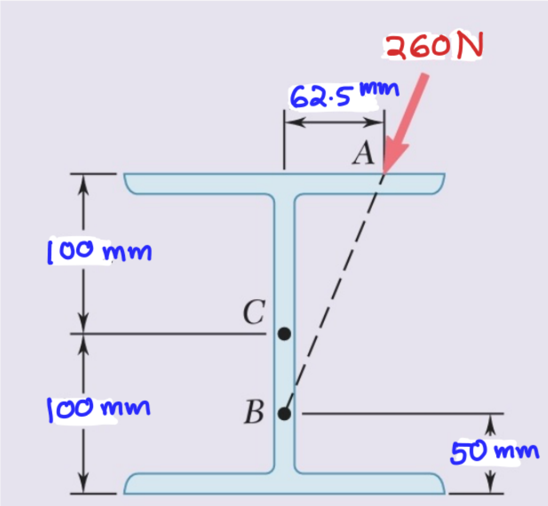
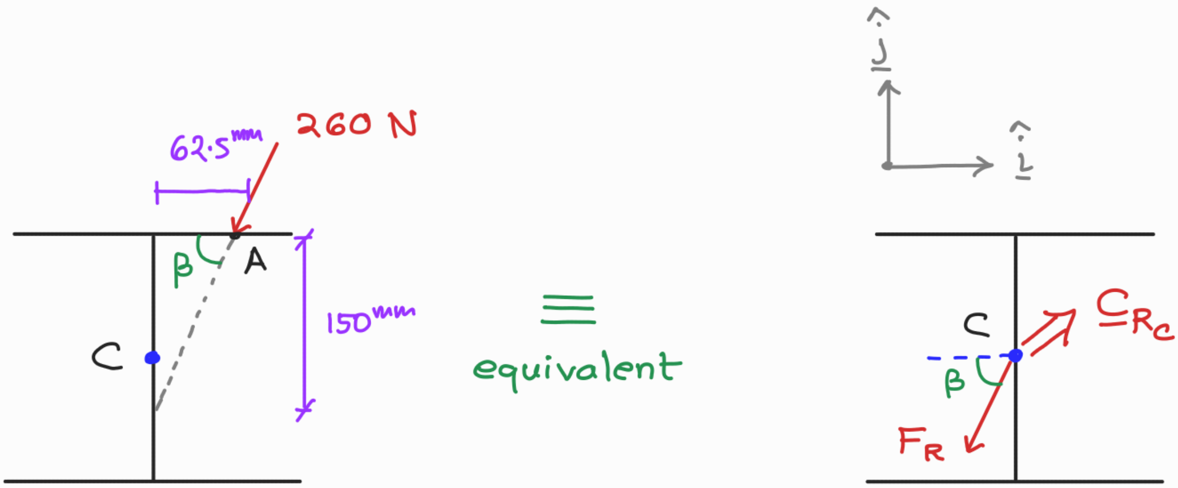
Tutorial 4 solutions

1)



A 260 N force is applied at A to the rolled-steel section.

Replace that force with an equivalent force-couple system at the center C of the section.



To get the equivalent force system, we need to satisfy two conditions

ⓐ $\underline{F}_R = \underline{F}_R'$

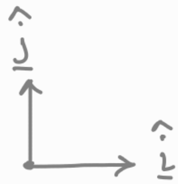
ⓑ $\underline{M}_A = \underline{M}_A'$

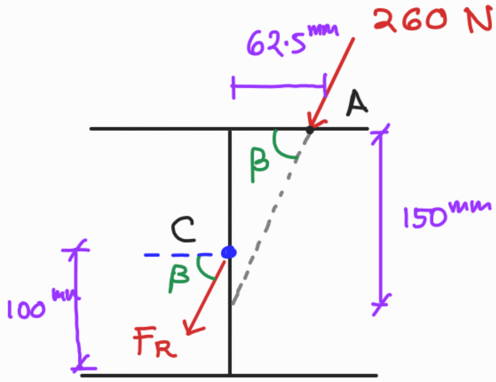
Recall

$$[\text{Force}_A] \xrightarrow[\text{to C}]{\text{transfer}} [\text{Same force at C}] + [\text{One couple}]$$

