yz & -x^2+xz-z & xy-y \end{vmatrix}$$

$$= \hat{i}\left(x-1-x+1\right) - \hat{j}\left(y-y\right) + \hat{k}\left(-2x+z+2x-z\right)$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}(0)$$

$$= 0 \quad \forall \; x, y, z$$

$\Rightarrow$ $\underline{F}$ is conservative

$\Rightarrow$ Work done by this force when a particle moves from point A to point B is path independent. We need to find V.

$W_{\cancel{A-B}} = -[V(2,3,2) - V(0,3,0)]$ Where V is the potential function of $\underline{F}$.

What is $V(x,y,z)$

$F_x = -\frac{\partial V}{\partial x}$ $\quad F_y = -\frac{\partial V}{\partial y}$ $\quad F_z = -\frac{\partial V}{\partial z}$.

$-2xy + yz = -\frac{\partial V}{\partial x} \Rightarrow \int(-2xy + yz)\,dx = -V(x,y,z) + f(y,z)$ (i)

$-x^2 + xz - z = -\frac{\partial V}{\partial y} \Rightarrow \int(-x^2 + xz - z)\,dy = -V(x,y,z) + g(z,x)$ (ii)

$xy - y = -\frac{\partial V}{\partial z} \Rightarrow \int(xy - y)\,dz = -V(x,y,z) + h(x,y)$ (iii)

Complete the integration of the left hand sides of (i), (ii), (iii)

$$-x^2y + yzx = -V(x,y,z) + f(y,z) \quad \text{(iv)}$$

$$-x^2y + xzy - zy = -V(x,y,z) + g(z,x) \quad \text{(v)}$$

$$xyz - yz = -V(x,y,z) + h(x,y) \quad \text{(vi)}$$

or, $V(x,y,z) = x^2y - xyz + f(y,z)$. (vii)

$$V(x,y,z) = x^2y - xyz + yz + g(z,x) \quad \text{(viii)}$$

$$V(x,y,z) = -xyz + yz + h(x,y) \quad \text{(ix)}$$

vii, viii, ix are simultaneously satisfied if we choose

$f(y,z) = yz$, $g(z,x) = 0$, $h(x,y) = x^2y$.

$$\Rightarrow \boxed{V(x,y,z) = x^2y - xyz + yz}$$

$$\therefore \; W_{A-B} = -\left[V(2,3,2) - V(0,3,0)\right]$$

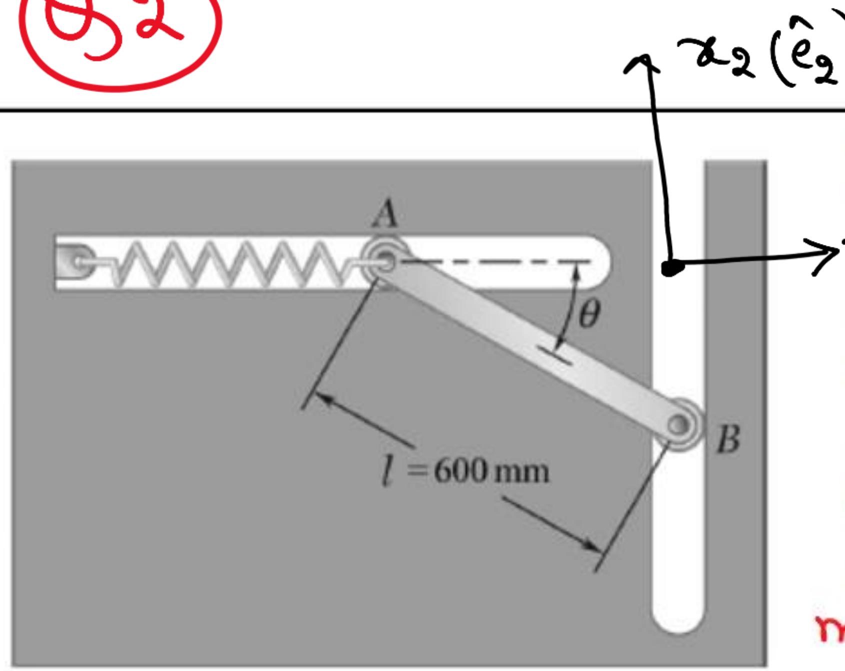
$$= -6 \text{ Nm}.$$

$$= -\left[\left(2^2 \cdot 3 - 2\cdot 3\cdot 2 + 3\cdot 2\right) - (0 - 0 + 0)\right]$$

$$= -\left[12 - 12 + 6\right]$$

$$= -\left[+6\right] = -6 \text{ Nm}.$$

Q2



PROBLEM 17.40

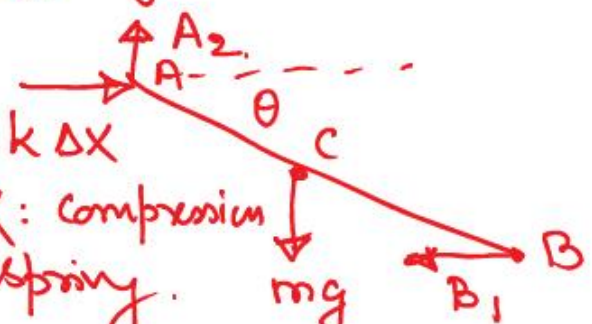
No friction force anywhere.

