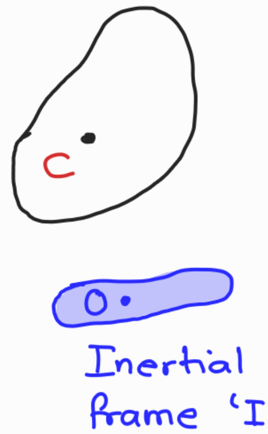
Statics of RBs

Statics deals with study of bodies at rest, meaning every material point of the body is at rest at all times

$$\Rightarrow \left.\begin{aligned} &\underline{v}_{c|I}(t) = \underline{0} \\ &\text{and} \\ &\underline{\omega}_{m|I}(t) = \underline{0} \end{aligned}\right\} \text{at all times 't'}$$



However, this is only a necessary condition!

Necessary and Sufficient Condition for an RB to be at REST :

1) $\underline{v}_{c|I}(t) = \underline{0}$ and $\underline{\omega}_{m|I}(t) = \underline{0}$ } Body is at rest

2) Resultant external force and net moment of the external forces about a point (say A) must be zero

$$\Rightarrow \left.\begin{aligned} &\underline{F}_R(t) = \sum \underline{F}_i(t) = \underline{0} \\ &\text{and} \\ &\underline{M}_A(t) = \sum \underline{r}_{iA} \times \underline{F}_i(t) + \sum \underline{C}_j(t) = \underline{0} \end{aligned}\right\} \text{at all times 't'}$$

If the necessary and sufficient conditions are satisfied, then the body is said to be in STATIC EQUILIBRIUM.

So assuming that the body is at rest ($\langle 1 \rangle$ is satisfied), two vector (or six scalar) eqns of $\underline{F}_R = \underline{0}$ and $\underline{M}_A = \underline{0}$ need to be satisfied for static equilibrium.

If $\hat{\underline{e}}_x - \hat{\underline{e}}_y - \hat{\underline{e}}_z$ is chosen as a working coordinate system, then

$$\boxed{F_{R_x} = 0, \quad F_{R_y} = 0, \quad F_{R_z} = 0, \quad M_{A_x} = 0, \quad M_{A_y} = 0, \quad M_{A_z} = 0}$$

For planar 2D case (say x-y plane): We have a COPLANAR force system

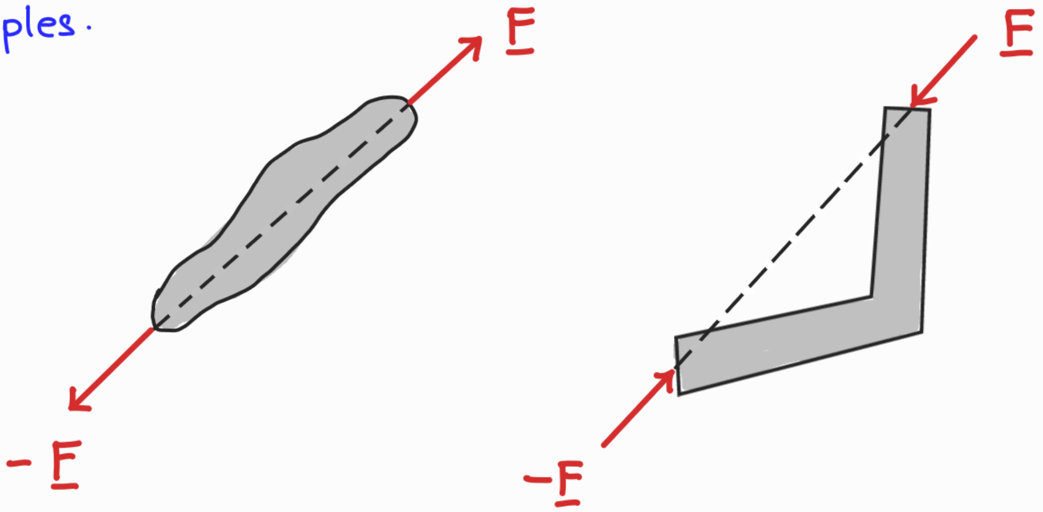
$\begin{cases} \text{forces lie in the } x\text{-}y \text{ plane} \\ \text{couples (if any) are along } \pm z \text{ direction} \end{cases}$

