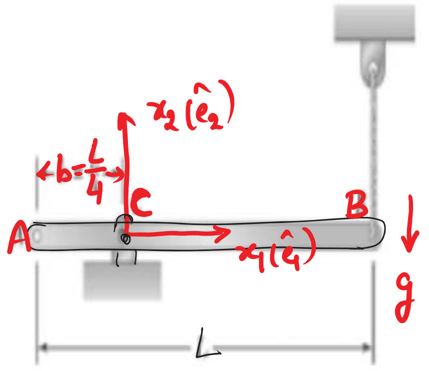
Set 7 A



PROBLEM 16.85

A uniform rod of length L and mass m is supported as shown. If the cable attached at end B suddenly breaks, determine (*a*) the acceleration of end B, (*b*) the reaction at the pin support.

$$\underline{a}_{B|I} = \frac{9g}{7}\downarrow \qquad C_2 = \frac{4mg}{7},\ C_1 = 0$$

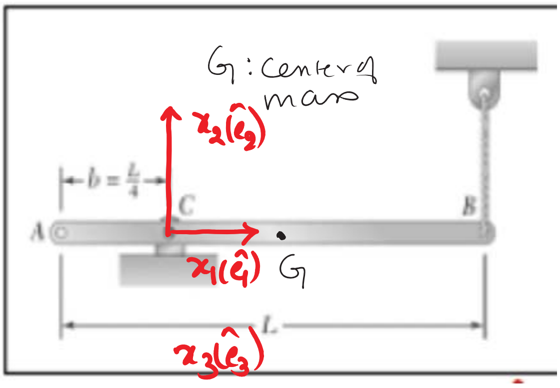
It is an example of "2D problems"

Governing equations of motion

$$F_1 = m(\underline{a}_{G|I})_1$$

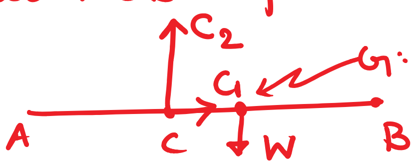
$$F_2 = m(\underline{a}_{G|I})_2$$

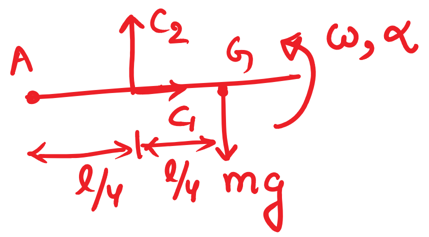
$$M_{A_3} - (\underline{r}_{GA} \times \underline{a}_{A|I})_3 m = I^A_{33}\alpha$$



WCSYS: $C\,x_1(\hat{e}_1)\,x_2(\hat{e}_2)\,x_3(\hat{e}_3)$

FBD of System = Rod ACB after the String has been broken



~~for~~ Euler's first axiom along $\hat{e}_1$ 

$C_1 = m(\underline{a}_{G|I})_1$ (i)

~~o~~ Euler's ~~2nd~~ 1st axiom along $\hat{e}_2$

$C_2 - mg = m(\underline{a}_{G|I})_2$ (ii)

Euler's 2nd axiom along $\hat{e}_3$ about point G

$$M_{G3} - (\cancel{\underline{r}_{G/G}}^{\,0} \times m\, \underline{a}_{G|I})_3 = I^G_{33}\alpha$$

$$-C_2 \frac{l}{4} = I^G_{33}\alpha \qquad I^G_{33} = ml^2/12$$

$$-C_2 l/4 = \frac{ml^2}{12}\alpha \rightarrow \text{(iii)}$$

note $\omega_{m|I} = 0$ because motion starts from rest

3 equ^ns but unknowns $C_1, C_2, (\underline{a}_{G|I})_1$, ~~...~~, α, $(\underline{a}_{G|I})_2$

What are $(\underline{a}_{G|I})_1$ and $(\underline{a}_{G|I})_2$? Use Kinematics to get relationship.

$\Rightarrow (\underline{a}_{G|I})_2 = \alpha \frac{l}{4}$ (iv), $(\underline{a}_{G|I})_1 = 0$ (v), These are 2 additional equations.

Now we have 5 unknowns ($C_1, C_2, (\underline{a}_{G|I})_1, (\underline{a}_{G|I})_2, \alpha$) and 5 equ^ns. Solve (HW) to arrive at _

$C_1 = 0,\quad C_2 = \frac{4mg}{7},\quad (a_{G|I})_1 = 0,\quad (a_{G|I})_2 = -\frac{12g}{28},\quad \alpha = -\frac{12g}{7\ell}.$