The ends of a 4.5 kg rod AB are constrained to move along slots cut in a vertical plane as shown. A spring of constant $k = 600$ N/m is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 0$, determine the angular velocity of the rod and the velocity of end B when $\theta = 30°$.

(friction not present).

mass of rod: m, K: spring constant

~~FB~~ System = rod. FBD.



ΔX: compression in spring.

B_1, A_2 are workless forces and the remaining two forces are conservative.

$\therefore (T+V)_f - (T+V)_i = 0$ where i: initial state $\theta = 0°$, f: final state $\theta = 30°$

$$V = V^{\text{spring force}} + V^{\text{gravitational force}}$$

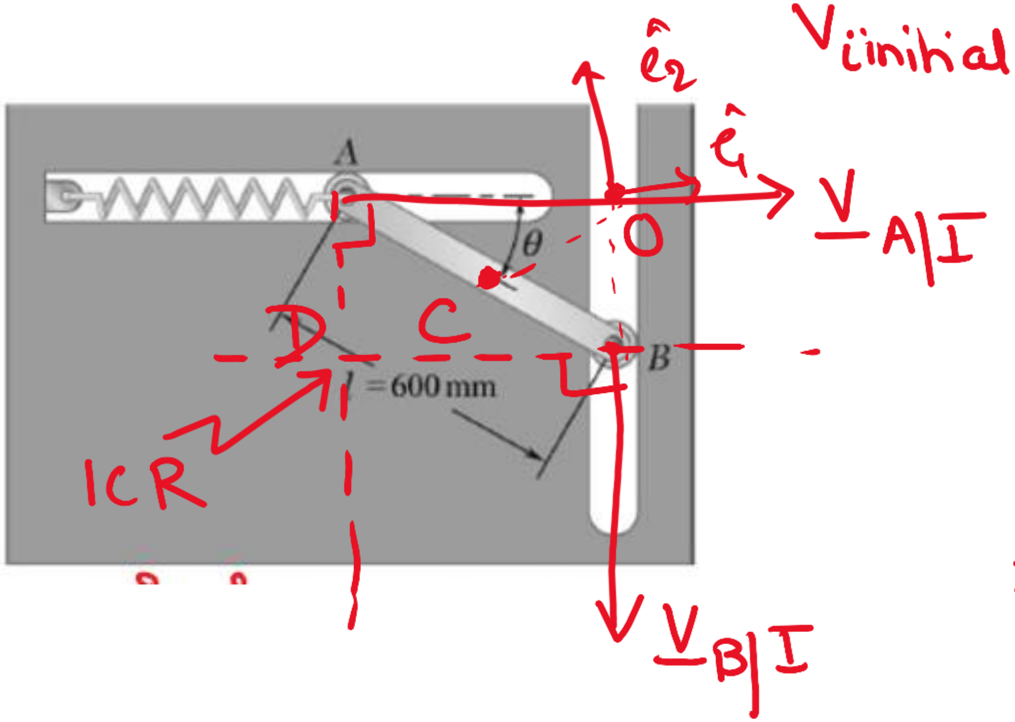
$$= -mg\frac{L}{2}\sin\theta + \frac{1}{2}kL^2(1-\cos\theta)^2.$$

$V^{\text{gravitational force}} = mg\,x_2$ ($x_2 \uparrow$ +ve)

$V^{\text{spring force}} = \frac{1}{2}k(x_i - x_0)^2$ $x_0 = 0$ here

In the initial state : $T_i = 0$ (motion starts from rest)

$V_{initial} = V_{initial}^{gravitation} + V_{initial}^{spring\ force} = 0 + 0$



Point D is the ICR of Rod AB

$$\Rightarrow \underline{v}_{C|I} = \underline{\omega}_{AB|I} \times \underline{r}_{CD}$$

$$\Rightarrow |\underline{v}_{C|I}| = \omega\, |\underline{r}_{CD}|$$

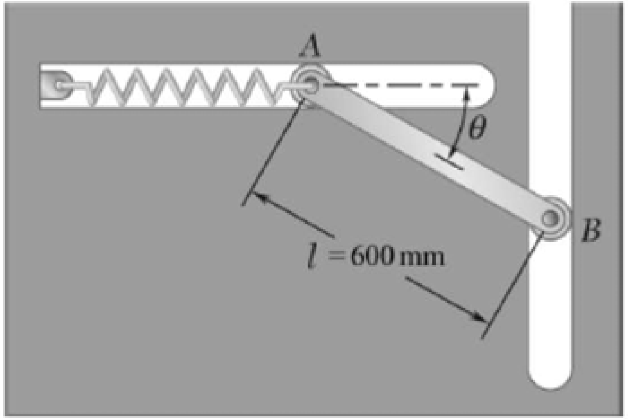
$$= \omega \frac{\sqrt{(L\cos\theta)^2 + (L\sin\theta)^2}}{2}.$$

$$|\underline{v}_{C|I}| = \omega L/2$$

$$\Rightarrow \underline{v}_{C|I} \cdot \underline{v}_{C|I} = \omega^2 L^2/4$$

$$\therefore T = \frac{1}{2} m \frac{L^2\omega^2}{4} + \frac{1}{2} \frac{mL^2\omega^2}{12} = mL^2\omega^2/6$$