and for static equilibrium in planar 2D, we need to have

$$\boxed{F_{Rx} = 0, \quad F_{Ry} = 0, \quad M_{A_z} = 0}$$

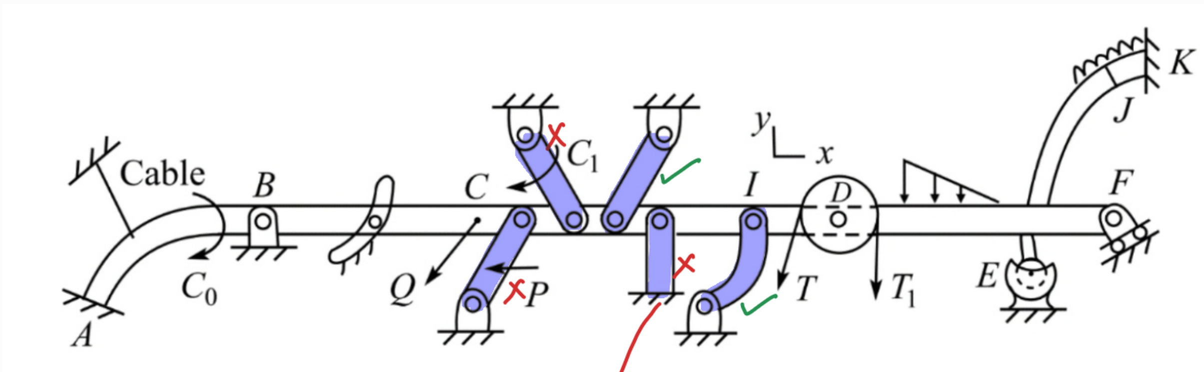
Two - force members

A frequently occuring equilibrium situation is when a body is in equilibrium under the action of two forces ONLY and no couples.



These bodies are called two-force members, and for them to be under static equilibrium, the forces must be equal, opposite, and collinear (have same line of action)

Can you identify two-force members?



Potential two-force members are shaded

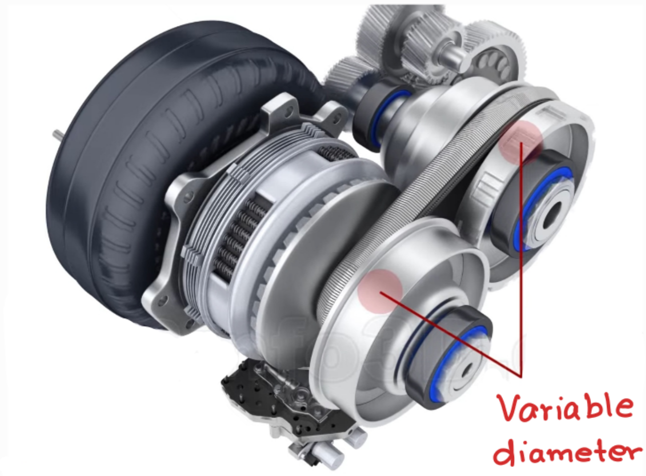
Why is this member not a two-force member?

(because of fixed support, there will be a reaction couple)