Let's get angle β : $\beta = \tan^{-1}\left(\frac{150}{62.5}\right)$

$= 1.176$ rad





Using (a), get force at C

$$(\underline{F}_R)_C = -260\cos\beta\,\hat{\underline{i}} - 260\sin\beta\,\hat{\underline{j}}$$

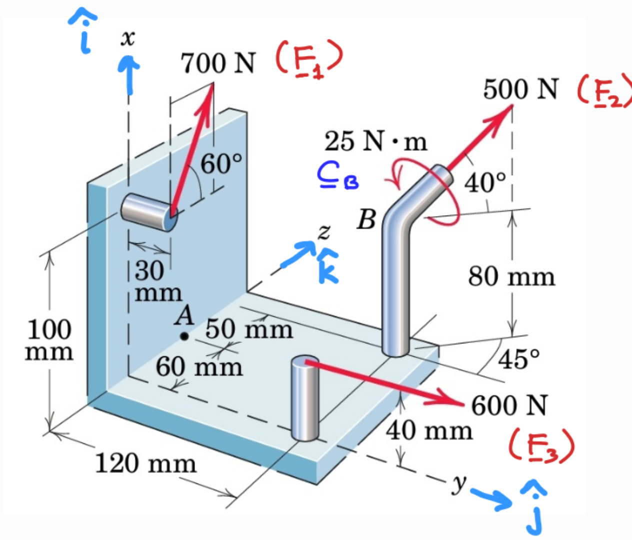
$$= -100\,\hat{\underline{i}} - 240\,\hat{\underline{j}} \quad (N)$$

Using (b), we get the resultant couple at C

$$(\underline{C}_R)_C = \underline{r}_{AC} \times \underline{F}_A = \left(\frac{62.5}{1000}\hat{\underline{i}} + \frac{100}{1000}\hat{\underline{j}}\right) \times \left(-100\,\hat{\underline{i}} - 240\,\hat{\underline{j}}\right)$$

$$= -5\,\hat{\underline{k}} \quad (Nm)$$

2) Find the equivalent force system at point A



The given system has three forces and a couple. First, we need to write the forces and the couple in terms of its components.

$$\underline{F}_1 = 700 \sin 60° \, \hat{i} + 700 \cos 60° \, \hat{k} \quad (N)$$

$$= 606.22 \, \hat{i} + 350 \, \hat{k} \; (N)$$

$$\underline{F}_2 = 500 \sin 40° \, \hat{i} + 500 \cos 40° \cos 45° \, \hat{j} + 500 \cos 40° \sin 45° \, \hat{k}$$

$$= 321 \, \hat{i} + 270.84 \, \hat{j} + 270.84 \, \hat{k} \quad (N)$$

$$\underline{F}_3 = 600 \, \hat{j} \quad (N)$$

$$\underline{C}_B = -25 \sin 40° \, \hat{i} - 25 \cos 40° \cos 45° \, \hat{j} - 25 \cos 40° \sin 45° \, \hat{k}$$

$$= -16.07 \, \hat{i} - 13.54 \, \hat{j} - 13.54 \, \hat{k} \quad (Nm)$$

Using the same idea of equivalent force system

$$\underline{F}_R = \sum_{i=1}^{3} \underline{F}_i$$

$$\Rightarrow \quad (\underline{F}_R)_A = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$= 927.22 \, \hat{i} + 870.84 \, \hat{j} + 620.84 \, \hat{k}$$

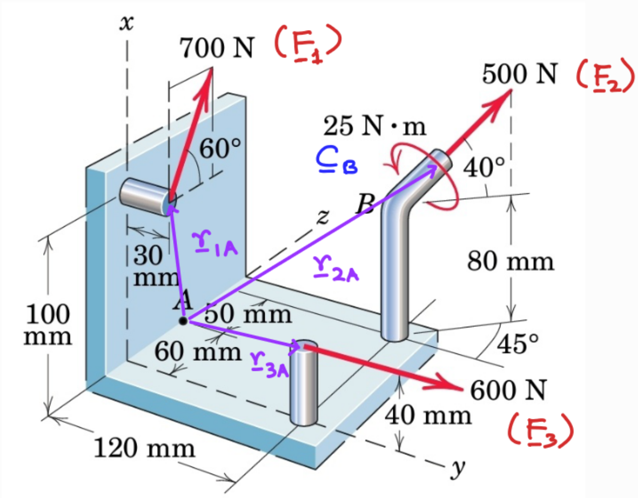
Using the equivalent of moment about pt A