Since $(T+V)_f = 0$, $\Rightarrow$ find ω



$$\frac{mL^2\omega^2}{6} = \frac{mgL\sin\theta}{2} - \frac{kL^2}{2}(1-\cos\theta)^2$$

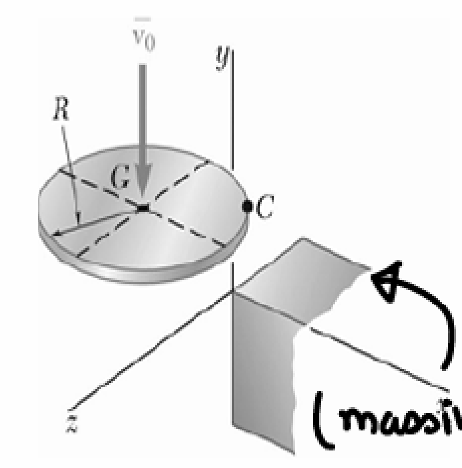
$$\omega = \sqrt{\frac{\frac{mgL\sin\theta}{2} - \frac{kL^2}{2}(1-\cos\theta)^2}{mL^2/6}}$$

Answer.

Q3

PROBLEM 18.29

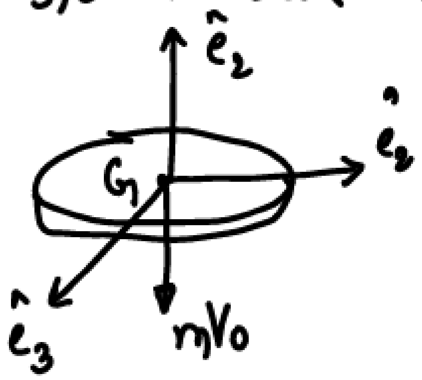
A circular plate of mass m is falling with a velocity $\bar{\mathbf{v}}_0$ and no angular velocity when its edge C strikes an obstruction. Assuming the impact to be perfectly plastic ($e = 0$), determine the angular velocity of the plate immediately after the impact.



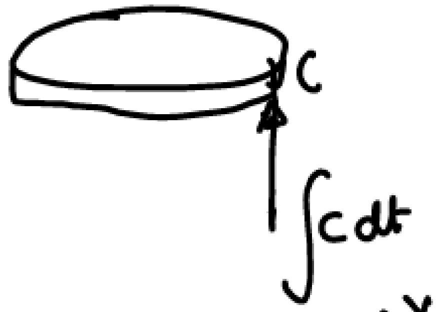
(massive, will undergo no change in its momentum / angular momentum)

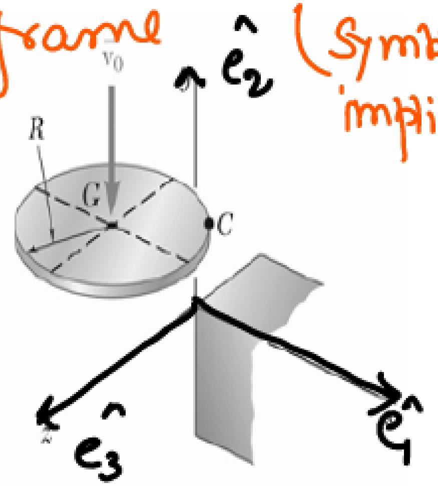
System = disk : Impact process

all velocity & angular momentum vectors are w.r.t. I frame (symbol implied)

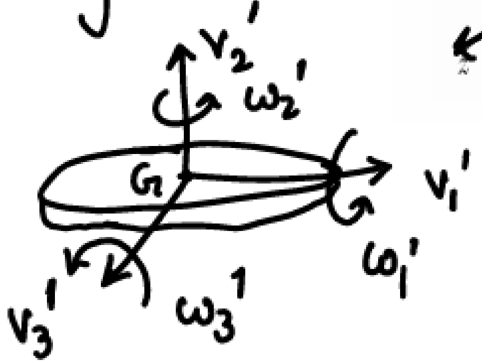


$+$





Before impact $=$



Find $\omega_1', \omega_2', \omega_3'$

Find $\omega_1', \omega_2', \omega_3'$

Unknowns : 7 unknowns. 6 equ^ns from momentum-impulse (MI) equ^ns

7^th equ^n: coeff. of resti. equ^n.

Linear-momentum impulse equ^n

$$\underline{V}_{G|I} = -v_0 \hat{e}_2$$

$$\int \underline{F}^{impulsive} dt = m\left[\underline{V}'_{G|I} - \underline{V}_{G|I} \right]$$

$$\left(\int c\,dt\right)\hat{e}_2 = m\left[v_1'\hat{e}_1 + v_2'\hat{e}_2 + v_3'\hat{e}_3 - (-v_0\hat{e}_2)\right]$$

$$\left(\int c\,dt\right)\hat{e}_2 = mv_1'\hat{e}_1 + (mv_2' + mv_0)\hat{e}_2 + v_3'\hat{e}_3$$

$\Rightarrow \quad \underbrace{v_1' = 0}, \quad \underbrace{v_3' = 0}, \quad \underbrace{\int c\,dt = m v_2' + m v_0} \quad \text{(i)}$

Now using & Angular momentum-impulse equ^n

$$\int \underline{M}_{A}^{\text{impulse}}\,dt = \left[m\,\underline{r}_{GA} \times \underline{v}_A(t_2) + \underline{H}_A(t_2)\right] - \left[m\,\underline{r}_{GA} \times \underline{v}_A(t_1) + \underline{H}_A(t_1)\right]$$

