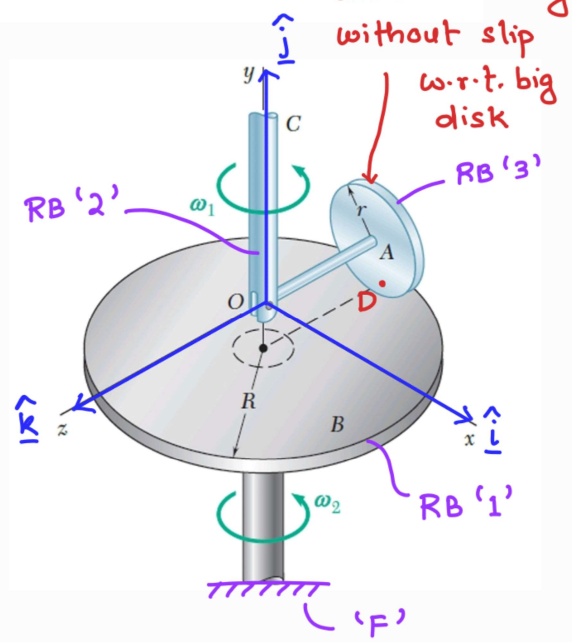
Tutorial 3 solution

1) In the system shown (Figure 1), disk A is free to rotate about the horizontal rod OA. Assuming that shaft OC and disk B rotate with constant angular velocities ω_1 and ω_2, respectively, both counterclockwise. Determine:

(a) The angular velocity of disk A w.r.t ground

(b) The angular acceleration of disk A w.r.t ground

The smaller disc is rolling without slip w.r.t. big disk



Notice this is a Category II problem where angular velocity & acceleration of the disk A is unknown and we will find the velocity of point of contact D from two sides to arrive at required equations.

① Identify the ref. frames

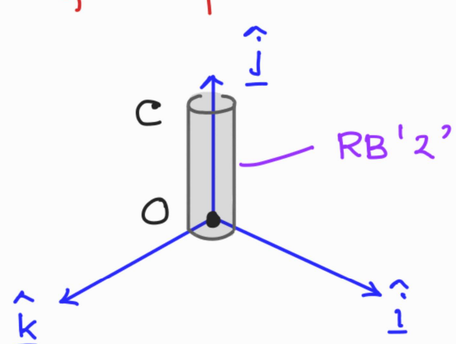
'F' → ground

'1' → disk B

'2' → shaft OCA

'3' → disk A

② Select a coordinate sys so that we can express our answers in terms of components



This csys is fixed to RB '2'

③ Use "rolling without slip" condition at point of contact D from two sides

From side COA (I)

$$\underline{v}_{D|F}^{(I)} = \cancel{\underline{v}_{D|3}}^{\,0} + \overset{?}{\underline{v}_{A|F}} + \overset{?}{\underline{\omega}_{3|F}} \times \underline{r}_{DA}$$

$$\underline{r}_{DA} = -r\,\hat{\underline{j}}$$

$$\underline{\omega}_{3|F} = \overset{?}{\underline{\omega}_{3|2}} + \underline{\omega}_{2|F}$$

$$= \omega\,\hat{\underline{k}} + \omega_1\,\hat{\underline{j}}$$

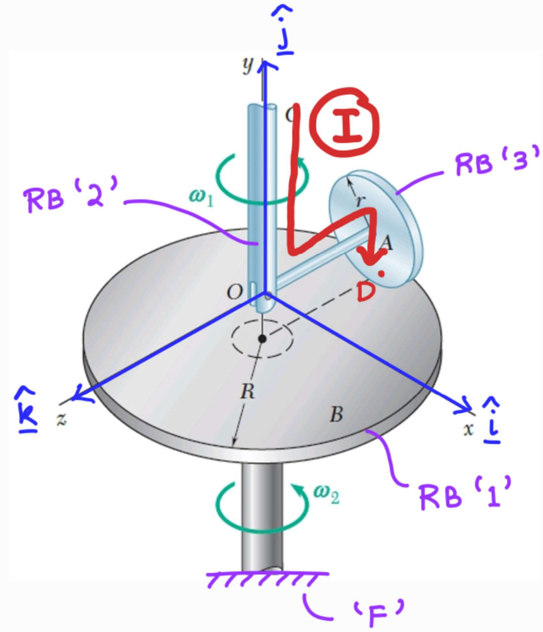
ω – unknown scalar

You can think of point A as lying at the end of shaft OCA:

$$\underline{v}_{A|F} = \overset{0}{\cancel{\underline{v}_{A|2}}} + \overset{0}{\cancel{\underline{v}_{O|F}}} + \underline{\omega}_{2|F} \times \underline{r}_{AO}$$

$$= \underline{0} + \underline{0} + \omega_1 \hat{\underline{j}} \times (-R\hat{\underline{k}})$$

$$= -\omega R \hat{\underline{i}}$$



$$\underline{v}_{D|F}^{\text{I}} = \underline{v}_{A|F} + \underline{\omega}_{3|F} \times \underline{r}_{DA}$$

$$= -\omega_1 R \hat{\underline{i}} + (\omega\hat{\underline{k}} + \omega_1\hat{\underline{j}}) \times (-r\hat{\underline{j}})$$

$$= -\omega_1 R \hat{\underline{i}} + \omega r \hat{\underline{i}}$$

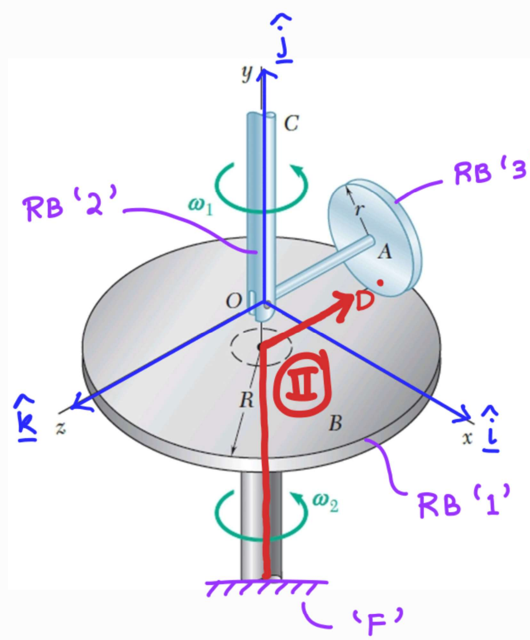
$$= (-\omega_1 R + \omega r)\hat{\underline{i}}$$

From side II

$$\underline{v}_{D|F}^{\text{II}} = \overset{0}{\cancel{\underline{v}_{D|1}}} + \overset{0}{\cancel{\underline{v}_{O|F}}} + \underline{\omega}_{1|F} \times \underline{r}_{DO}$$

$$= \omega_2 \hat{\underline{j}} \times (-R\hat{\underline{k}})$$

$$= -\omega_2 R \hat{\underline{i}}$$



Matching $\underline{v}_{D|F}$ from RB '3' and that from RB '1', we get:

$$\underline{v}_{D|F}^{\text{I}} = \underline{v}_{D|F}^{\text{II}}$$

$$\Rightarrow (-\omega_1 R + \omega r)\hat{\underline{i}} = -\omega_2 R \hat{\underline{i}}$$

$$\Rightarrow \boxed{\omega = (\omega_1 - \omega_2)\frac{R}{r}}$$

∴ Angular velocity of disk A rel. to ground:

$$\underline{\omega}_{3|F} = \omega_1 \hat{\underline{j}} + (\omega_1 - \omega_2)\frac{R}{r}\hat{\underline{k}}$$

We already know: $\underline{\omega}_{3|F} = \underline{\omega}_{3|2} + \underline{\omega}_{2|F}$

We can take time-derivative directly w.r.t F

$$\dot{\underline{\omega}}_{3|F} = \frac{d}{dt}\left(\underline{\omega}_{3|2} + \underline{\omega}_{2|F}\right)\Big|_F$$

$$= \overline{\dot{\underline{\omega}}_{3|2}}\Big|_F + \overline{\dot{\underline{\omega}}_{2|F}}\Big|_F$$

$$= \overline{\dot{\underline{\omega}}_{3|2}}\Big|_2 + \underline{\omega}_{2|F} \times \underline{\omega}_{3|2} + \dot{\underline{\omega}}_{2|F}$$

$$= \dot{\underline{\omega}}_{3|2} + \underline{\omega}_{2|F} \times \underline{\omega}_{3|2} + \dot{\underline{\omega}}_{2|F}$$

this represents the angular acc. of RB '3' as seen by an observer attached to RB '1' (and is $\underline{0}$)