$$(\underline{C}_R)_A = \sum_{i=1}^{3} (\underline{r}_{iA} \times \underline{F}_i) + \underline{C}_B$$

$$\underline{r}_{1A} = 0.1\,\hat{i} + 0.03\,\hat{j} - 0.06\,\hat{k} \;(m)$$

$$\underline{r}_{2A} = 0.08\,\hat{i} + 0.12\,\hat{j} + 0.05\,\hat{k} \;(m)$$

$$\underline{r}_{3A} = 0.04\,\hat{i} + 0.12\,\hat{j} - 0.06\,\hat{k} \;(m)$$



$$\underline{r}_{1A} \times \underline{F}_1 = (0.1\,\hat{i} + 0.03\,\hat{j} - 0.06\,\hat{k}) \times (606.22\,\hat{i} + 350\,\hat{k})$$

$$= 10.5\,\hat{i} - 71.37\,\hat{j} - 18.19\,\hat{k} \;(Nm)$$

$$\underline{r}_{2A} \times \underline{F}_2 = (0.08\,\hat{i} + 0.12\,\hat{j} + 0.05\,\hat{k}) \times (321\,\hat{i} + 270.84\,\hat{j} + 270.84\,\hat{k})$$

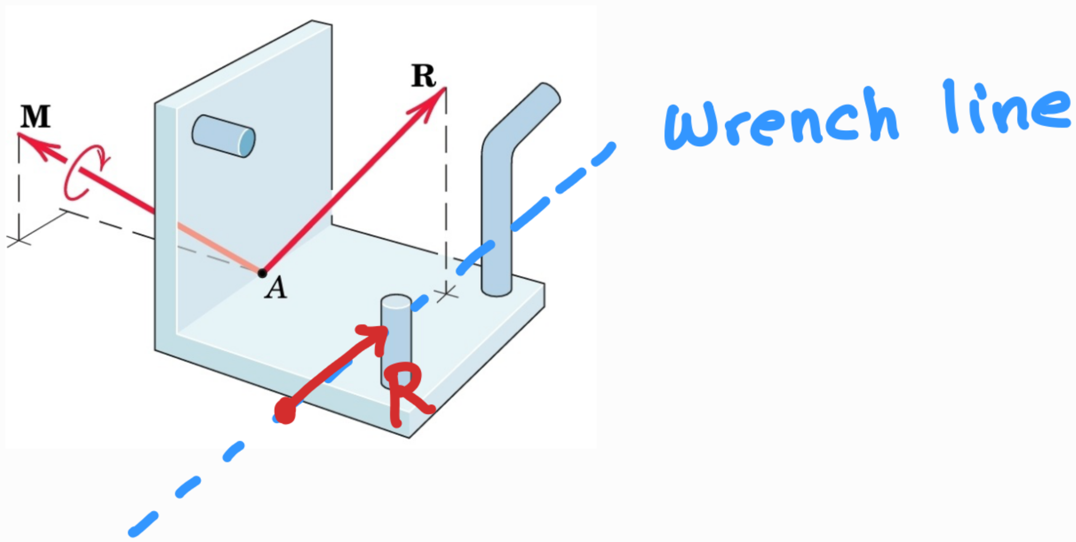
$$= 18.96\,\hat{i} - 5.62\,\hat{j} - 16.85\,\hat{k} \;(Nm)$$

$$\underline{r}_{3A} \times \underline{F}_3 = (0.04\,\hat{i} + 0.12\,\hat{j} - 0.06\,\hat{k}) \times 600\,\hat{j}$$