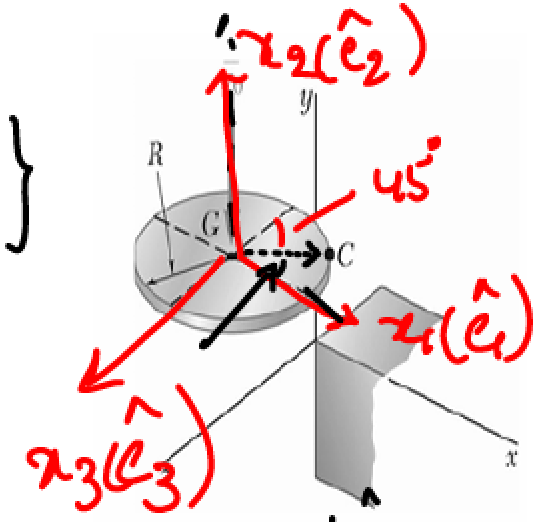
Choose $A = G$ & $\underline{r}_{GA} = 0$.

then $\int \underline{M}_G^{\text{impulse}}\,dt$

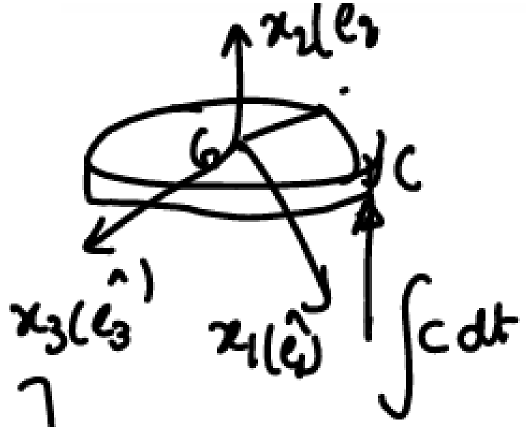
LHS $=$ $\left(\underline{r}_{CG} \times \int c\,dt\,\hat{e}_2\right)$. $\quad \underline{r}_{CG} = \frac{R}{\sqrt{2}}\{\hat{e}_1 - \hat{e}_3\}$

$$= \frac{R}{\sqrt{2}}(\hat{e}_1 - \hat{e}_3) \times \hat{e}_2 \int c\,dt$$

$$= \frac{R\int c\,dt}{\sqrt{2}}\left[\hat{e}_3 + \hat{e}_1\right] \quad \text{(iv)}$$



$\frac{R\int c\,dt}{\sqrt{2}}\left[\hat{e}_3+\hat{e}_1\right]$ (iv)



RHS = $\underline{H}_{G}(t_2) - \underline{H}_{G}(t_1)$

$$\underline{H}_G(t_2) = \begin{bmatrix} \frac{mR^2}{4} & 0 & 0 \\ 0 & \frac{mR^2}{2} & 0 \\ 0 & 0 & \frac{mR^2}{4} \end{bmatrix} \begin{bmatrix} \omega_1' \\ \omega_2' \\ \omega_3' \end{bmatrix}$$

$$\underline{\omega}_{m|I} = \omega_1'\hat{e}_1 + \omega_2'\hat{e}_2 + \omega_3'\hat{e}_3$$

$$\underline{H}_{G_I}(t_2) = \frac{mR^2}{4}\omega_1'\hat{e}_1 + \frac{mR^2}{2}\omega_2'\hat{e}_2 + \frac{mR^2}{4}\omega_3'\hat{e}_3 \quad \text{(v)}$$

$\underline{H}_G(t_1) = 0$, mateing (iv) and (v)

$$\Rightarrow \frac{R\int c\,dt}{\sqrt{2}}\hat{e}_1 + \frac{R\int c\,dt}{\sqrt{2}}\hat{e}_3 = \frac{mR^2}{4}\omega_1'\hat{e}_1 + \frac{mR^2}{2}\omega_2'\hat{e}_2 + \frac{mR^2}{4}\omega_3'\hat{e}_3$$

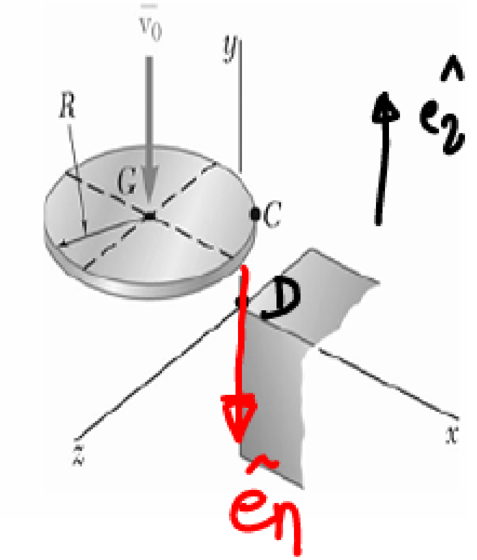
$$\Rightarrow \quad \omega_2' = 0 \quad , \quad \frac{R\int c\,dt}{\sqrt{2}} = \frac{mR^2}{4}\omega_1' \ \text{(vii)}, \quad \frac{R\int c\,dt}{\sqrt{2}}\hat{e}_3 = \frac{mR^2}{4}\omega_3'\hat{e}_3 \ \text{(viii)}$$

