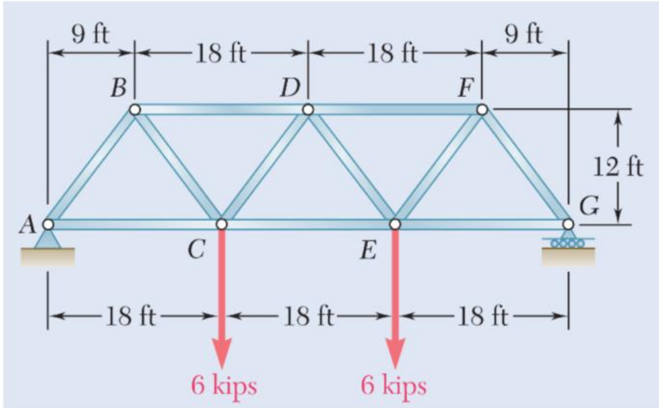
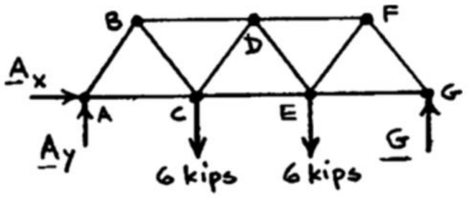
Part A solution

1) Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression



SOLUTION

Free body: Truss: $\Sigma F_x = 0\text{:} \quad A_x = 0$



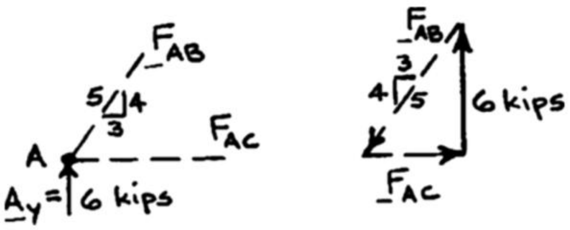
Due to symmetry of truss and loading,

$$A_y = G = \frac{1}{2}\text{ total load} = 6 \text{ kips}\uparrow$$

Free body: Joint A: $\dfrac{F_{AB}}{5} = \dfrac{F_{AC}}{3} = \dfrac{6\text{ kips}}{4}$

$F_{AB} = 7.50$ kips C ◀

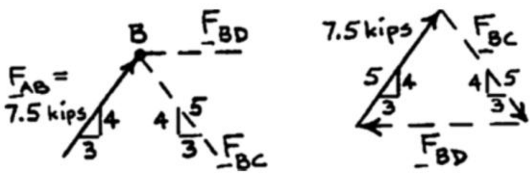
$F_{AC} = 4.50$ kips T ◀



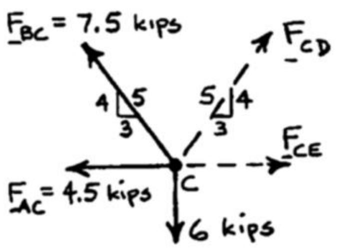
Free body: Joint B: $\dfrac{F_{BC}}{5} = \dfrac{F_{BD}}{6} = \dfrac{7.5\text{ kips}}{5}$

$F_{BC} = 7.50$ kips T ◀

$F_{BD} = 9.00$ kips C ◀



Free body: Joint C:



$$+\uparrow \Sigma F_y = 0\text{:} \quad \frac{4}{5}(7.5) + \frac{4}{5}F_{CD} - 6 = 0$$

$F_{CD} = 0$ ◀

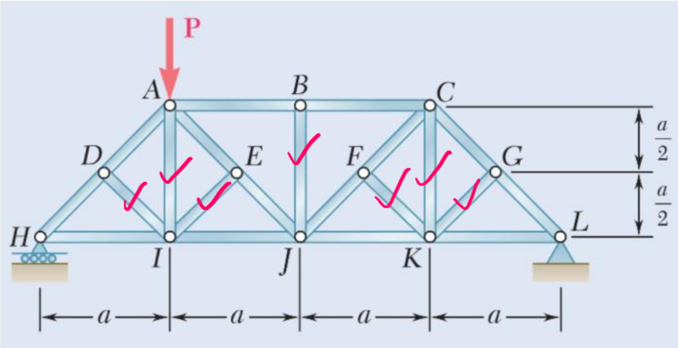
$$\xrightarrow{+} \Sigma F_x = 0\text{:} \quad F_{CE} - 4.5 - \frac{3}{5}(7.5) = 0$$