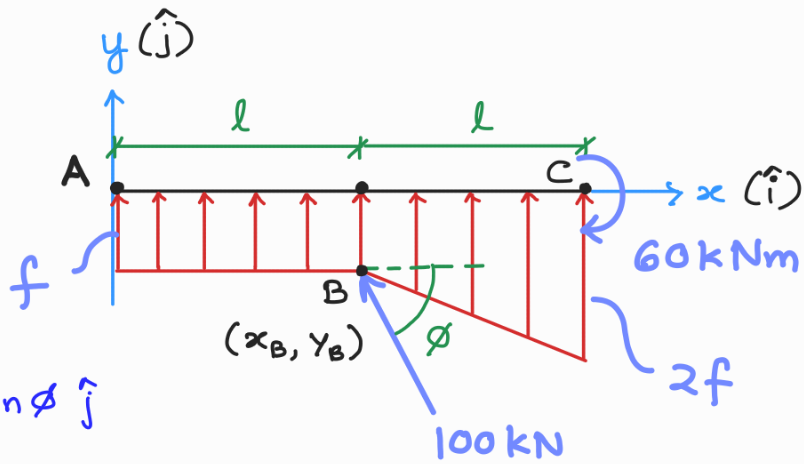
$$= 36\,\hat{i} + 24\,\hat{k} \;(Nm)$$

$$\therefore \quad (\underline{C}_R)_A = \left(65.4\hat{i} - 76.99\hat{j} - 11.04\hat{k}\right) + \left(-16.07\hat{i} - 13.54\hat{j} - 13.54\hat{k}\right)$$

$$= 49.33\hat{i} - 90.53\hat{j} - 24.58\hat{k} \quad (\mathrm{Nm})$$



3) Find the equivalent force system at point A and the wrench location.



Given: $l = 6$ m, $f = 10$ kN

$\cos\phi = 0.8 \Rightarrow \sin\phi = 0.6$

$x_B = 6$ m, $y_B = -5$m

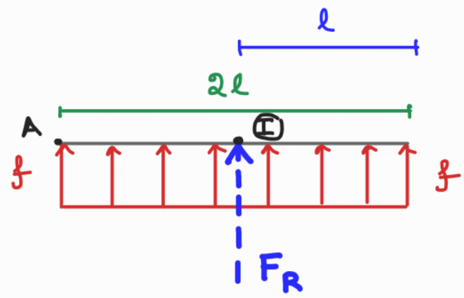
$$\underline{F}_B = -100\cos\phi\,\hat{i} + 100\sin\phi\,\hat{j}$$

$$= -80\,\hat{i} + 60\,\hat{j} \quad (\text{kN})$$

$$\underline{C}_C = -60\,\hat{k} \quad (\text{kNm})$$

This problem tests how to find equivalence of distributed force systems. The given parallel and coplanar distributed force can be split up into two sets:

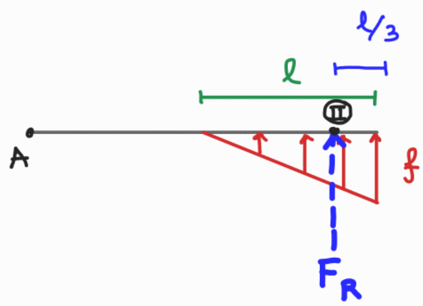
(I) Rectangular / Uniform distribution of magnitude 'f' and length '$2l$'



Resultant, $F_R^{(I)} = f(2l)\,\hat{j} = 120\,\hat{j}$ (kN)

Moment arm from A, $\underline{r}_{(I)A} = l\,\hat{i}$ (m)

(II) Triangular distribution of maximum magnitude 'f' and length 'l'



Resultant, $F_R^{(II)} = \frac{1}{2} f\, l\,\hat{j} = 30\,\hat{j}$ (kN)

Moment arm from A, $\underline{r}_{(II)A} = \left[l + \left(l - \frac{l}{3}\right)\right]\hat{i} = \frac{5l}{3}\,\hat{i}$ (m)

