APL100 Problem Set 1 (Part A)

If not mentioned explicitly, velocity and acceleration are to be found with respect to the ground frame.

1. **Theme of the problem: Finding $\underline{v}_{P|F}$ and $\underline{a}_{P|F}$ using a frame-fixed Cartesian CSYS**. A pin P moves in a fixed parabolic slot whose equation is given by, $x = cy^2$, and in a straight horizontal slot as shown in Fig. 1. The straight slot translates in the y-direction at a constant acceleration a_0, starting from rest at t=0, when the pin is at the origin. Find the position, velocity, and acceleration of P at time t.

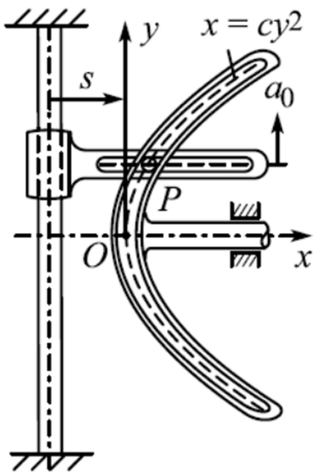

Figure 1

2. **Theme of the problem: Finding $\underline{v}_{P|F}$ and $\underline{a}_{P|F}$ using the cylindrical-polar CSYS**. A particle moves along a logarithmic spiral in the $x-y$ plane, given by the equation, $r = ce^{\phi}$, where c is a constant. If $\dot{\phi} = \frac{b}{r^2}$, where, $b > 0$ and is a constant, find the velocity and the acceleration of the particle when the coordinates of its location are (r, θ). Also, find the radius of curvature (ρ) of its trajectory at this location. Note the radius of curvature ρ, may be obtained as $\rho = \frac{|\underline{v}_{P|F}|^3}{|\underline{v}_{P|F} \times \underline{a}_{P|F}|}$ (this expression will be derived in one of the lecture sessions soon).

3. **Theme of the problem: Finding $\underline{v}_{P|F}$ and $\underline{a}_{P|F}$ using the cylindrical-polar CSYS** Consider the fly-ball governor shown in the Fig. 2. Arms OA and OB are hinged to the shaft at O, however, when the sleeve, S, moves up, the angle θ increases, and the balls move radially outward and upward. At this instant, $\theta = 40°$; $\omega = 2$rad/s; $\dot{\omega} = 0.4$rad/s^2 and the sleeve, S is moving up at a speed $v = 2$m/s which is changing at the rate $a = -0.1$m/s^2 (w.r.t. the ground frame). OA = 0.5m and OC = 0.3m.
Find the velocity and acceleration of ball B, with respect to ground, at this instant.

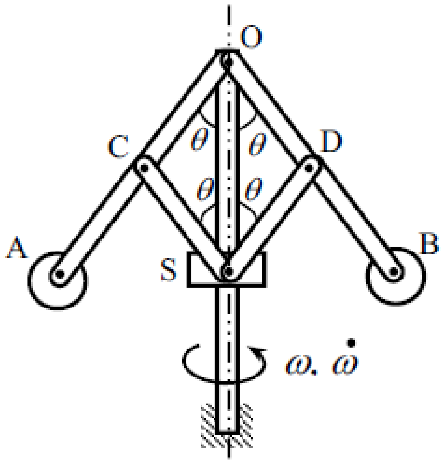

Figure 2: Question 3

4. **Theme of the problem: Finding $\underline{v}_{P|F}$ and $\underline{a}_{P|F}$ using the path CSYS** A small block of mass, $m = 0.5$kg, slides down a hill whose shape may be approximated by $y = Hcos(\pi x/L)$, where $H = 200$m and $L = 800$m. Its speed at the position shown in Fig. 3 is 40m/s. Find the rate of increase of speed in this position if the coefficient of friction between the hill and the block is 0.2.

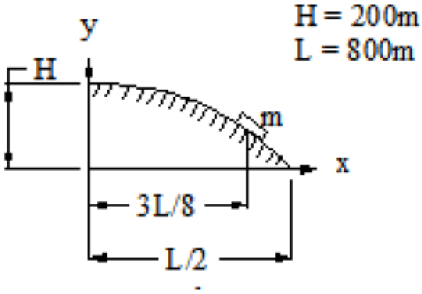

Figure 3: Question 4.

