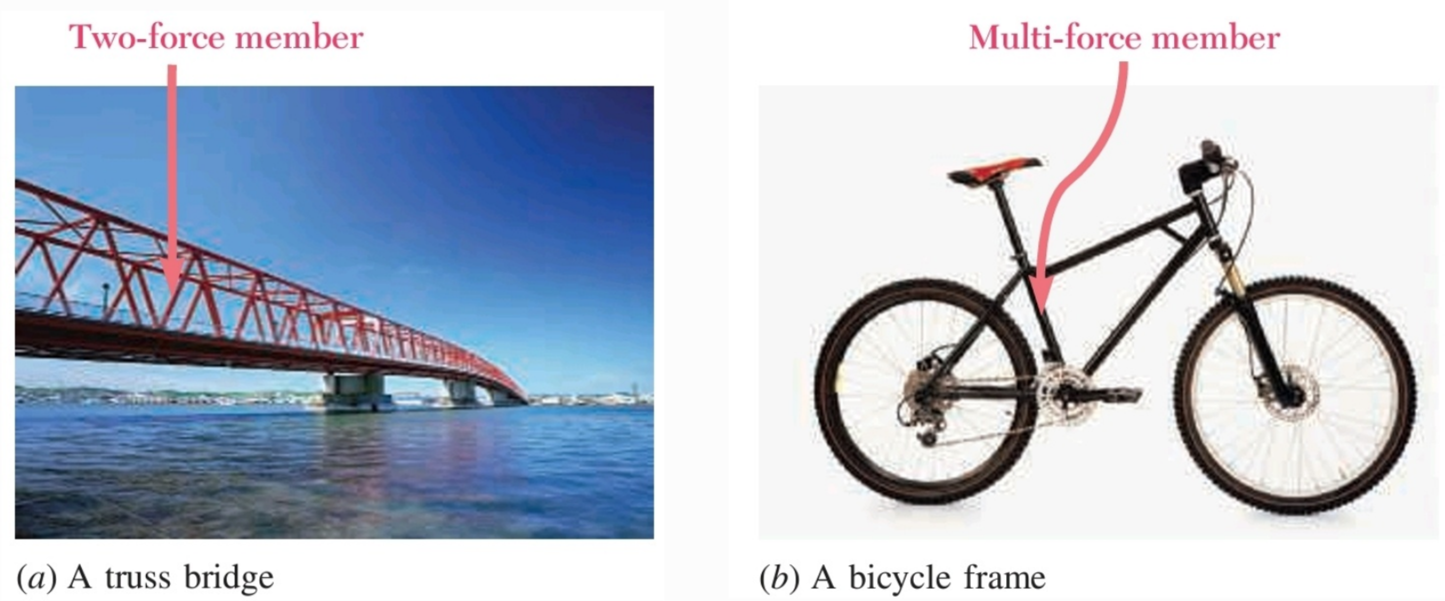
Truss ⇝ Which structures can be called trusses?

* They consist of slender straight members

* The members are connected at the ends of each other by pin connections

* ALL members of a truss structure are **two-force members** (in contrast to frames)

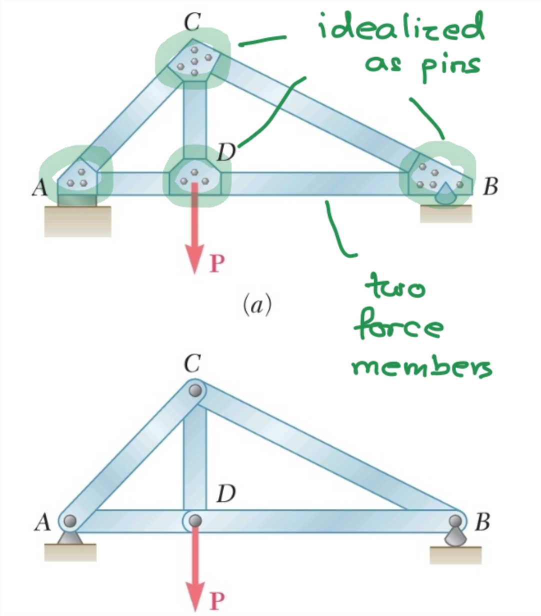


(a) A truss bridge (b) A bicycle frame

A typical truss consists of straight members connected at joints, and

→ Straight members modeled as two-force members

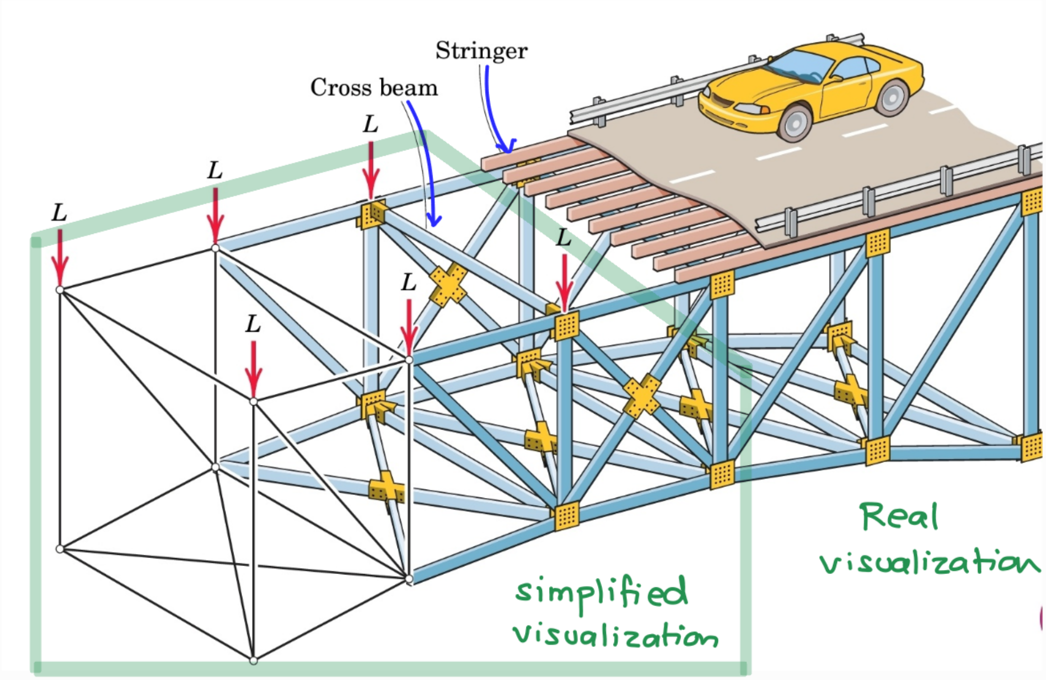
→ Joints modeled as pins!