–

co-eff of restitution equ^{n} 0

D ∈ massive body participatg in impact

$$(\underline{V}_D')_n - (\underline{V}_c')_n = \cancel{e}\left[\qquad \right]$$



$$(\underline{V}_D')_n = (\underline{V}_c')_n$$

0

(massive body $\Rightarrow$ $(\underline{V}_c')_n = 0$

$$(\underline{V}_c')\cdot\hat{e}_2 = 0$$

$(-c)$

$\underline{V}'_c$ using m as I.F. and G as I.P.

$$\underline{V}'_c = \underline{V}'_G + \underline{\omega}'_{m|I} \times \underline{r}_{CG} + 0$$

$$= v_2'\hat{e}_2 + (\omega_1'\hat{e}_1 + \omega_3'\hat{e}_3) \times (\hat{e}_1 - \hat{e}_3)\frac{R}{\sqrt{2}}.$$

$$\underline{V}'_c = v_2'\hat{e}_2 + \omega_1'\frac{R}{\sqrt{2}}\hat{e}_2 + \omega_3'\frac{R}{\sqrt{2}}\hat{e}_2$$

so $(\underline{V}'_c)\cdot\hat{e}_2 = 0 \Rightarrow$

$$\Rightarrow v_2' + (\omega_1' + \omega_3')\frac{R}{\sqrt{2}} = 0 \qquad (ix)$$

Soln (i), vii, viii and (ix)

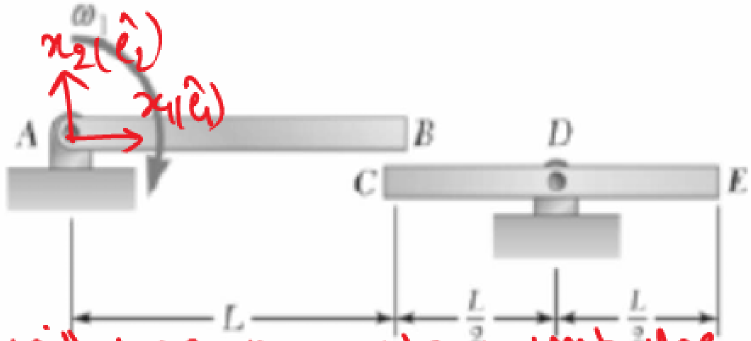
$$\left.\omega_1' = \frac{2\sqrt{2}}{5}\frac{V_0}{R}, \quad \omega_2' = 0, \quad \omega_3' = \frac{2\sqrt{2}}{5}\frac{V_0}{R}\right\}$$ Solve & verify.

$$\underline{\omega}' = 2\sqrt{2}/5\; V_0/R\,[\hat{e}_1 + \hat{e}_3]$$

Example of "2D" problem

all velocity & angular momentum vectors are w.r.t. frame I (symbol implied)

PROBLEM 17.146



A slender rod *CDE* of length *L* and mass *m* is attached to a pin support at its midpoint *D*. A second and identical rod *AB* is rotating about a pin support at *A* with an angular velocity ω_1 when its end *B* strikes end *C* of rod *CDE*. Denoting by *e* the coefficient of restitution between the rods, determine the angular velocity of each rod immediately after the impact.

We will use momentum-impulse equn set for AB and CDE.

Rod AB: the impact process is represented as

ω_1 F A B $+$ $\int A\,dt$ A $\int B\,dt$ B $=$ ω_1' A B

Rod AB: $\underline{V}_F = -\frac{\omega_1 L}{2}\hat{e}_2$ $\qquad \underline{V}_F' = -\frac{\omega_1' L}{2}\hat{e}_2$ {w.r.t. ground frame is implied}

$$\int \underline{F}^{\text{impulsive}}\,dt = m\{\underline{V}_F' - \underline{V}_F\}$$

along $\hat{e}_1$: $\quad 0 = m\{(\underline{V}_F')_1 - (\underline{V}_F)_1\}$ component along $\hat{e}_1$

$0, \quad = m(0-0)$

along $\hat{e}_2$:

$$\int A\,dt + \int B\,dt = m\{(\underline{V}_F')_2 - (\underline{V}_F)_2\}$$ component along $\hat{e}_2$

$$\int A\,dt + \int B\,dt = m\left\{-\frac{\omega_1' L}{2} - \left(-\frac{\omega_1 L}{2}\right)\right\}$$

$$\int A\,dt + \int B\,dt = \frac{mL}{2}(\omega_1 - \omega_1') \qquad \rightarrow (i)$$

Moment of momentum-impulse about point F

$$\int (\underline{M}_F^{\text{impulsive}})_3\,dt = (\underline{H}_F')_3 - (\underline{H}_F)_3$$ where

$\underline{H}_F$ represents ang. momentum of AB about F or

$$-\int A\,dt\,\frac{L}{2} + \frac{L}{2}\int B\,dt = I^F_{33}\left\{-\omega_1' - (-\omega_1)\right\}$$

(counterclockwise is +ve)

$$\frac{L}{2}\left\{\int B\,dt - \int A\,dt\right\} = \frac{mL^2}{12}(\omega_1-\omega_1')$$

or $\int B\,dt - \int A\,dt = \frac{mL}{6}(\omega_1-\omega_1')$ (ii)