$\dot{\underline{\omega}}_{3|2} = \underline{0}$

is given as zero

$\dot{\underline{\omega}}_{2|F} = \underline{0}$

$$\therefore \quad \dot{\underline{\omega}}_{3|F} = \underline{\omega}_{2|F} \times \underline{\omega}_{3|2}$$

$$= \omega_1 \hat{\underline{j}} \times (\omega_1 - \omega_2)\frac{R}{r}\hat{\underline{k}}$$

$$= \omega_1 (\omega_1 - \omega_2)\frac{R}{r}\hat{\underline{i}}$$

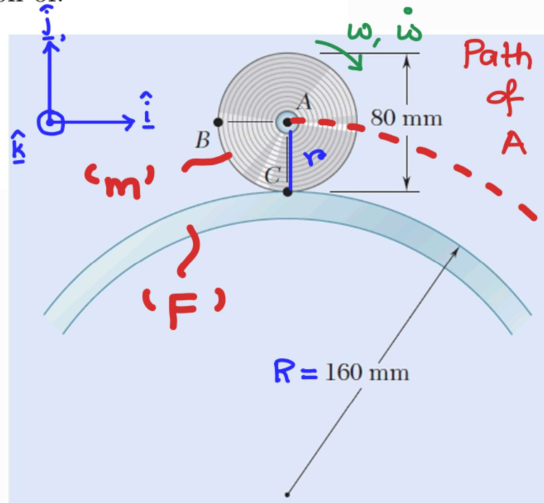
2> A wheel rolls without slipping on a fixed cylinder. Knowing that at the instant shown (Figure 2) the angular velocity of the wheel is 10 rad/s clockwise and its angular acceleration is 30 rad/s^2 counterclockwise, determine the acceleration of:

(assumed +)

(a) Point A,

(b) Point B,

(c) Point C.



$r = 40$ mm , $\underline{\omega}_{m|F} = -\omega_o \hat{k} = -10\hat{k}$

$R = 160$ mm , $\underline{\dot{\omega}}_{m|F} = \alpha_o k = 30\hat{k}$

(i) Note that the path of point A follows a circle of radius $= R + r$

Problem amenable to use of path csys.

This problem is best solved using path csys because :

(a) the given data has info about the radii of curvature,

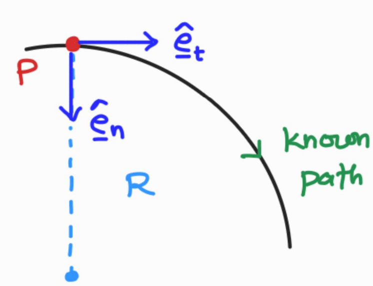
(b) the path of point A is known.

(ii) Rolling without slip $\Rightarrow$ $\underline{v}_{C|F} = \underline{0}$ [pt C on the wheel in contact has zero velocity relative to 'F', and instantaneous motion of the wheel is just a rotation around contact pt. C]

Acceleration of a point P relative to frame F in path csys

$$\underline{a}_{P|F} = \underbrace{\ddot{s}\,\underline{\hat{e}}_t}_{\text{tangential acceleration}} + \underbrace{\boxed{\frac{\dot{s}^2}{R}}}_{\text{normal acceleration}}\underline{e}_n$$

velocity



$\dot{s} \rightarrow$ velocity of the point of interest

For velocity of pt A, we use the <u>known velocity at C</u> :

at that instant relative to 'F'

$$\underline{v}_{A|F} = \cancel{\underline{v}_{C|F}}^{\,0} + \underline{\omega}_{m|F} \times \underline{r}_{AC}$$

$$= \underline{0} + (-\omega_0)\hat{\underline{k}} \times (-r\,\hat{\underline{j}})$$

$$= -\omega_0 r\,\hat{\underline{i}} = \dot{s}\,\hat{\underline{e}}_t$$

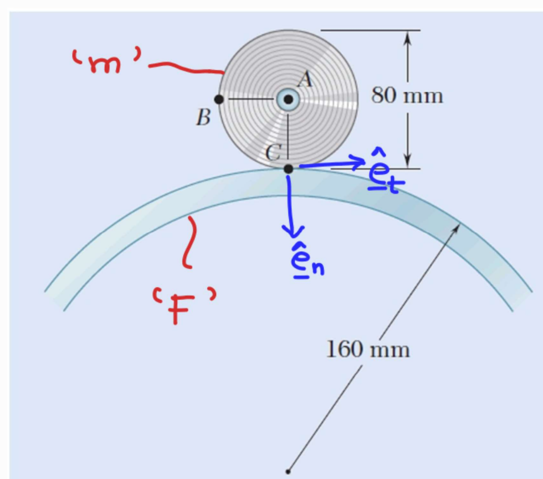
Since pt A travels along a circle of radius $(R+r)$,

we may also write $\underline{v}_{A|F} = \dot{s}\,\hat{\underline{e}}_t$ using path csys.