$+\uparrow F_{CE} = +9$ kips

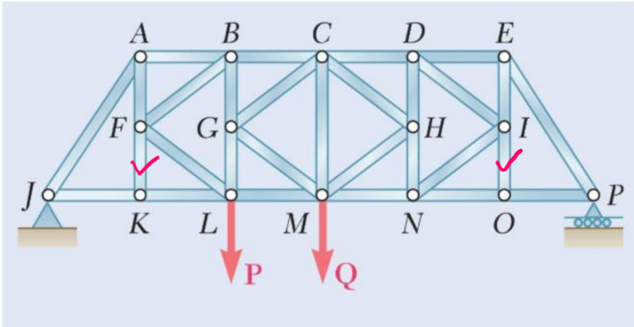
$F_{CE} = 9.00$ kips T ◀

Truss and loading is symmetrical about ℄.

2) For the given loading, determine the zero-force members in each of the two trusses shown



(a)



(b)

SOLUTION

Truss (*a*):

FB: Joint B: $F_{BJ} = 0$

FB: Joint D: $F_{DI} = 0$

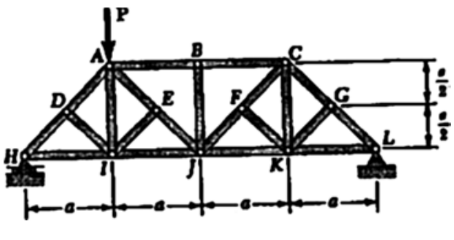
FB: Joint E: $F_{EI} = 0$

FB: Joint I: $F_{AI} = 0$

FB: Joint F: $F_{FK} = 0$

FB: Joint G: $F_{GK} = 0$

FB: Joint K: $F_{CK} = 0$



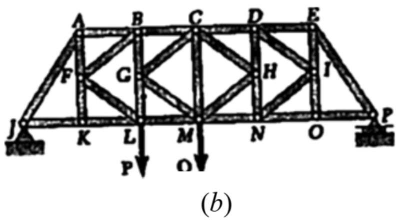
(*a*)

The zero-force members, therefore, are $AI, BJ, CK, DI, EI, FK, GK$ ◀

Truss (*b*):

FB: Joint K: $F_{FK} = 0$

FB: Joint O: $F_{IO} = 0$

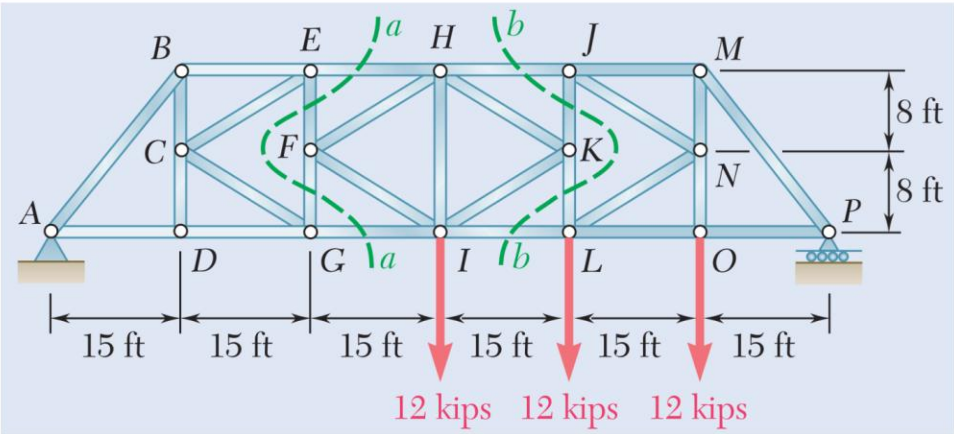


(*b*)