Add (ii) and (i)

$$2\int B\,dt = \frac{mL}{6}(\omega_1-\omega_1') + \frac{mL}{2}(\omega_1-\omega_1')$$

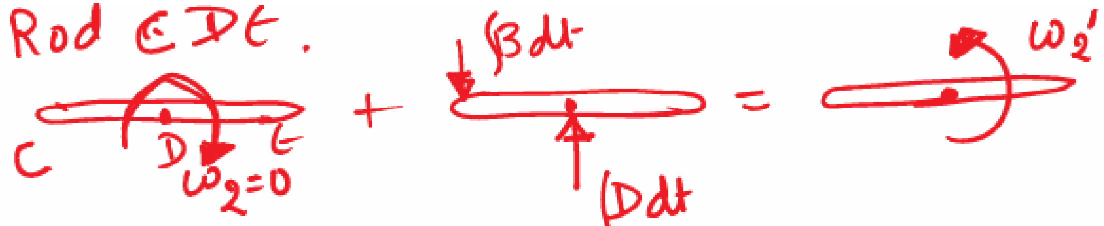
$$\int B\,dt = mL(\omega_1-\omega_1')\left[\frac{1}{12}+\frac{1}{4}\right]$$

$$\int B\,dt = \frac{mL(\omega_1-\omega_1')}{3} \rightarrow \text{(iii)}$$

We need more eqns to find all the unknowns.

Go to RB (CDE)

Rod CDE.



Angular momentum impulse eqn. about point D (centre of mass of rod CDE).

$$\int B\,dt\,\frac{L}{2} = (\underline{H}_D')_3 - (\underline{H}_D)_3$$

Where $\underline{H}_D$ is ang. momentum of rod CDE about D (w.r.t. ground)

$$\frac{L}{2}\int B\,dt = I^D_{33}\,\omega_2' - I^D_{33}(0)$$

$$\frac{\cancel{L}}{\cancel{2}}\int B\,dt = \frac{mL^{\cancel{2}}}{\cancel{12}\;6}\,\omega_2' \rightarrow \text{(iv)}$$

Sub $\int B\,dt$ from (iii)

$$\int B\,dt = \frac{\cancel{mL}}{\cancel{3}}(\omega_1-\omega_1') = \frac{\cancel{mL}}{\cancel{6}\;2}\,\omega_2'$$

or $2\omega_1 - 2\omega_1' = \omega_2' \rightarrow$ (v)

next, use coeff. of restitution between points B and C.

$(\underline{v}_c')_n - (\underline{v}_B')_n = e\{(\underline{v}_B)_n - (\underline{v}_c)_n\}$

where 'n' means component along $\hat{e}_2$

$\omega_2' \frac{L}{2} - (\omega_1' L) = e\{\omega_1 L - 0\}$

$\omega_2' - 2\omega_1' = 2e\omega_1$

$\omega_2' = 2e\omega_1 + 2\omega_1' \longrightarrow (vi)$

Sub (vi) in (v)

$\cancel{2}\omega_1 - \cancel{2}\omega_1' = \cancel{2}e\omega_1 + \cancel{2}\omega_1'$

$(1-e)\omega_1 = 2\omega_1'$

or $\boxed{\omega_1' = \frac{\omega_1}{2}(1-e)}$

and $\omega_2' = 2e\omega_1 + \cancel{2}\frac{\omega_1}{\cancel{2}}(1-e)$

$= 2e\omega_1 + \omega_1 - \omega_1 e.$

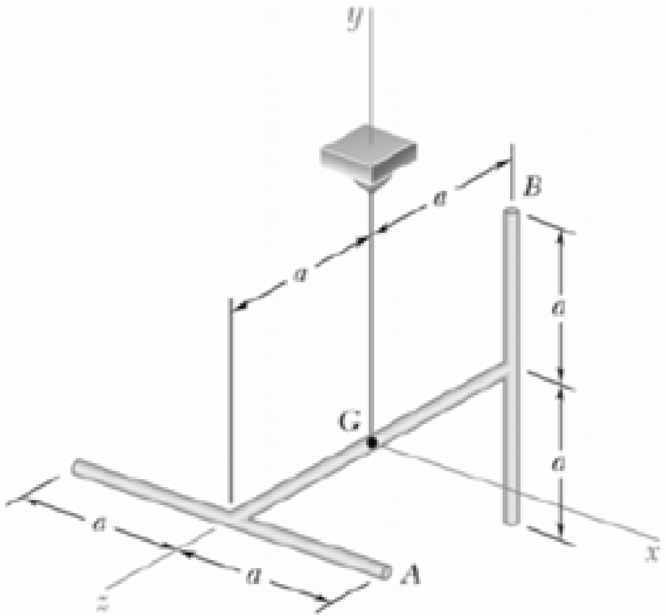
$\boxed{\omega_2' = \omega_1(1+e)}$

In vector form:

$$\left.\begin{aligned} \underline{\omega}_{AB|I} &= -\frac{\omega_1}{2}(1-e)\,\hat{e}_3 \\ \underline{\omega}_{CDE|I} &= \omega_1(1+e)\,\hat{e}_3 \end{aligned}\right\}$$

Answer

Set 9 B:

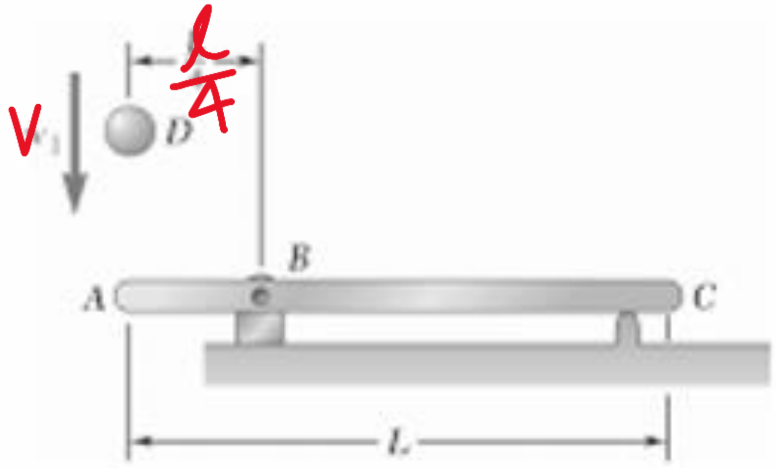


PROBLEM 18.25

Three slender rods, each of mass m and length $2a$, are welded together to form the assembly shown. The assembly is hit at A in a vertical downward direction. Denoting the corresponding impulse by $\mathbf{F}\,\Delta t$, determine immediately after the impact (a) the velocity of the mass center G, (b) the angular velocity of the rod.

$$\boldsymbol{\omega} = (3F\Delta t/8ma)(\mathbf{i} - 4\mathbf{k}) \blacktriangleleft$$

$$\bar{\mathbf{v}} = 0 \blacktriangleleft$$

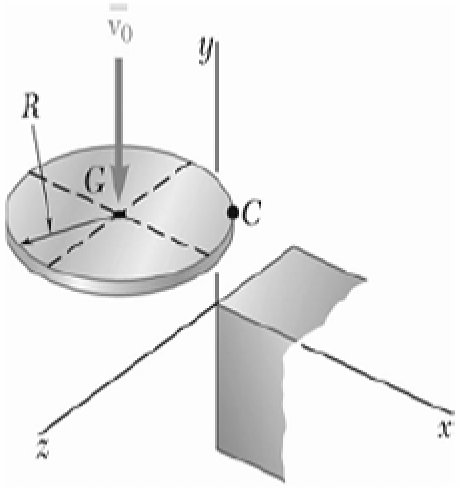


PROBLEM 17.127

Member ABC has a mass of 2.4 kg and is attached to a pin support at B. An 800-g sphere D strikes the end of member ABC with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750$ mm and that the coefficient of restitution between the sphere and member ABC is 0.5, determine immediately after the impact (*a*) the angular velocity of member ABC, (*b*) the velocity of the sphere.

$\omega' = 3.00$ rad/s ↻ ◀

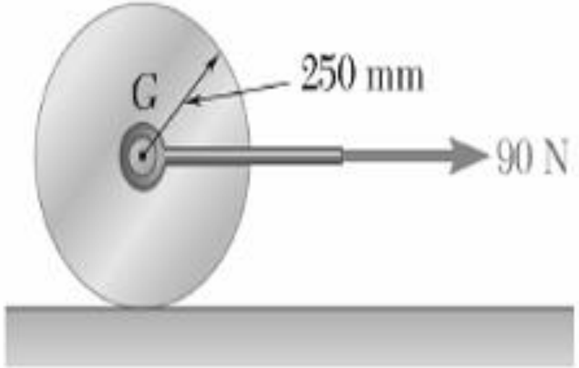
$v'_D = 0.938$ m/s ↑ ◀



PROBLEM 18.51

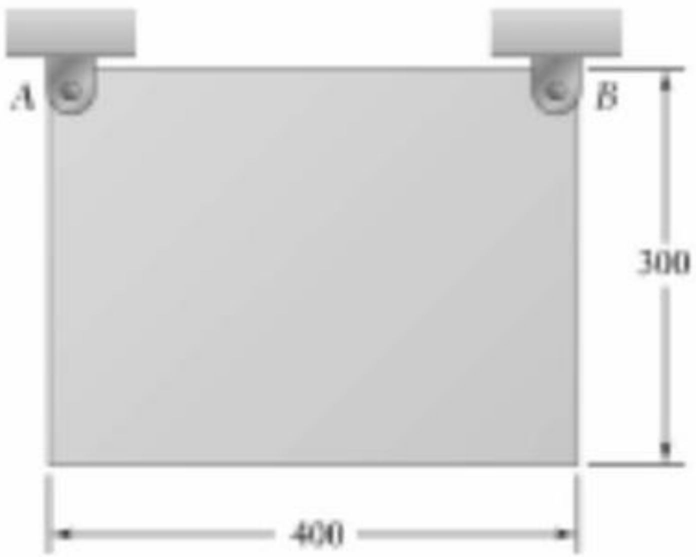
Determine the kinetic energy lost when edge C of the plate of Problem 18.29 hits the obstruction.

$$T_0 - T = \frac{1}{10} m\bar{v}_0^2 \blacktriangleleft$$



PROBLEM 17.27

A 20-kg uniform cylindrical roller, initially at rest, is acted upon by a 90-N force as shown. Knowing that the body rolls without slipping, determine (*a*) the velocity of its center *G* after it has moved 1.5 m, (*b*) the friction force required to prevent slipping.