- The weight of the roadway & vehicles is transferred to the longitudinal stringers

- Stringers transfer load to cross beams

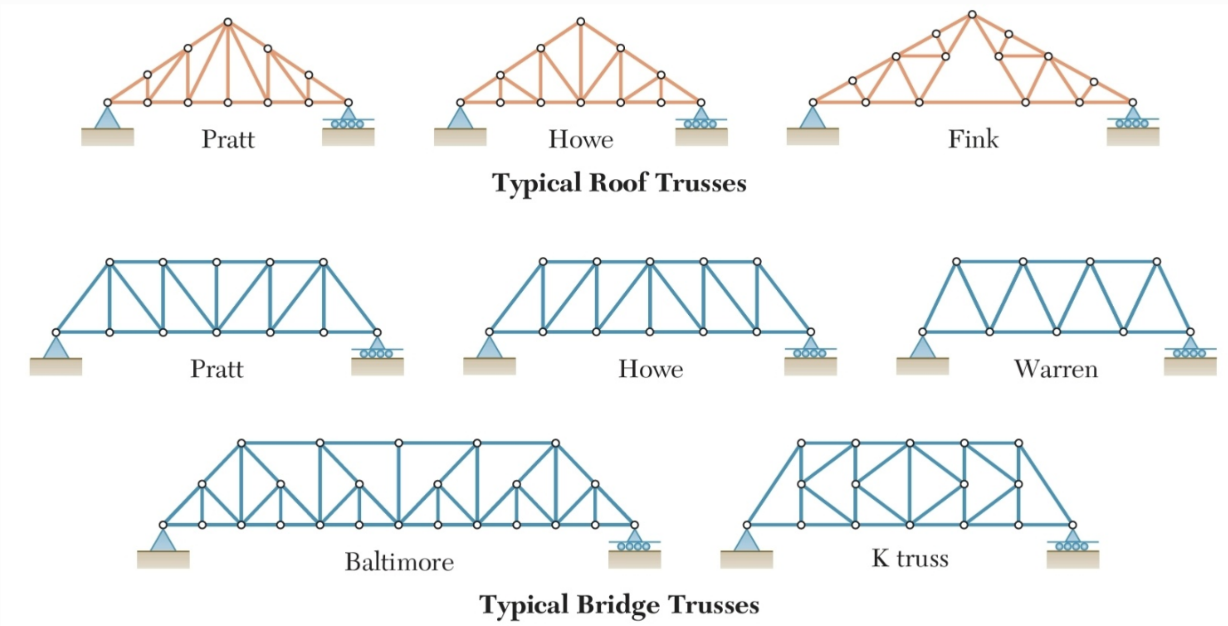
- The loads from cross-beams are transformed to the two vertical sides of the truss structure



A section of a bridge structure made of truss

Advantages: They are light-weight (compared to the loads they can withstand), and are easy to assemble

Typical trusses



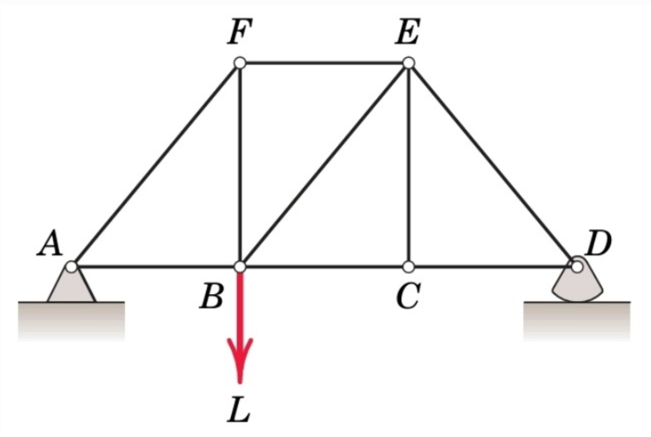
Typical Roof Trusses

Typical Bridge Trusses

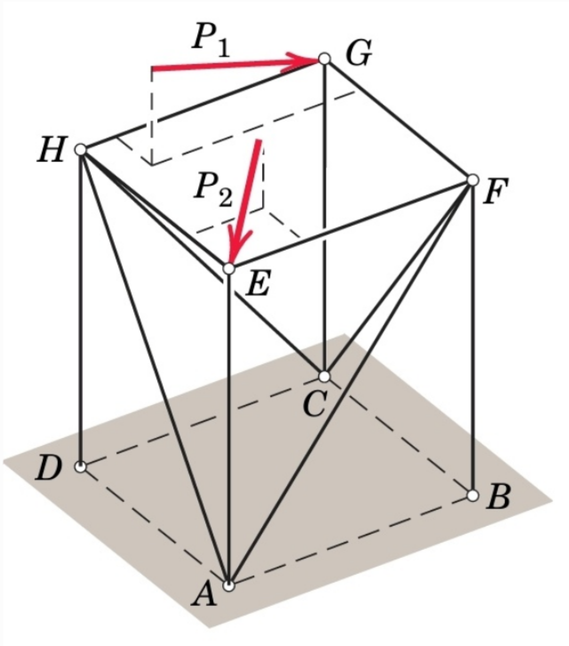
2D trusses vs 3D trusses

2D Truss: Entire truss lies in one plane and the applied forces lie in the same plane

3D Truss: (also called space truss) Non-planar truss, applied force system is non-planar