Note that at the given instant $\hat{\underline{e}}_t$ coincides with $\hat{\underline{i}}$

$$\therefore \quad \dot{s} = -\omega_0 r$$

Acceleration of pt A,



$$\underline{a}_{A|F} = \ddot{s}\,\hat{\underline{e}}_t + \frac{\dot{s}^2}{R+r}\,\hat{\underline{e}}_n$$

$$= \overset{?}{\ddot{s}}\,\hat{\underline{e}}_t + \frac{(\omega_0 r)^2}{R+r}\,\hat{\underline{e}}_n$$

To find $\ddot{s}$, we can use the acceleration at C and use the fact that the tangential component of acc. at C = 0

$$\Rightarrow \quad \underline{a}_{C|F} \cdot \hat{\underline{e}}_t = 0$$

$$\underline{a}_{C|F} = \cancel{\underline{a}_{C|m}}^{\,0} + \boxed{\underline{a}_{A|F}}^{?} + \underline{\dot{\omega}}_{m|F} \times \underline{r}_{CA} + \underline{\omega}_{m|F} \times (\underline{\omega}_{m|F} \times \underline{r}_{CA})$$

$$= \ddot{s}\,\hat{\underline{e}}_t + \frac{(\omega_0 r)^2}{R+r}\hat{\underline{e}}_n + \alpha_0 \hat{\underline{e}}_b \times -r\hat{\underline{e}}_n$$

$$+ (-\omega_0 \hat{\underline{e}}_b) \times \left[(-\omega_0 \hat{\underline{e}}_b) \times (-r\hat{\underline{e}}_n)\right]$$

$$= \ddot{s}\,\hat{\underline{e}}_t + \frac{(\omega_0 r)^2}{R+r}\hat{\underline{e}}_n + \alpha_0 r\,\hat{\underline{e}}_t - \omega_0^2 r\,\hat{\underline{e}}_n$$

$$\underline{a}_{C|F} \cdot \hat{\underline{e}}_t = 0$$

$$\Rightarrow (\ddot{s} + \alpha_0 r) = 0 \quad \Rightarrow \quad \ddot{s} = -\alpha_0 r$$

$$\therefore \quad \boxed{\underline{a}_{A|F} = -\alpha_0 r\,\hat{\underline{e}}_t + \frac{(\omega_0 r)^2}{R+r}\hat{\underline{e}}_n}$$

$\therefore$ Acceleration of point C :

$$\underline{a}_{C|F} = \frac{(\omega_0 r)^2}{R+r}\hat{\underline{e}}_n - \omega_0^2 r\,\hat{\underline{e}}_n$$

($\because$ tangential comp. is zero)

$$= \frac{\omega_0^2 r^2 - \omega_0^2 r^2 - \omega_0^2 r R}{R+r}\hat{\underline{e}}_n$$

$$= -\frac{\omega_0^2}{\left(\frac{1}{R} + \frac{1}{r}\right)}\hat{\underline{e}}_n$$

Acceleration of point B :

$$\underline{v}_{B|F} = \cancelto{0}{\underline{v}_{B|m}} + \underline{v}_{A|F} + \dot{\underline{\omega}}_{m|F} \times \underbrace{\underline{r}_{BA}}_{-r\hat{\underline{e}}_t} + \underline{\omega}_{m|F} \times (\underline{\omega}_{m|F} \times \underline{r}_{BA})$$

$$= -\alpha_0 r\, \hat{\underline{e}}_t + \frac{(\omega_0 r)^2}{R+r}\hat{\underline{e}}_n + \alpha_0 \hat{\underline{e}}_b \times -r\hat{\underline{e}}_t$$

$$+ \left(-\omega_0 \hat{\underline{e}}_b\right) \times \left[\left(-\omega_0 \hat{\underline{e}}_b\right) \times \left(-r\hat{\underline{e}}_t\right)\right]$$

$$= -\alpha_0 r\, \hat{\underline{e}}_t + \frac{(\omega_0 r)^2}{R+r}\hat{\underline{e}}_n - \alpha_0 r\, \hat{\underline{e}}_n + \omega_0^2 r\, \hat{\underline{e}}_t$$

$$= \left(\omega_0^2 r - \alpha_0 r\right)\hat{\underline{e}}_t + \left[\frac{(\omega_0 r)^2}{R+r} - \alpha_0 r\right]\hat{\underline{e}}_n$$

Note : $\hat{\underline{e}}_t \rightarrow \hat{\underline{i}}$

$\hat{\underline{e}}_n \rightarrow -\hat{\underline{j}}$

Plug in the numerical values, and you should get the answers provided.