APL100 Problem Set 2 (Part A)

1. **Theme of the problem: Finding velocity and acceleration of the same particle with respect to different reference frames**

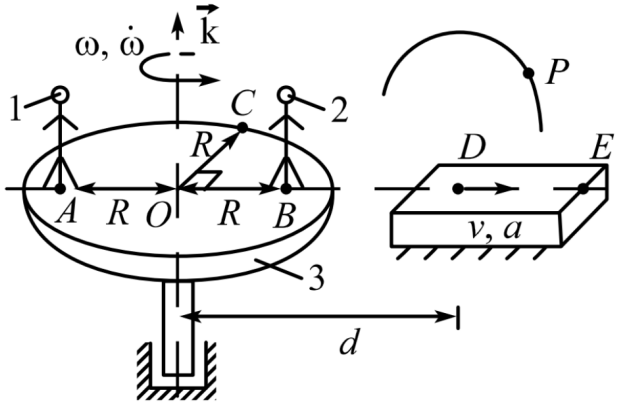
   Two persons 1 and 2, strapped to a platform at opposite positions along AB (the points A and B are fixed to the platform), observe the motion of projectile P and an ant D moving w.r.t. ground along OE at speed v m/s, which is increasing at the rate α m/s^2. The platform itself rotates relative to the ground frame with its angular velocity being $\omega\hat{k}$ rad/s and its angular acceleration being $\dot{\omega}\hat{k}$ rad/s^2 relative to the ground frame. C is another point that has been fixed on the platform.

   (a) Do the two persons observe the same velocity and acceleration of the projectile P? Justify your answer.

   (b) Find the velocity and acceleration of ant D observed with respect to the platform.

   (c) Find the velocity of point B relative to point A with respect to (i) the platform frame and (ii) the ground frame.

   (d) Find the velocity and acceleration of point C as seen by the first person.



2. **Theme of the problem: Multiple RBs connected and rotating relative to each other, and we have to find the velocity and acceleration (w.r.t. the ground) of a particle which is fixed to one of these RBs**

   An amusement park ride, shown in the figure below, consists of several connected rotating parts:

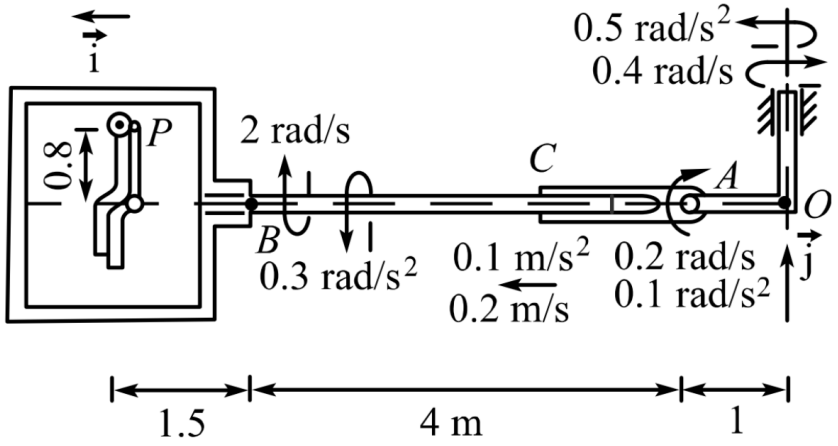
   (i) The cockpit (where passengers sit) rotates relative to the telescopic arm AB at a given rate.

   (ii) The telescopic arm AB swings relative to the arm OA at another specified rate.

   (iii) The length of the telescopic arm AB changes over time at a known rate.

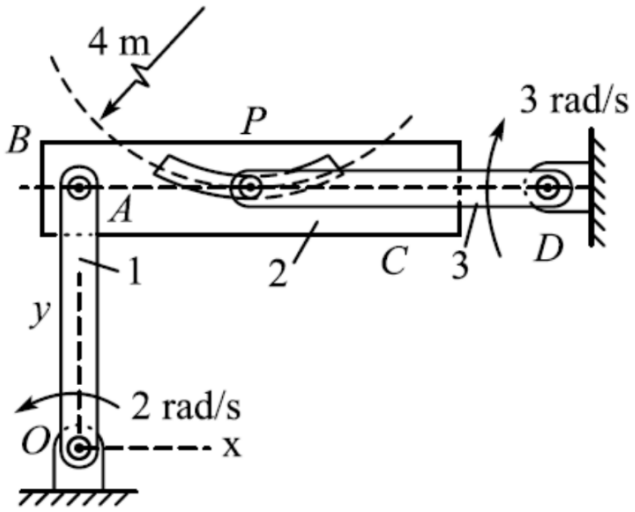
   (iv) The arm OA rotates around a fixed vertical axis at a given rate.

Point P represents the center of the passenger's eye, who is seated in the cockpit. Find the acceleration of P w.r.t. the ground frame.