One we have α, we can readily find $a_{B|I}$

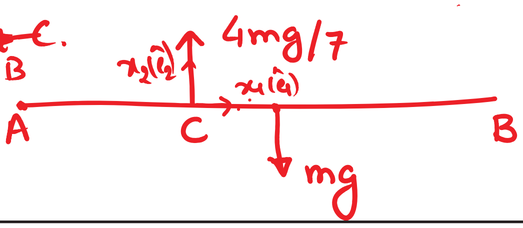
$$a_{B|I} = \alpha\hat{e}_3 \times \left(\frac{3\ell}{4}\hat{e}_1\right) = 3\frac{\alpha\ell}{4}\hat{e}_2 = -\frac{9g}{7}\hat{e}_2$$

$$\boxed{a_{B|I} = -\frac{9g}{7}\hat{e}_2}$$

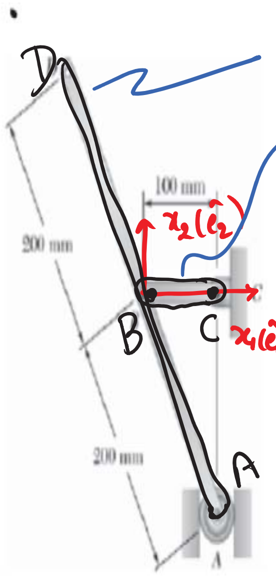
$$\Rightarrow \boxed{a_{B|I} = -9g/7\,\hat{e}_2}$$

~~Reaction at~~ C.

FBD of ACB



Reaction force at C acting on ACB is $4mg/7\ \hat{e}_2$. at this instant.



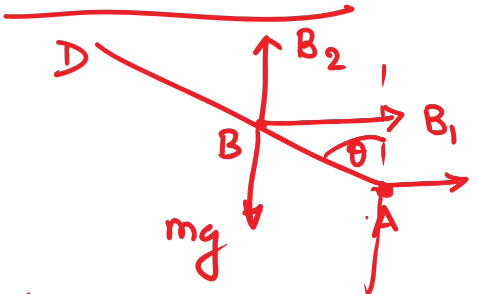
PROBLEM 16.124

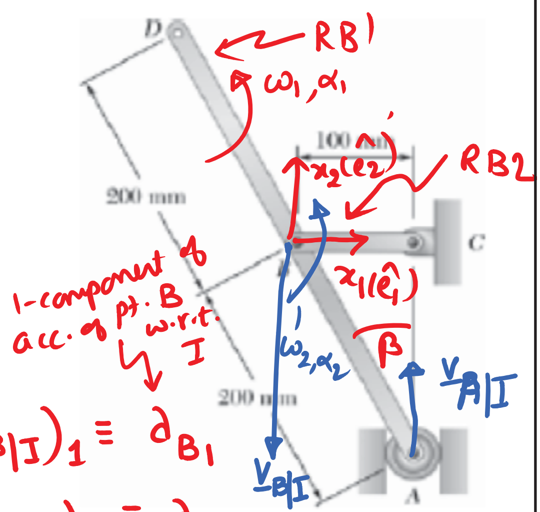
Multiple rigid bodies in "2D" motion

The 4-kg uniform rod *ABD* is attached to the crank *BC* and is fitted with a small wheel that can roll without friction along a vertical slot. Knowing that at the instant shown crank *BC* rotates with an angular velocity of 6 rad/s clockwise and an angular acceleration of 15 rad/s² counterclockwise, determine the reaction at *A*. and at B. being exerted on Rod ABD

Two RBs ① and ②

FBD of ABD (RB1)





Euler 1st axiom for RB 1

$B_1 + A_1 = m_1 a_{B1}$ (†) $\quad (a_{B|I})_1 \equiv a_{B1}$

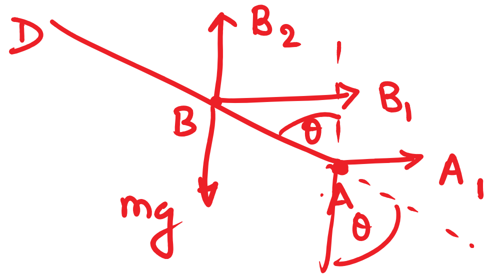
$B_2 - m_1 g = m_1 a_{B2}$ (†*) $\quad (a_{B|I})_2 \equiv a_{B2}$.

Euler's 2nd axiom for RB 1 about point B (which is CM)

$M_{B_3} = I^B_{33}\alpha_1$ (note $\omega_1 = 0$, because $\underline{v}_{B|I} \parallel \underline{v}_{A|I}$ and $\cos\beta \neq 0$)

$M_{B_3} = I^{B}_{33}\alpha_1$

$\frac{A_1 l \cos\theta}{2} = \frac{m_1 l^2}{12}\alpha_1$ $\quad$ +



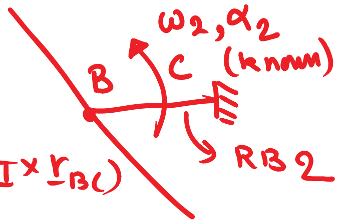
Unknowns : B_1, B_2, A_1, α.

equns : 3

Let's investigate kinematics ~~for~~ to get more equations

l = length of ABD

from the side of C, $\underline{a}_{B|I}$ can be found. using 2 as I.F. and C as I.P.



known $\quad$ known

$\underline{a}_{B|I} = \cancel{\underline{a}_{C|I}} + \dot{\underline{\omega}}_{2|I} \times \underline{r}_{BC} + \underline{\omega}_{2|I} \times (\underline{\omega}_{2|I} \times \underline{r}_{BC}) + \cancel{2\underline{\omega}_{2|I} \times \underline{v}_{B|2}} + \cancel{\underline{a}_{B|2}}$