The total resultant force of the equivalent force system is:

$$\underline{F}_R = \underline{F}_B + \underline{F}_R^{\text{I}} + \underline{F}_R^{\text{II}}$$

$$= -80\hat{i} + 60\hat{j} + 120\hat{j} + 30\hat{j}$$

$$= -80\hat{i} + 210\hat{j} \quad (kN)$$

The total resultant couple of the equivalent force system at point A is:

$$(\underline{C}_R)_A = \underline{r}_B \times \underline{F}_B + \underline{r}_{\text{I}A} \times \underline{F}_R^{\text{I}} + \underline{r}_{\text{II}A} \times \underline{F}_R^{\text{II}} + \underline{C}_c$$

$$= (x_B\hat{i} + y_B\hat{j}) \times (-80\hat{i} + 60\hat{j}) + (l\hat{i}) \times (120\hat{j})$$

$$+ \left(\frac{5}{3} l\hat{i}\right) \times (30\hat{j}) - 60\hat{k}$$

$$= (6\hat{i} - 5\hat{j}) \times (-80\hat{i} + 60\hat{j}) + (6\hat{i}) \times (120\hat{j})$$

$$+ \left(\frac{5}{3} \cdot 6\hat{i}\right) \times (30\hat{j}) - 60\hat{k}$$

$$= -40\hat{k} + 720\hat{k} + 300\hat{k} - 60\hat{k}$$

$$= 920\hat{k} \quad (Nm)$$

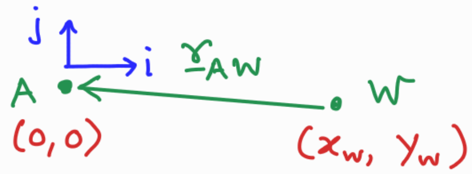
Note that $(\underline{C}_R)_A$ is perpendicular to $\underline{F}_R$, i.e

$$\underline{C}_{R_A}^{\parallel} = 0$$

Therefore, the wrench position W can be found as

minus signs since directed negatively

$$\underline{r}_{AW} \times \underline{F}_R + \underline{C}_{R_A}^{\,h} = \underline{0}$$

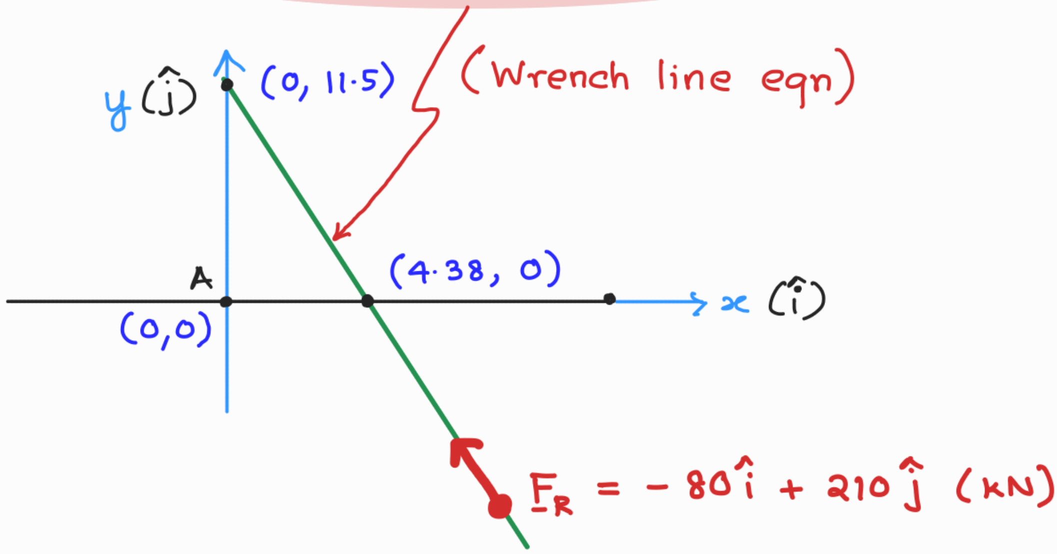


$$\Rightarrow \quad (-x_w \hat{i} - y_w \hat{j}) \times (-80\hat{i} + 210\hat{j}) + 920\hat{k} = \underline{0}$$

$$\Rightarrow \quad (-210x - 80y + 920)\hat{k} = \underline{0}$$

Evaluating only the kth component from both sides, we get the equation of the WRENCH LINE:

$$21x_w + 8y_w - 92 = 0$$



You can place $\underline{F}_R$ at any point on the wrench line and you will still get the same resultant couple $\underline{C}_{R_A}^{\,h}$ at A.

This is because the location of the force does not matter as long as the force appears along the same line of action!