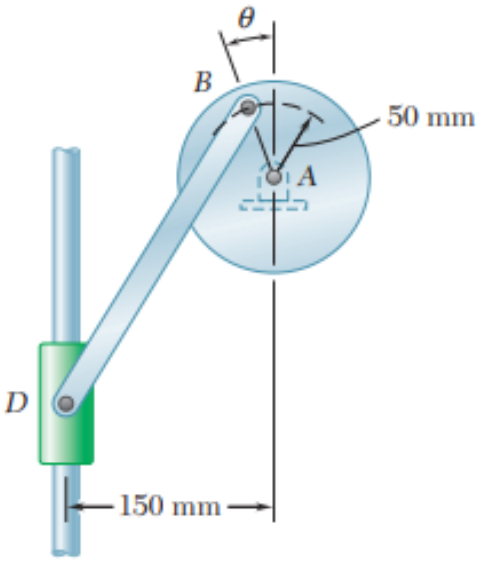
3. **Theme of the problem: Multiple RBs are connected and rotating relative to each other and we wish to find angular velocity/angular acceleration of one of these RBs relative to the ground frame**

   Members OA and DP rotate at constant rate of 2 rad/s and 3 rad/s. The pin at P can slide in the circular slot of radius 4 m in plate AC. Find the angular acceleration of the plate AC in the given configuration when AP=1 m. OA = DP = 2 m



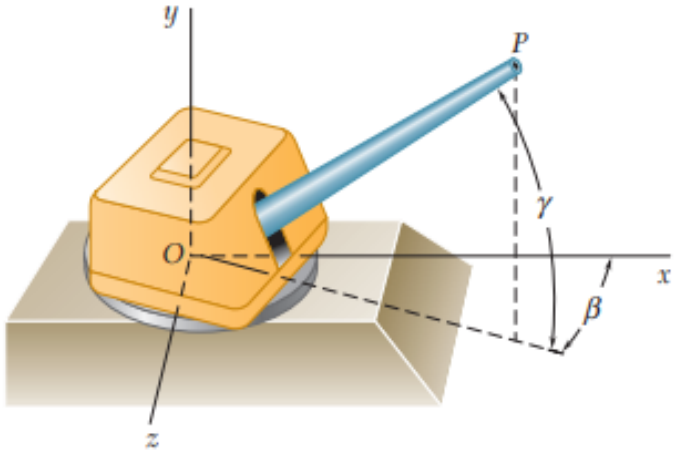
Problem Set 2 (Part B)

1. The disk shown has a constant angular velocity of 500 rpm (counter-clockwise) relative to the ground frame (G). Knowing that rod BD is 250 mm long, determine the acceleration of collar D w.r.t. the ground frame when (a) $\theta = 90\,\text{deg}$, (b) $\theta = 180\,\text{deg}$



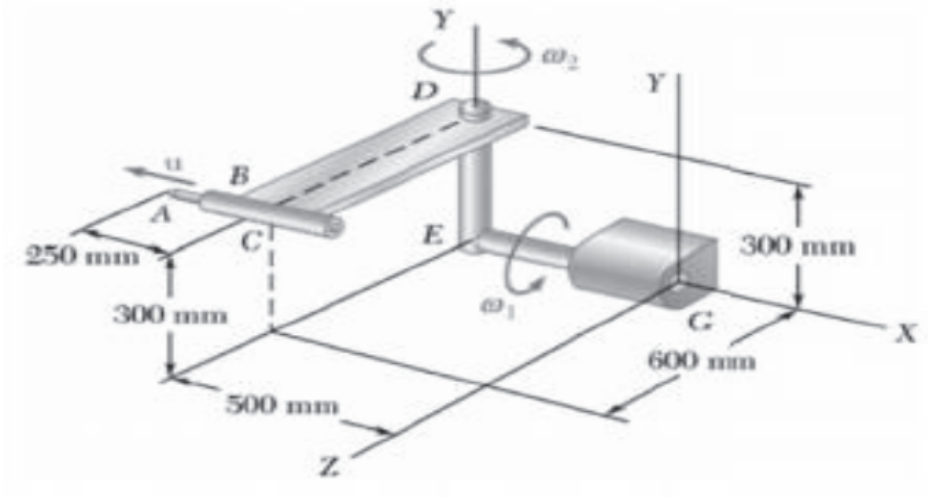
$\underline{a}_{D|G} = 59.8\ \text{m/s}^2 \uparrow$
$\underline{a}_{D|G} = 190.6\ \text{m/s}^2 \uparrow$

2. A gun barrel of length $OP = 4$ m is mounted on a turret as shown. To keep the gun aimed at a moving target, the azimuth angle β is being increased at the rate $\frac{d\beta}{dt} = 30\text{deg/s}$ and the elevation angle γ is being increased at the rate $\frac{d\gamma}{dt} = 10\text{deg/s}$. For the position $\beta = 90\,\text{deg}$ and $\gamma = 30\,\text{deg}$, determine (a) the angular velocity of the barrel relative to the ground frame, (b) the angular acceleration of the barrel relative to the ground frame, (c) the velocity and acceleration of point P with respect to the ground frame (G).



$\underline{\omega}_{barrel|G} = -0.1745\hat{i} - 0.524\hat{j}\ \text{rad/s}$
$\underline{\dot{\omega}}_{barrel|G} = -0.0914\hat{k}\ \text{rad/s}^2$
$\underline{v}_{P|G} = -1.818\hat{i} + 0.605\hat{j} - 3.49\hat{k}\ \text{m/s}$

3. The position of the stylus tip A is controlled by the robot shown. In the position shown the stylus moves at a constant speed $u = 180$ mm/s w.r.t. the solenoid BC. At the same time, arm CD rotates at a constant rate $\omega_2 = 1.6$ rad/s relative to component DEG. Knowing that the entire robot rotates about the X axis at the constant rate $\omega_1 = 1.2$ rad/s relative to the ground, determine (a) the velocity of A,(b) the acceleration of A w.r.t. the ground frame.



$\underline{v}_{A|G} = (0.78)\hat{i} - (0.72)\hat{j} + (0.76)\hat{k}$ m/s
$\underline{a}_{A|G} = (0.64)\hat{i} - (1.392)\hat{j} - (1.824)\hat{k}$m/s^2

For more practice problems: Chapter 15 of Vector Mechanics for Engineers, Beer and Johnston, 15th Edition.