← given

$\dot{\underline{\omega}}_{2|I} = 15 e_3$

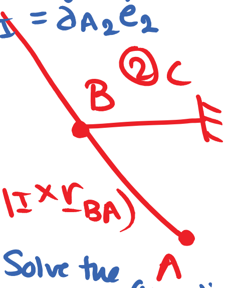
$\underline{\omega}_{2|I} = -6\hat{e}_3$

$\underline{a}_{B|I} = 3.6\hat{e}_1 - 1.5\hat{e}_2 \quad (*)$ ✓

Now writing expression for $\underline{a}_{B|I}$ from the side of A

Since point A can move only in the vertical direction, $\underline{a}_{A|I} = a_{A_2}\hat{e}_2$

Using RB 1 as I.F. and A as I.P. $\quad$ ①



$a_{A_2}\hat{e}_2$ $\quad$ (a_{A_2} is scalar)

$\underline{a}_{B|I} = \underline{a}_{A|I} + \dot{\underline{\omega}}_{1|I} \times \underline{r}_{BA} + \underline{\omega}_{1|I} \times (\underline{\omega}_{1|I} \times \underline{r}_{BA}) + 2\underline{\omega}_{1|I} \times \cancel{\underline{V}_{B|1}} + \cancel{\underline{a}_{B|1}}$

← unknown

$\dot{\underline{\omega}}_{1|I} = \alpha_1\hat{e}_3 \quad \underline{\omega}_{1|I} = 0$

Solve the question ~~Verify~~ yourself & verify

$\underline{a}_{B|I} = a_{A_2}\hat{e}_2 - .2\alpha_1 \sin\theta\, \hat{e}_2 - .2\alpha_1 \cos\theta\, \hat{e}_1 \rightarrow (**)$

Match $(*)$ & $(**)$ and solve for α_1

$\alpha_1 = -20.78 \text{ rad s}^{-1}$, (can ~~cal~~ find a_{A_2} as well) use α_1 to find A_1 (†) equ^n

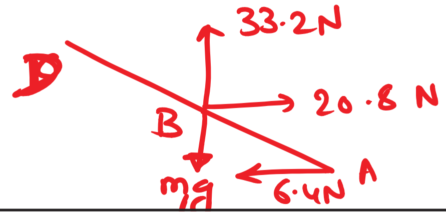
$$A_1 \frac{l}{2}\cos\theta = \frac{m_1 l^2}{12}\alpha_1 \qquad \alpha_1 \text{ known.}$$

$$\Rightarrow \boxed{A_1 = -6.4\,N} \quad \text{(Verify)}$$

Now use (††) and (†*) to arrive at

$$\left.\begin{array}{l} B_1 = 20.8\,N. \\ B_2 = 33.2\,N \end{array}\right\} \text{Verify (HW).}$$

FBD of Rod ABD



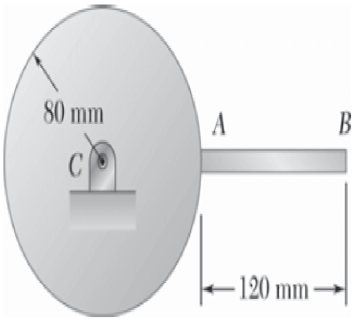
Reaction force exerted on ABD at

$B = 20.8\,\hat{e}_1 + 33.2\,\hat{e}_2$ N Answer.

and at A $-6.4\,\hat{e}_1$ N

Set 7 B

Single RB

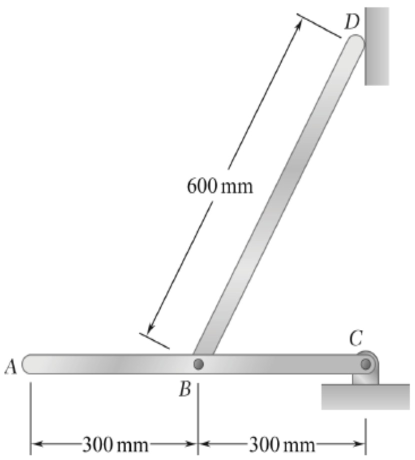


PROBLEM 16.87

A 1.5-kg slender rod is welded to a 5-kg uniform disk as shown. The assembly swings freely about C in a vertical plane. Knowing that in the position shown the assembly has an angular velocity of 10 rad/s clockwise, determine (*a*) the angular acceleration of the assembly, (*b*) the components of the reaction at C.

$\alpha = 43.6\ \text{rad/s}^2$ ◀ $C_x = 21.0\ \text{N} \leftarrow$ ◀ $C_y = 54.6\ \text{N} \uparrow$ ◀

Two RBs



PROBLEM 16.134

Two 4-kg uniform bars are connected to form the linkage shown. Neglecting the effect of friction, determine the reaction at D immediately after the linkage is released from rest in the position shown.

$D = 1.618\ \text{N} \leftarrow$ ◀

For more problems, B&J Chapter 16