The zero-force members, therefore, are FK and IO ◀

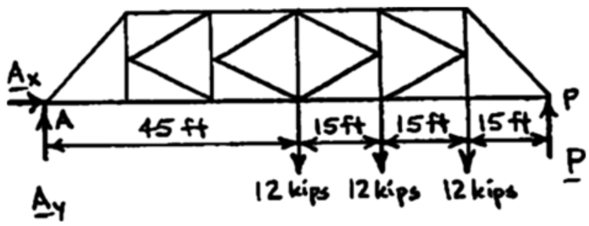
All other members are either in tension or compression.

3) Determine the force in members EH and GI of the truss shown.
(*Hint*: Use section aa.)



SOLUTION

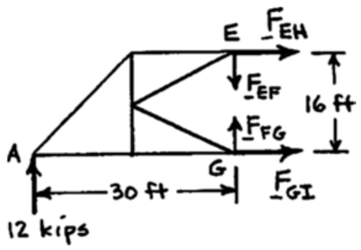
<u>Reactions</u>:



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_P = 0: \quad 12 \text{ kips}(45 \text{ ft}) + 12 \text{ kips}(30 \text{ ft}) + 12 \text{ kips}(15 \text{ ft}) - A_y(90 \text{ ft}) = 0$$

$$\mathbf{A}_y = 12 \text{ kips} \uparrow$$

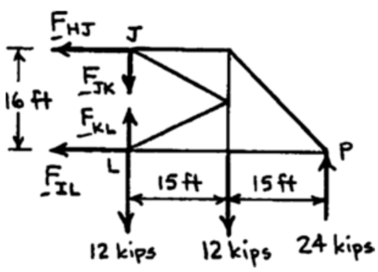


$$+\uparrow \Sigma F_y = 0: \quad 12 \text{ kips} - 12 \text{ kips} - 12 \text{ kips} - 12 \text{ kips} + P = 0 \qquad \mathbf{P} = 24 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_G = 0: \quad -(12 \text{ kips})(30 \text{ ft}) - F_{EH}(16 \text{ ft}) = 0$$

$$F_{EH} = -22.5 \text{ kips} \qquad F_{EH} = 22.5 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{GI} - 22.5 \text{ kips} = 0 \qquad F_{GI} = 22.5 \text{ kips} \quad T \blacktriangleleft$$



$$\mathbf{A}_x = 0; \quad \mathbf{A}_y = 12.00 \text{ kips} \uparrow; \quad \mathbf{P} = 24.0 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_L = 0: \quad F_{HJ}(16 \text{ ft}) - (12 \text{ kips})(15 \text{ ft}) + (24 \text{ kips})(30 \text{ ft}) = 0$$

$$F_{HJ} = -33.75 \text{ kips} \qquad F_{HJ} = 33.8 \text{ kips} \quad C \blacktriangleleft$$

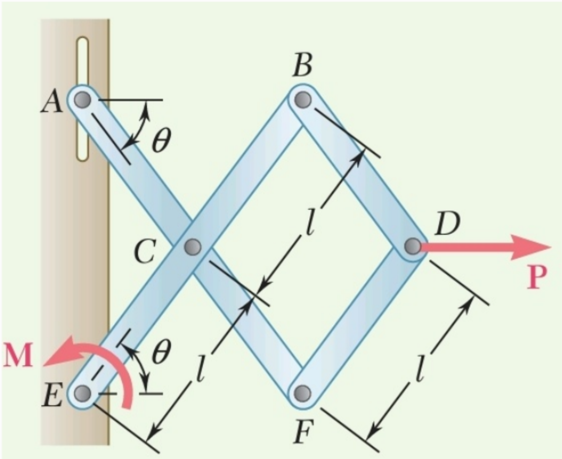
$$\xrightarrow{+} \Sigma F_x = 0: \quad 33.75 \text{ kips} - F_{IL} = 0$$

$$F_{IL} = +33.75 \text{ kips} \qquad F_{IL} = 33.8 \text{ kips} \quad T \blacktriangleleft$$

A)

Using the method of virtual work, determine the magnitude of the couple **M** required to maintain the equilibrium of the mechanism shown.