2D (plane) truss



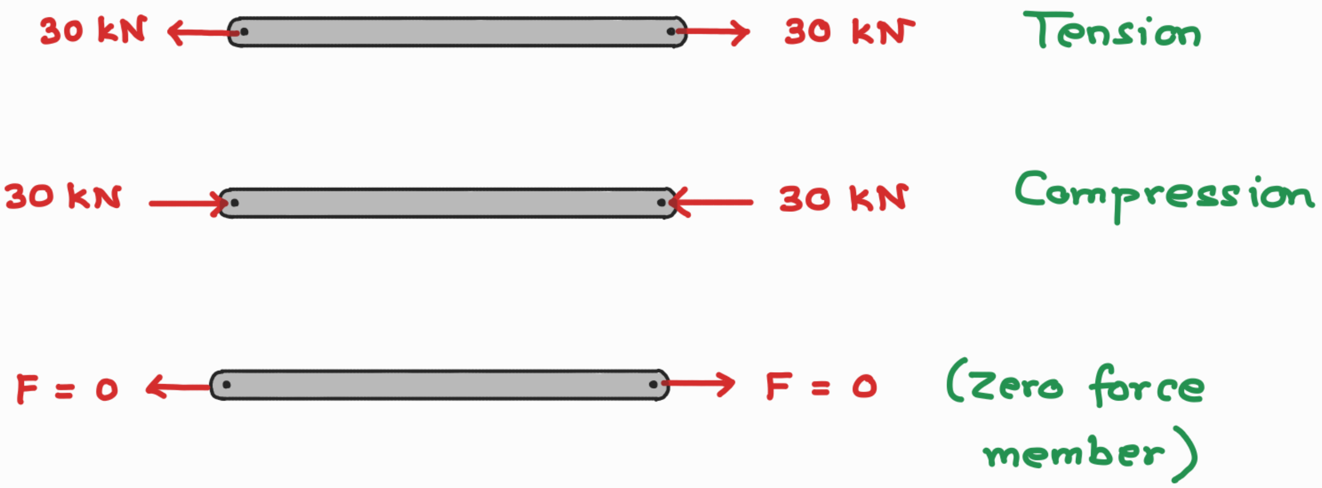
3D (space) truss

Note: All external loads are applied at pins — never anywhere between or at the ends of the members

In this course, we are going to be looking at 2D plane trusses only!

Analysis of Trusses

Since all members are straight and two-force members, they can either be in compression or tension or may turn out to be zero-force members



Analysis of trusses

1) finding support reactions
2) finding member forces (in all of them or some of them)

There are two methods for analyzing trusses:

1) Method of joints : is for finding forces in members by using static equilibrium of each pin joint

Uses only two equilibrium equations $\Sigma F_x = 0$, $\Sigma F_y = 0$

Preferred when finding forces in ALL members

a) Method of sections : is for finding member forces by using static equilibrium of a part of truss

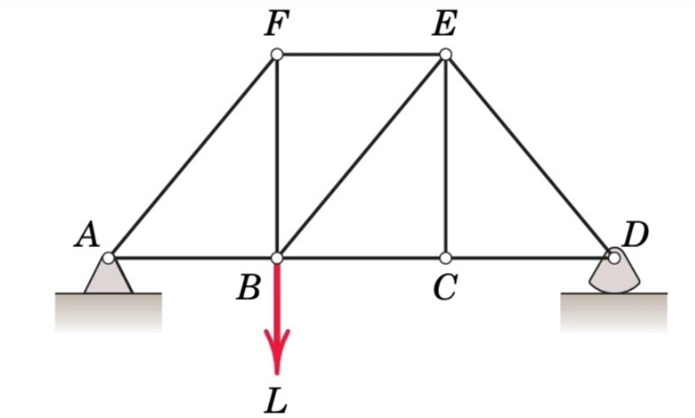
Takes advantage of the $\Sigma M_o = 0$ (moment eqn) as well

Preferred when finding forces in some specific members

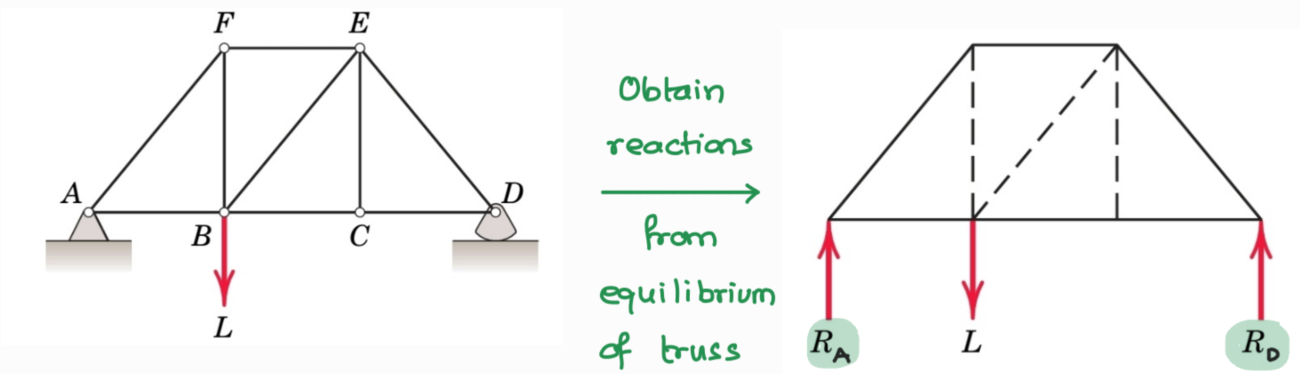
Method of Joints

Idea : Since the entire truss is in equilibrium, each pin joint must be in equilibrium.

Consider the truss as shown.