Pulley & Belt Friction

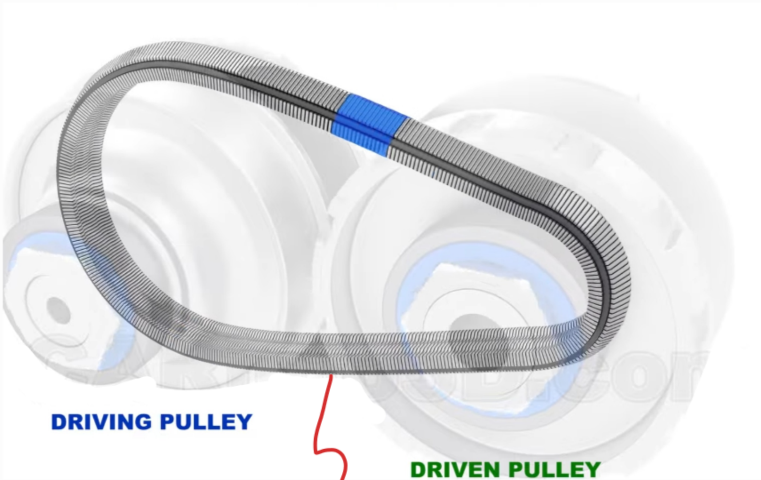
In some types of machines, we wish to maximize the effects of friction, such as in brakes, belt drives, and clutches.

Example of CVT (continuous variable transmission) in cars



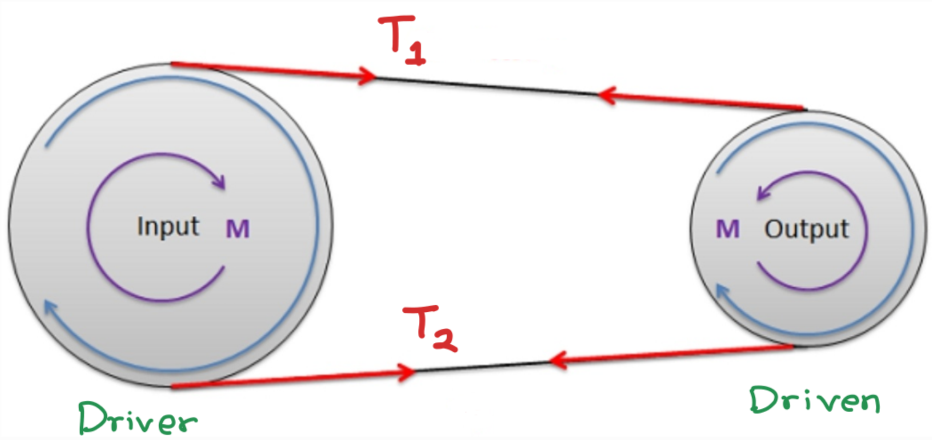
Variable diameter pulleys

(much like your bicycle gears)



belt

The friction between the metal belt and the pulleys is a major factor in the design process



Schematic of a continuously variable transmission

We wish to find a relationship between T_1 and T_2 when friction is present between the belt and the pulley

Friction (Review)

Let's recall Coulomb's dry friction between two surfaces in contact.