Assume members are weightless

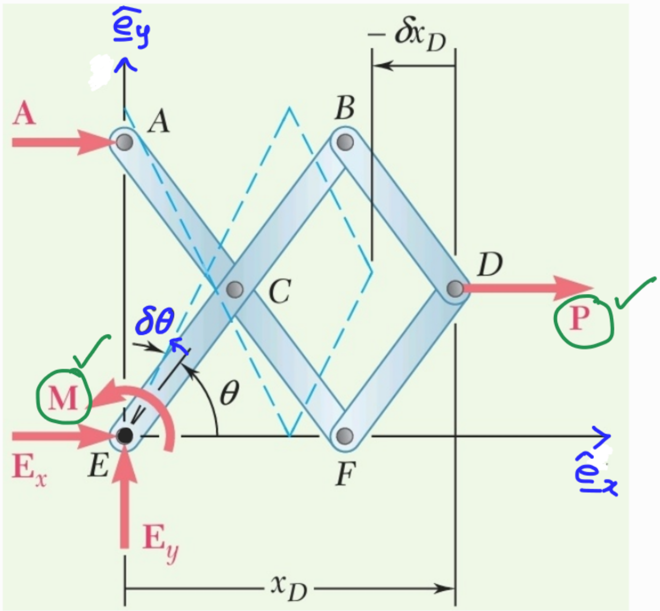


1) Identify DOF → θ (DOF = 1)

2) Draw the deflected config. by inducing virtual displacement

$\theta \rightarrow \theta + \delta\theta$

3) Identify forces that do non-zero virtual work



Force : $\underline{P} = P\,\hat{\underline{e}}_x$

Couple : $\underline{M} = M\,\hat{\underline{e}}_z$

Forces A_x, E_x, E_y, and the internal forces are workless

4) Choose a csys and determine $\delta \underline{r}_D$

Choose origin at E with the csys shown in figure

$$\underline{r}_D = x_D\,\hat{\underline{e}}_x + y_D\,\hat{\underline{e}}_y$$

$$= 3l\cos\theta\,\hat{\underline{e}}_x + l\sin\theta\,\hat{\underline{e}}_y$$

$$\delta\underline{r}_D = \left(-3l\sin\theta\,\hat{\underline{e}}_x + l\cos\theta\,\hat{\underline{e}}_y\right)\delta\theta$$

5) Express the virtual work of each force and couple in the PVW equation in terms of δq (here $\delta\theta$)

$$\delta W = \underline{P} \cdot \delta \underline{r}_D + M\,\delta\theta$$

$$= (P\,\hat{\underline{e}}_x) \cdot (-3\ell \sin\theta\,\hat{\underline{e}}_x + \ell\cos\theta\,\hat{\underline{e}}_y)\,\delta\theta + M\,\delta\theta$$

$$= (-3P\ell\sin\theta + M)\,\delta\theta$$

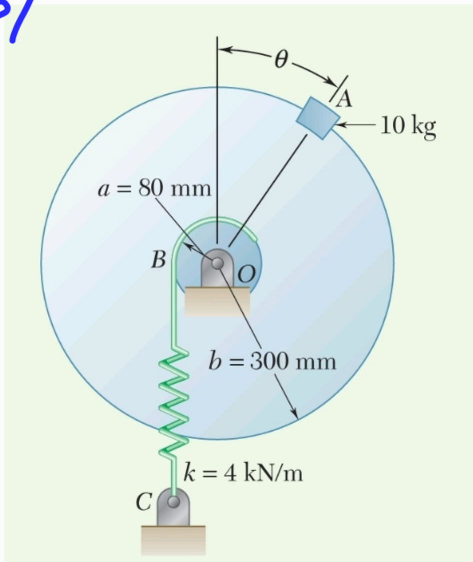
6) Factor out the common displacement from all the terms, and solve for the unknown force or couple.