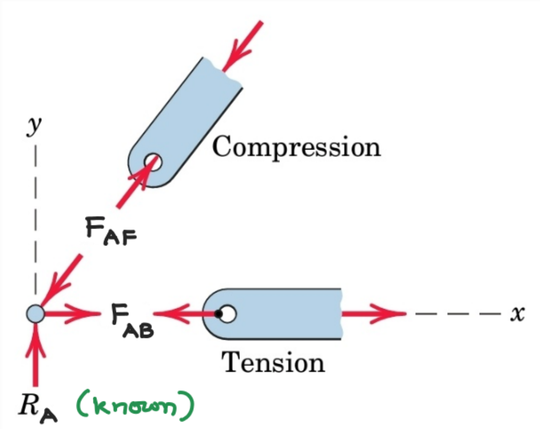


We begin the analysis with ANY JOINT where atleast one known load exists and where not more than two unknown forces are present



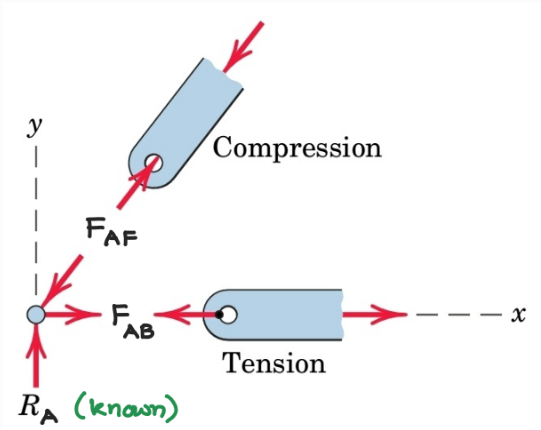
The solution may be started with the pin joint at the left end; draw its FBD.

With joints indicated by letters, designate the force in each member by the two letters defining the ends of the member



The directions of the forces are usually unknown; they can be assumed based on intuition, else assume they are in tension. If your assumption is wrong, the magnitude of the force will turn out to be negative.

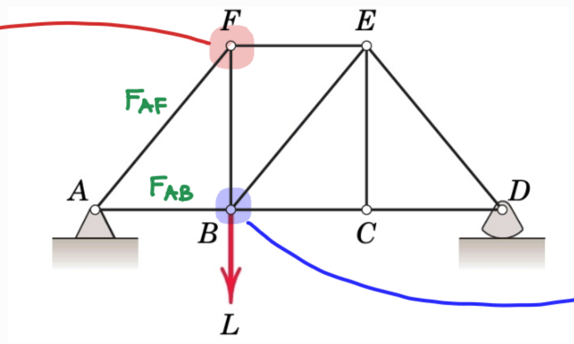
The FBD of the members AF & AB are also shown to clearly indicate the laws of action and reaction.



Analyzing the joint A, one can obtain the unknown magnitudes of F_{AB} and F_{AF} by first considering $+\uparrow \sum F_y = 0$ to find F_{AF} and then using $\xrightarrow{+} \sum F_x = 0$ to find F_{AB}.

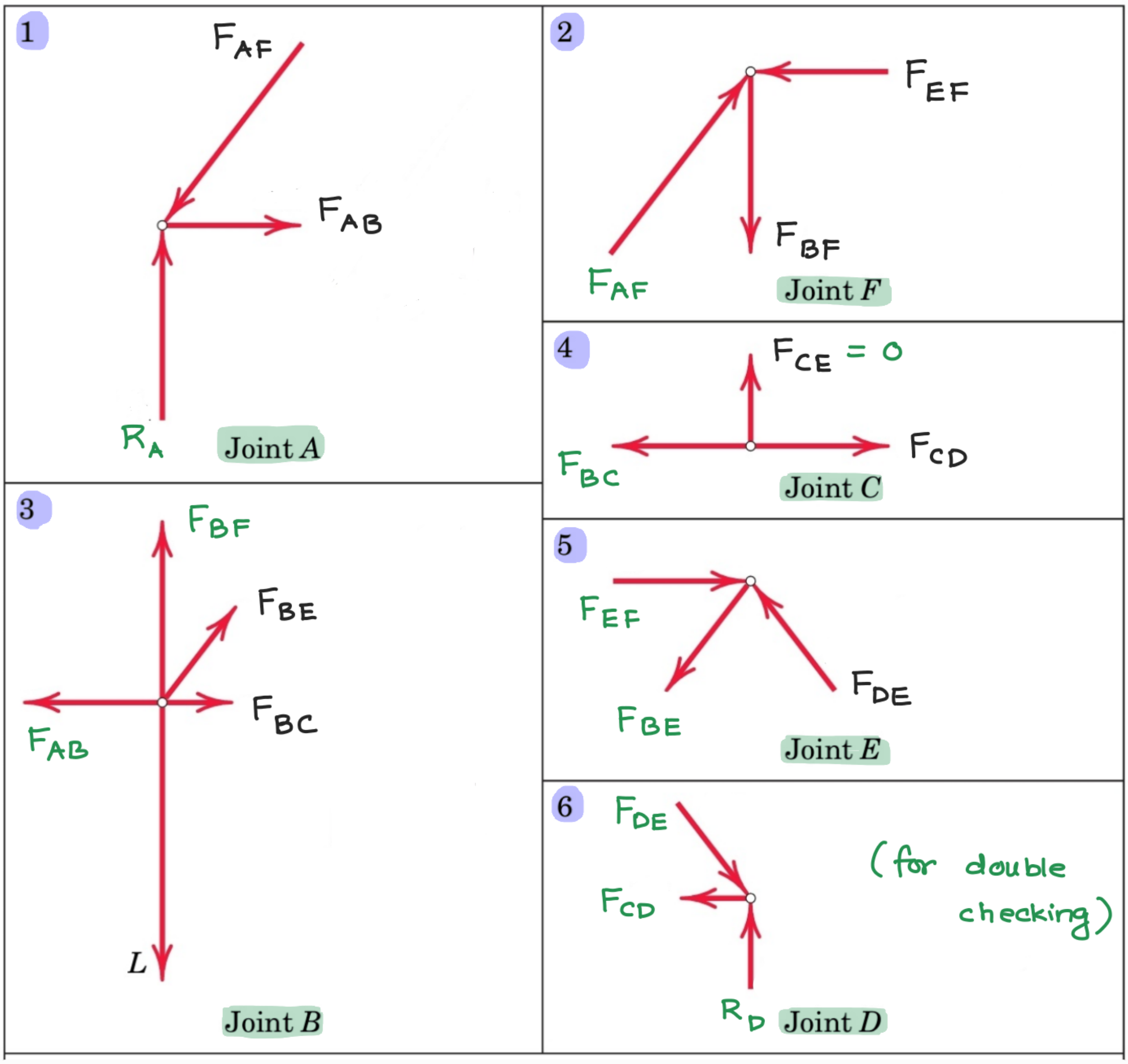
We proceed to the next joint having no more than two unknowns