Set 1, Part B

1. A particle moves along the spiral shown (Figure 4); determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$, where b is a constant. (Beer & Johnston, Problem 11.173)

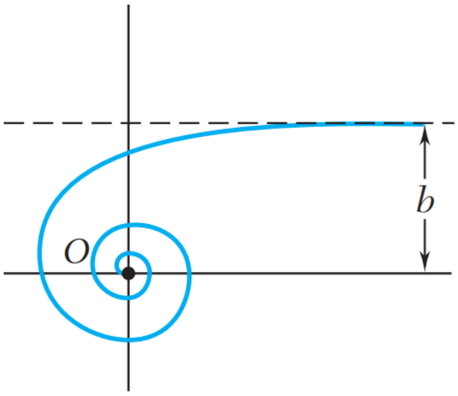

Figure 4

Answer:

$$\frac{b}{\theta^2}\sqrt{1+\theta^2}\,\dot{\theta}$$

2. Consider a cylindrical-polar coordinate system with its origin at point A of a truck (Figure 5) which is stationary w.r.t. ground. The end Point B of a boom is originally 5m from the Point A, when the driver starts to retract the boom with constant $\ddot{r} = -1.0\,\mathrm{m/s^2}$ and turn it with a constant $\ddot{\theta} = -0.5\,\mathrm{rad/s^2}$. At $t = 2\,\mathrm{s}$, determine (a) the velocity of Point B, (b) the acceleration of Point B, (c) the radius of curvature of the trajectory of point B. (Beer & Johnston, Problem 11.192)

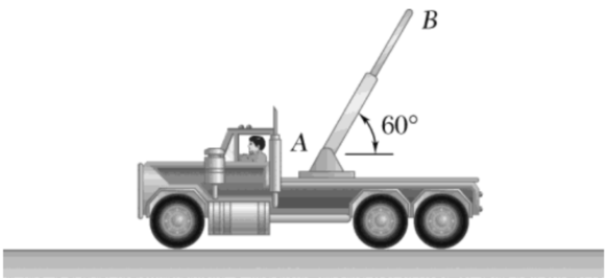

Figure 5

Answer:

(a)Velocity of Point B is $-2\hat{e}_r - 3\hat{e}_\theta$. (b)Acceleration of Point B is $-4\hat{e}_r + 2.5\hat{e}_\theta$. (c)Radius of curvature is 2.76m.

3. At the bottom of a loop in the vertical plane (Figure 6), the airplane (assumed to be a point mass in this problem) has a horizontal velocity of 150m/s and is speeding up at a

rate of 25m/s^2. The radius of curvature of the loop is 2000m. The plane is being tracked by a radar which is fixed to the ground (shown in the figure). Using a cylindrical polar coordinate CSYS with its origin coinciding with the radar, find the values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ for this instant. (Beer & Johnston, Problem 11.192)

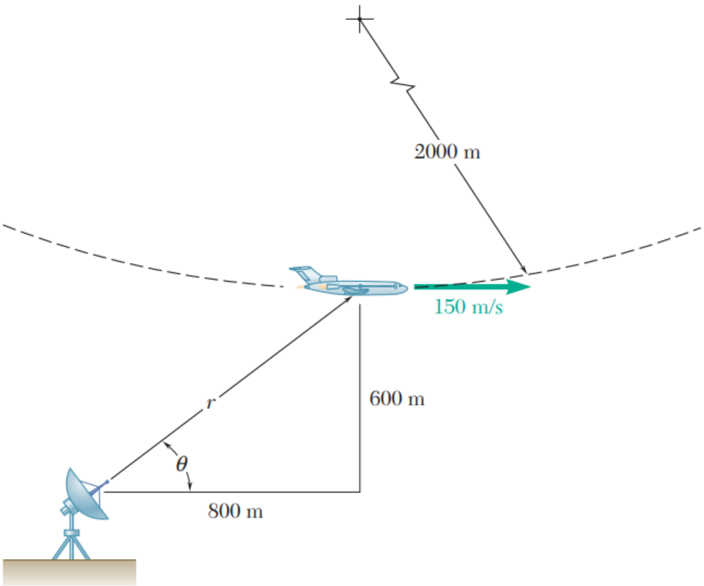

Figure 6

Answer:

$\dot{r} = 120\,\mathrm{m/s}, \quad \ddot{r} = 34.8\,\mathrm{m/s^2}, \quad \dot{\theta} = -0.09\,\mathrm{rad/s}, \quad \ddot{\theta} = 0.0156\,\mathrm{rad/s^2}.$

For more practice problems, see Chapter 11 of *Vector Mechanics for Engineers: Statics and Dynamics (in SI Units)* by Beer & Johnston: McGraw Hill Education, 12th edition.