$$\delta W = 0$$

$$\Rightarrow -3P\ell\sin\theta + M = 0 \qquad [\because \delta\theta \text{ is arbitrary}]$$

$$\Rightarrow \boxed{M = 3P\ell\sin\theta}$$

5)

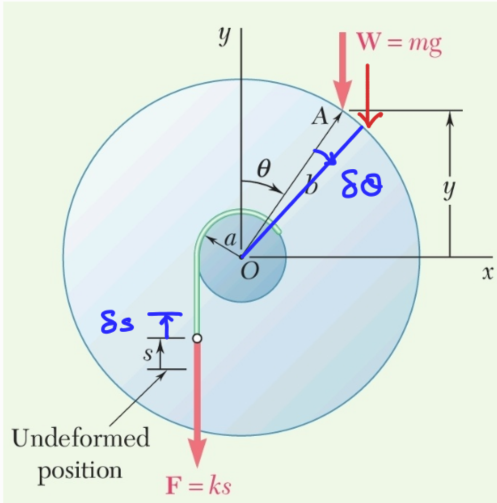


A 10-kg block is attached to the rim of a 300-mm-radius disk as shown. Knowing that spring BC is unstretched when $\theta = 0$, determine the position or positions of equilibrium, using PVW.

Solu :

1) # of DOFs = 1 $(q \equiv \theta)$

2) Draw FBD of the virtually displaced configuration



$\delta s \rightarrow$ virtual deflection of spring

$\delta\theta \rightarrow$ " rotation of the DOF

3) Identify the forces that do non-zero virtual work

$$\left.\begin{aligned} \underline{W} &= -mg\ \hat{e}_y \\ \underline{F} &= -ks\ \hat{e}_y \end{aligned}\right\}$$ conservative forces $\Rightarrow$ can use $\frac{dV}{dq} = 0$

Weight of the disk does no virtual work

4) Choose a coordinate system and determine total potential energy in terms of virtual disp at DOF 'q'

Spring : $V_s = \frac{1}{2} k s^2$

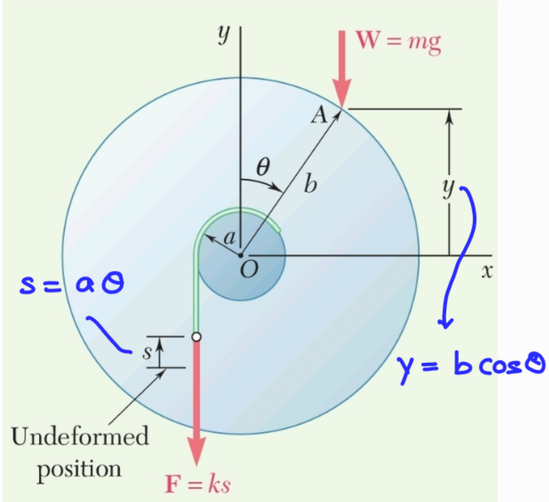
Block : $V_b = mgy$

Total PE, $V(\theta)$

$$= V_s + V_b$$

$$= \frac{1}{2} k s^2 + mgy$$

$$= \frac{1}{2} k\, a^2 \theta^2 + mg\, b\cos\theta$$



5) For static equilibrium, set $\dfrac{dV(q)}{dq} = 0$

$$\frac{dV(\theta)}{d\theta} = 0 \Rightarrow ka^2\theta - mg\sin\theta = 0$$

$$\Rightarrow \sin\theta = \frac{ka^2}{mgb}\theta$$

Solve by trial & error for θ $\Rightarrow$ $\theta = 0.902$ rad