Joint F can be analyzed next



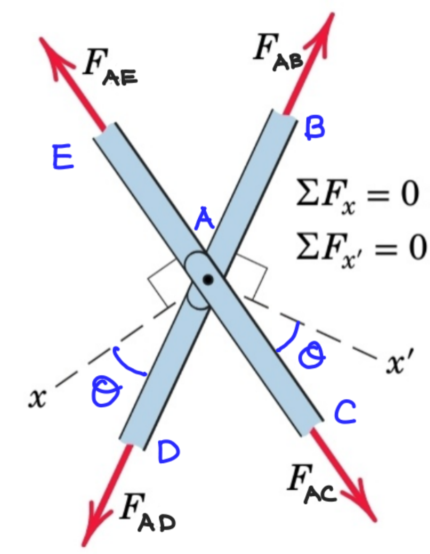
Joint B has three unknowns, therefore we skipped it for now

Proceeding in this fashion, we analyze subsequently joints B, C, E, and D in that order.



Special conditions (in analysis of trusses)

1) Case I : When two pairs of collinear members are joined at a pin joint (4 member + 1 pin)



Given

- AE, AC along a st. line
- AD, AB along a st. line

Inference (by applying $\Sigma F_x = 0$, $\Sigma F_{x'} = 0$)

$$F_{AE} = F_{AC} , \quad F_{AB} = F_{AD}$$

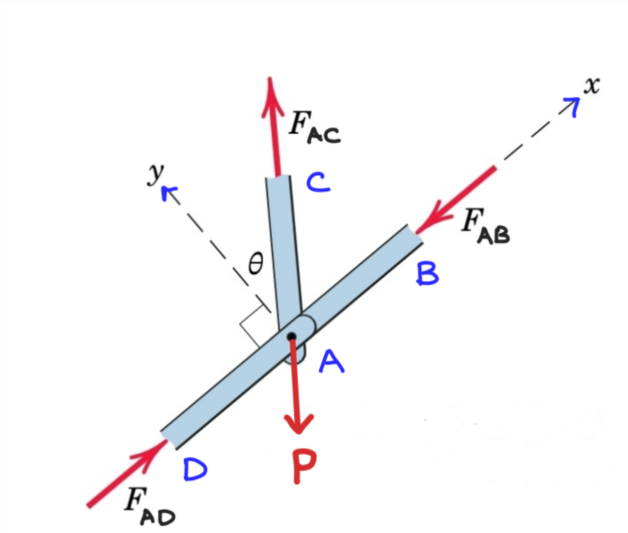
$$\Sigma F_x = 0 \Rightarrow (F_{AC} - F_{AE}) \cos 90^\circ + (F_{AD} - F_{AB}) \cos\theta = 0$$

$$\Rightarrow F_{AD} = F_{AB} \quad (\because \cos\theta \neq 0)$$

$$\Sigma F_{x'} = 0 \Rightarrow (F_{AC} - F_{AE}) \cos\theta + (F_{AD} - F_{AB}) \cos 90^\circ = 0$$

$$\Rightarrow F_{AC} = F_{AE} \quad (\because \cos\theta \neq 0)$$

2) Case II : When two collinear members are at a pin joint and a third member is added for stability of truss



Given:

- Members AD and AB are along st. line
- Applied force P and AC are along st. line

Inference:

$F_{AB} = F_{AD}$, $F_{AC} = P$

$+\nwarrow \Sigma F_y = 0$

$\Rightarrow (F_{AC} - P) \cos\theta = 0$

$\Rightarrow F_{AC} = P$

$+\nearrow \Sigma F_x = 0$

$\Rightarrow F_{AD} - F_{AB} = 0$

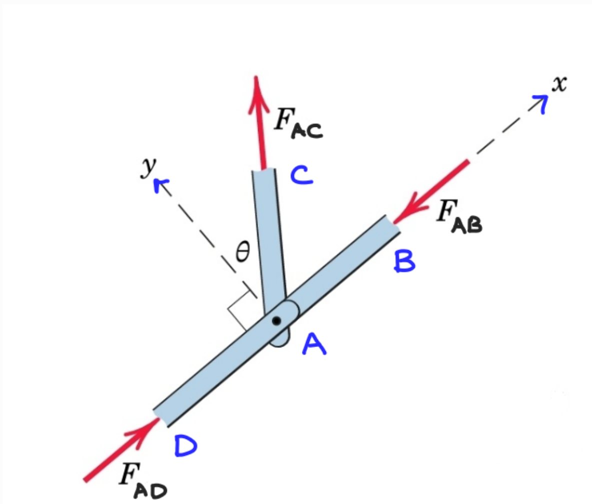
$\Rightarrow F_{AD} = F_{AB}$

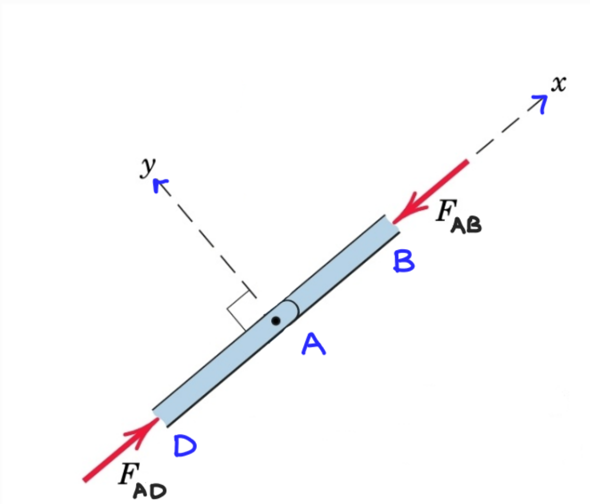
Special cases of Case II:

1) Applied external force $P = 0$: then $F_{AC} = 0$

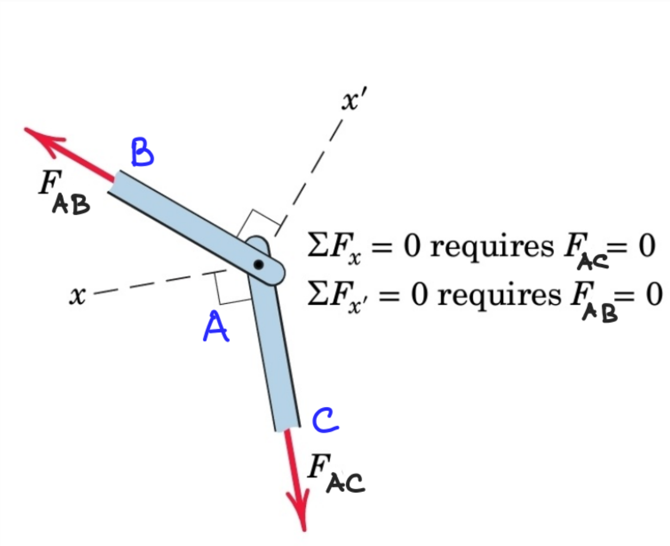
AC is a zero-force member

2) Member AC is absent and $P = 0$: $F_{AB} = F_{AD}$





3) Case III : When two non-collinear members are joined at a pin in the absence of an externally applied load, the forces in the members must be zero



$F_{AC} = 0$, $F_{AD} = 0$

Spot the zero-force members (by identifying special conditions)

↓

helps to do faster analysis

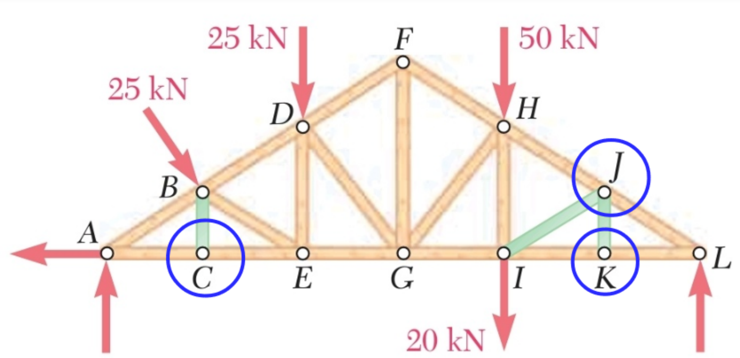


Fig. 6.13 An example of loading on a Howe truss; identifying special loading conditions.

Apply case 2 (two collinear members with a third member without external force) at pins C, K, and J

At C, $F_{BC} = 0$, $F_{AC} = F_{CE}$

At K, $F_{JK} = 0$, $F_{IK} = F_{KL}$

At J, $F_{IJ} = 0$, $F_{HJ} = F_{JL}$

Further applying Case 2 at joint I, $F_{GI} = F_{IK}$, $F_{HI} = 20$

You CANNOT apply Case 2 to joint B and say that $F_{AB} = F_{BD}$ because external load of 25 kN is not collinear with member BE

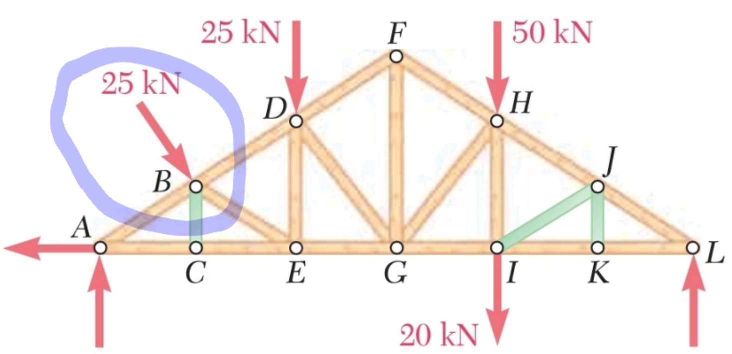
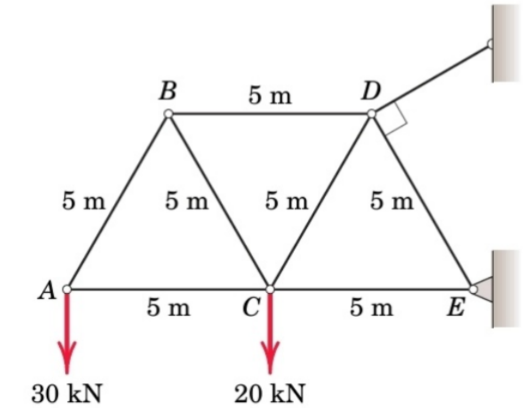


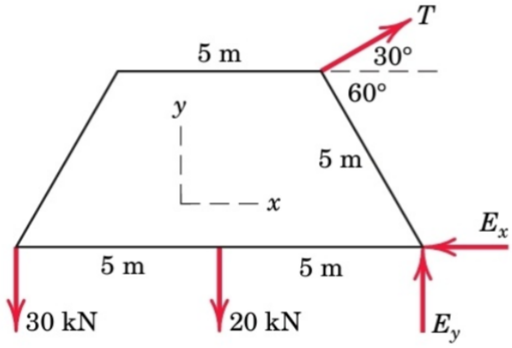
Fig. 6.13 An example of loading on a Howe truss; identifying special loading conditions.

Compute the force in each member of the loaded cantilever truss by the method of joints.