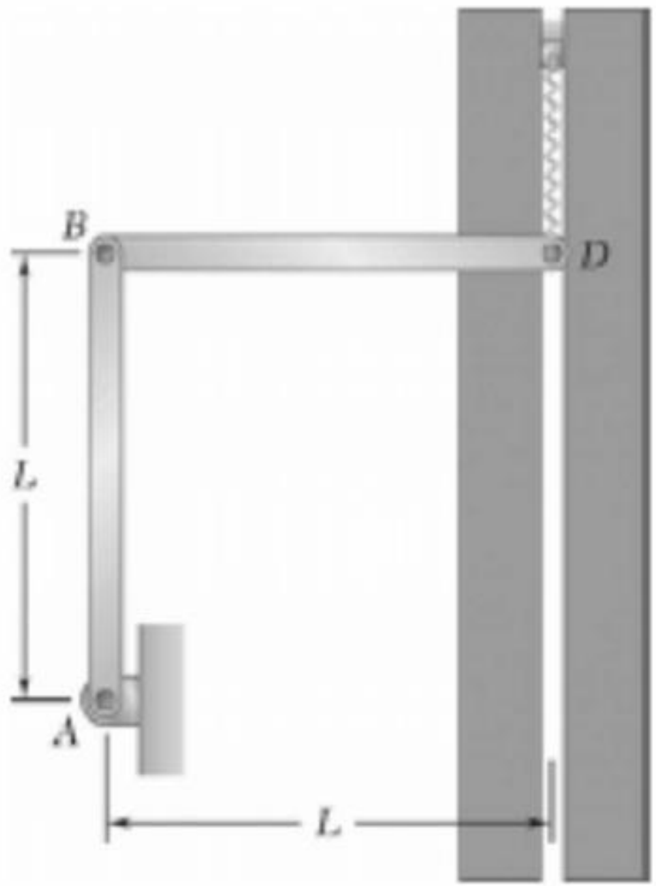


PROBLEM 17.137

A 300× 400 mm-rectangular plate is suspended by pins at A and B. The pin at B is removed and the plate swings freely about pin A. Determine (*a*) the angular velocity of the plate after it has rotated through 90°, (*b*) the maximum angular velocity attained by the plate as it swings freely.

$\omega_2 = 3.43$ rad/s ◄

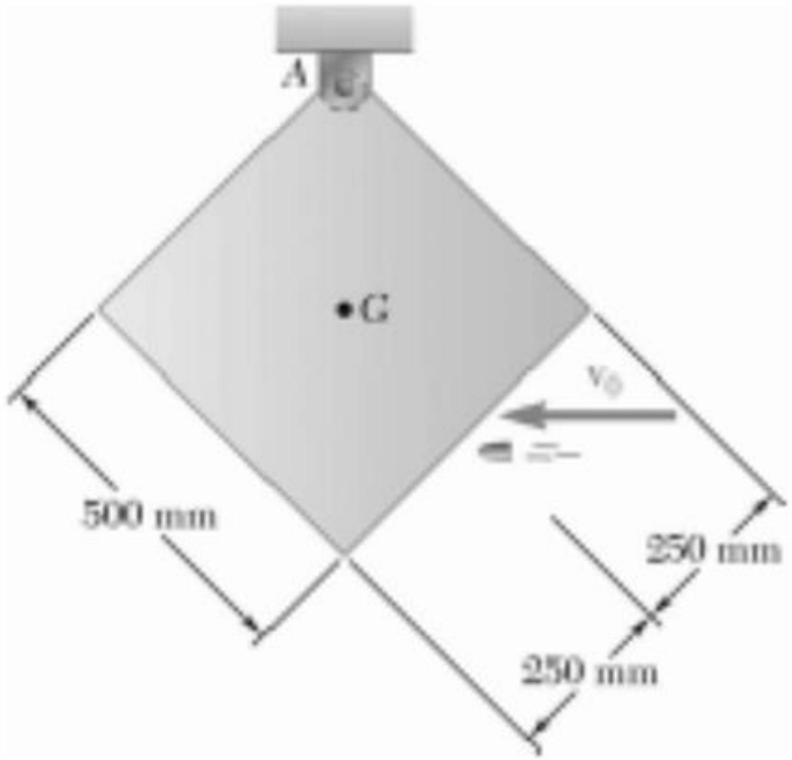
$\omega_3 = 4.85$ rad/s ◄



PROBLEM 17.42

Each of the two rods shown is of length $L = 1$ m and has a mass of 5 kg. Point D is connected to a spring of constant $k = 20$ N/m and is constrained to move along a vertical slot. Knowing that the system is released from rest when rod BD is horizontal and the spring connected to Point D is initially unstretched, determine the velocity of Point D when it is directly to the right of Point A.

$\mathbf{v}_D = 2.69 \text{ m/s} \downarrow$ ◀



PROBLEM 17.141

A 35-g bullet *B* is fired horizontally with a velocity of 400 m/s into the side of a 3-kg square panel suspended from a pin at *A*. Knowing that the panel is initially at rest, determine the components of the reaction at *A* after the panel has rotated 45°.

$\mathbf{A}_x = 189.7\text{ N} \longrightarrow$ ◀

$\mathbf{A}_y = 7.36\text{ N}\uparrow$ ◀