Solution If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80$ kN
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3$ kN
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10$ kN



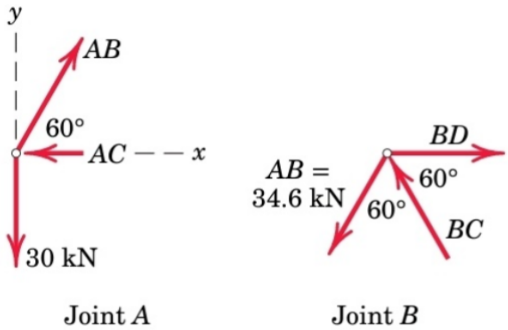
Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A. Equilibrium requires

$[\Sigma F_y = 0]$	$0.866AB - 30 = 0$	$AB = 34.6$ kN T	*Ans.*
$[\Sigma F_x = 0]$	$AC - 0.5(34.6) = 0$	$AC = 17.32$ kN C	*Ans.*

where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C. The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$[\Sigma F_y = 0]$	$0.866BC - 0.866(34.6) = 0$	$BC = 34.6$ kN C	*Ans.*
$[\Sigma F_x = 0]$	$BD - 2(0.5)(34.6) = 0$	$BD = 34.6$ kN T	*Ans.*



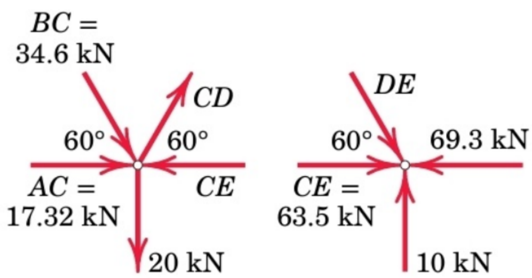
Joint C now contains only two unknowns, and these are found in the same way as before:

$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7$ kN T *Ans.*

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5$ kN C *Ans.*



Finally, from joint E there results

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \qquad DE = 11.55$ kN C *Ans.*

and the equation $\Sigma F_x = 0$ checks.

Summary of method of joints

- Makes use of only two static equilibrium equations
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$

- Applicable because all forces intersect at a point

- The moment equation ($\Sigma M_* = 0$) is not needed

- Limitation: Requires progressing joint-by-joint

- Limitation: To find the force in a specific member, one may need to analyze many joints sequentially

Method of Sections

- Involves cutting the truss into two parts and analyzing one part as a free body

- Makes use of all three equilibrium equations

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma M_* = 0$$

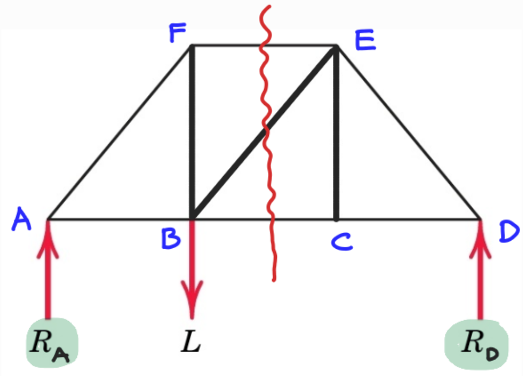
- Enables direct calculation of internal forces of specific members, without analyzing every joint

Guidelines for sectioning:

- Cut no more than three members with unknown forces
- This ensures that the resulting system of equations is solvable (as we have three equilibrium equations)
- Choose sections that allow straightforward application of the moment equation for simplification

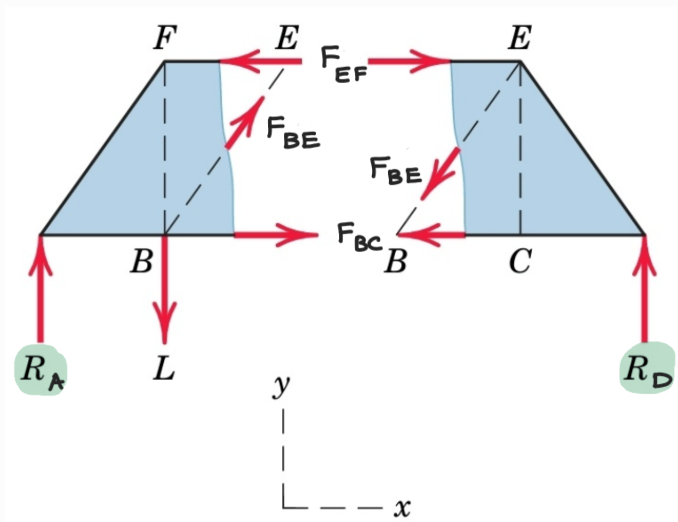
Lets demonstrate this method using a truss example

Step 1: Determine external reactions



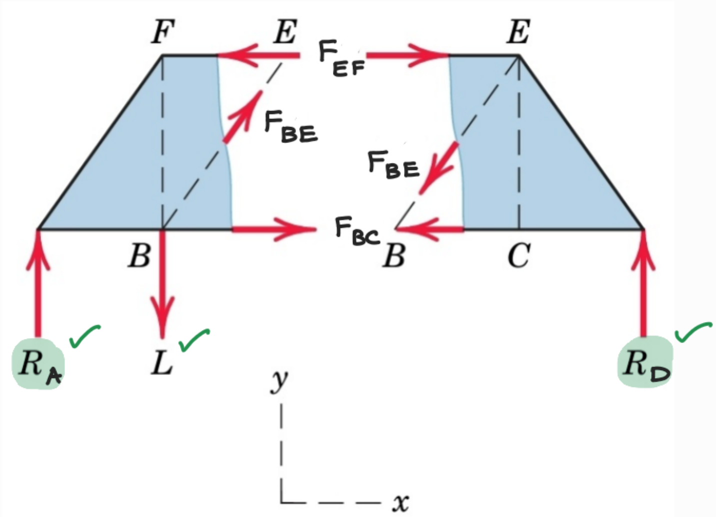
Step 2: To find the force in a specific member (e.g. F_{BE})

cut an imaginary section through the truss, such that no more than three members with unknown forces are cut



Step 3: Each part of truss must be in equilibrium. The cut members exert equal and opposite forces on the two separated parts

Use moment equations to solve for specific unknowns



$\circlearrowleft + \sum M_B = 0 \rightarrow$ Get F_{EF}

$\circlearrowleft + \sum M_E = 0 \rightarrow$ Get F_{BC}

$+\uparrow \sum F_y = 0 \rightarrow$ Get F_{BE}

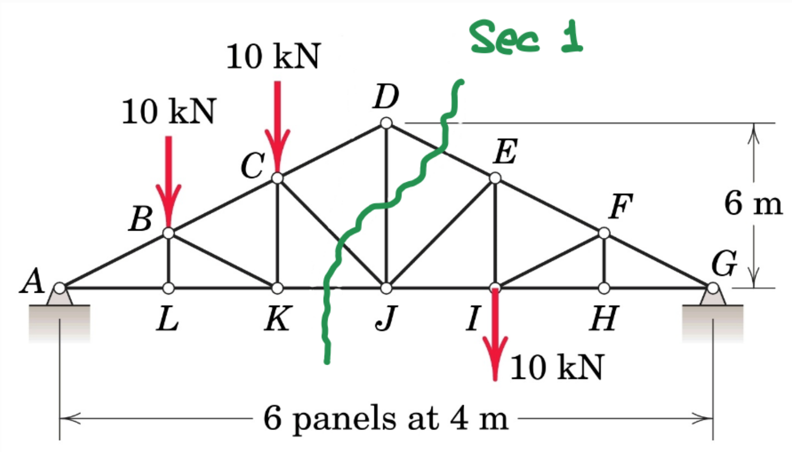
Some considerations to keep in mind for method of sections

- The sectioned part of truss is treated as single RBs in equilibrium
- Preferably, cut sections through members, not joints!
- Choose the part of truss with fewer unknowns to simplify calculations

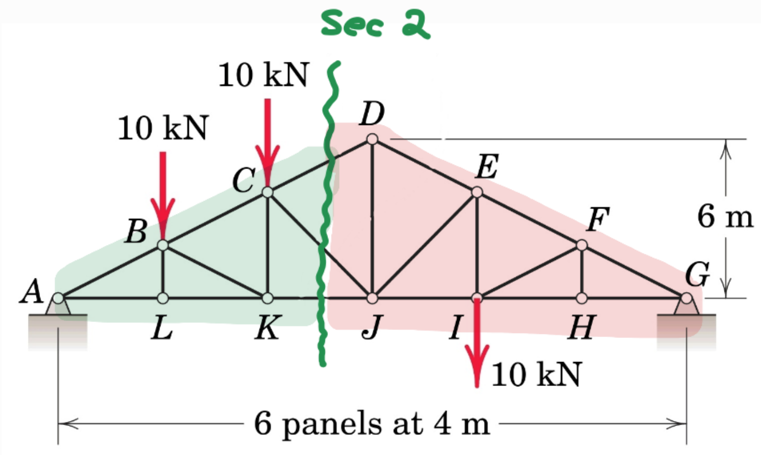
Example: Calculate force in member DJ. Neglect horizontal components of force at supports

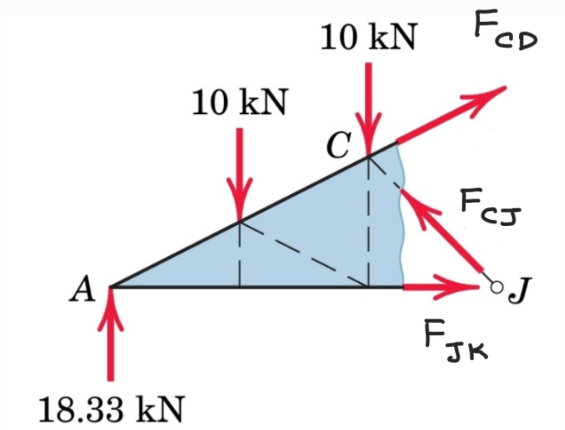
Can we choose a section 1?

NO, because it cuts through 4 (>3) members with unknown forces



Let's therefore choose a different section 2



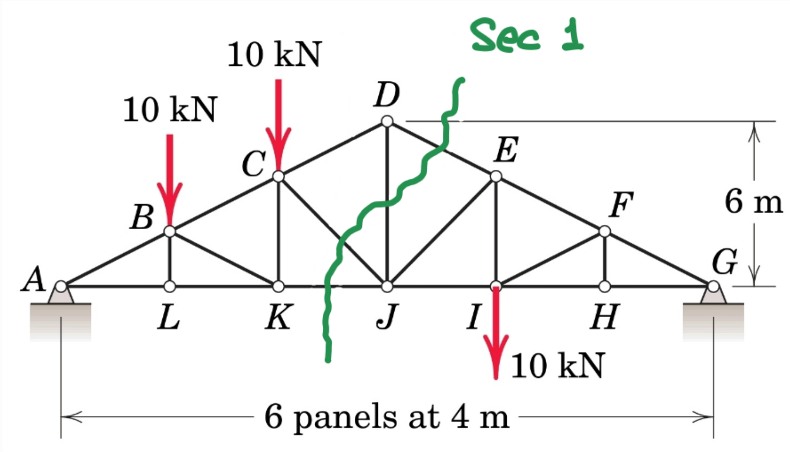


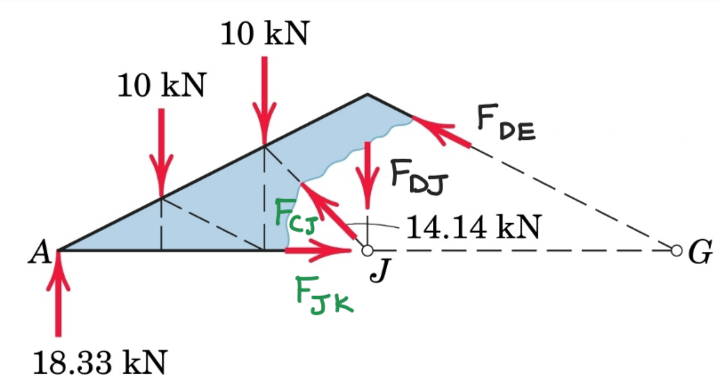
$\circlearrowleft + \ \Sigma M_C = 0 \rightarrow$ Get F_{JK}

$\circlearrowleft + \ \Sigma M_A = 0 \rightarrow$ Get F_{CJ}

$\circlearrowleft + \ \Sigma M_J = 0 \rightarrow$ Get F_{CD}

Then, choose section 1





$\overset{+}{\circlearrowleft} \sum M_G = 0 \rightarrow$ Get F_{DJ}

$F_{DJ} = 16.67$ kN (T)