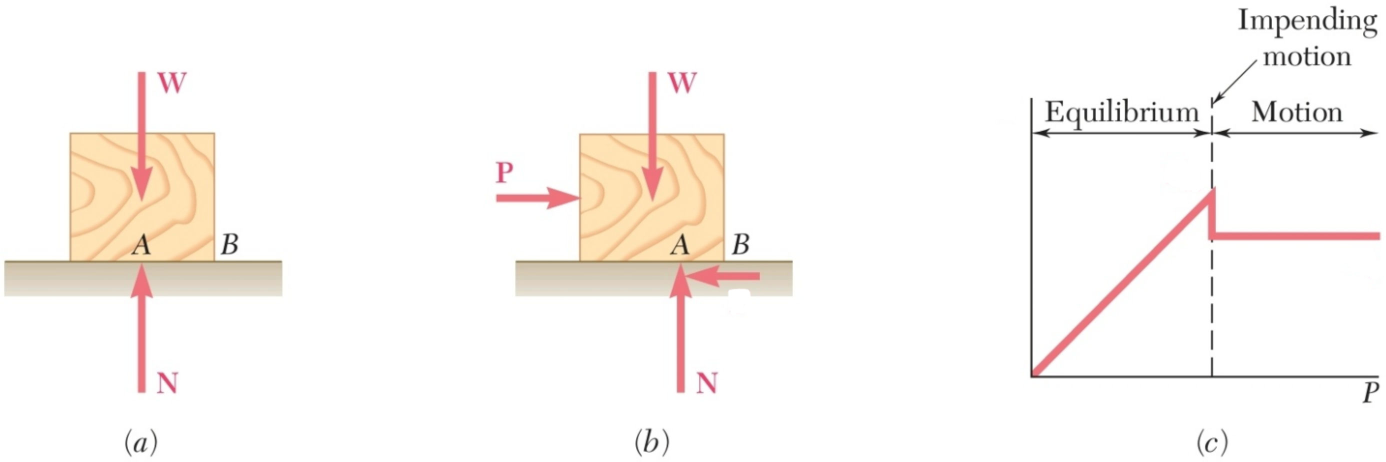


Fig. 8.1 (*a*) Block on a horizontal plane, friction force is zero; (*b*) a horizontally applied force **P** produces an opposing friction force **F**; (*c*) graph of **F** with increasing **P**.

1) Axiom of static friction: If 'no-slip' condition is valid for two RBs in contact, then the magnitude of frictional force between the surface of interaction is:

$$|\underline{f}| < \mu_s |\underline{N}|$$

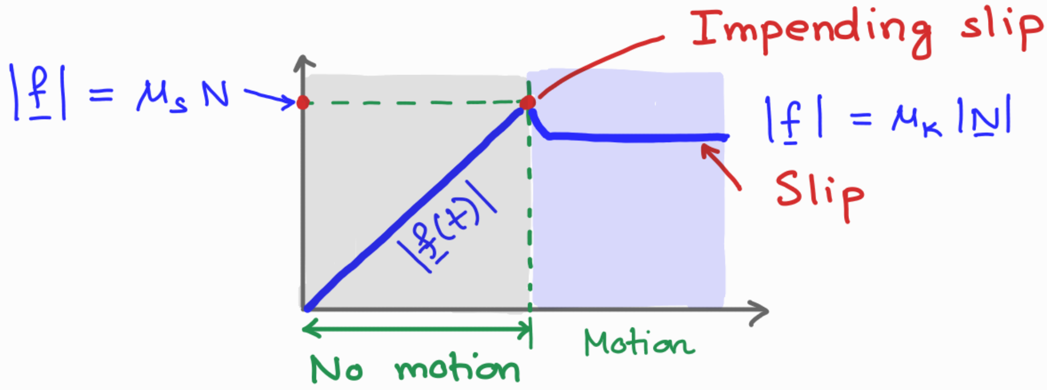
magnitude of resultant frictional force

coeff. of static friction

magnitude of resultant normal force

If "impending slip" condition is valid for the RBs, then maximum frictional force is encountered:

$$|\underline{f}| = \mu_s |\underline{N}|$$



$$\boxed{\mu_k \leq \mu_s}$$

2) Axiom of dynamic friction: If two RBs in contact are slipping / sliding relative to each other, then the magnitude of the frictional force is:

coeff. of kinetic friction

$$|\underline{f}| = \mu_k |\underline{N}|$$

Belt Friction

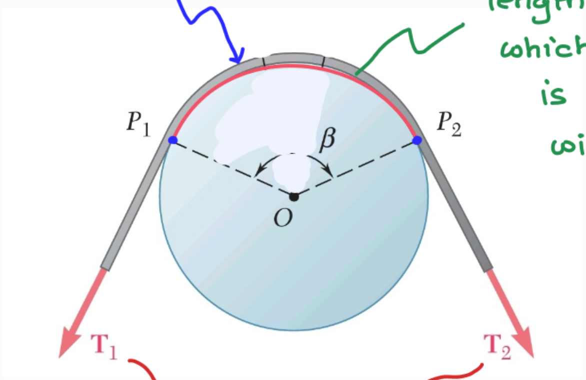
Coulomb friction in belts over pulleys serve many different purposes in engineering, such as transmitting a torque / moment from a driver pulley (connected to engine) to a driven pulley (attached to wheels).

Let's consider a flat belt passing over a fixed cylindrical drum. We want to determine the relation between the values T_1 and T_2 of the tension in the two parts of the belt where the belt is just about to slide towards the right

↳ impending motion

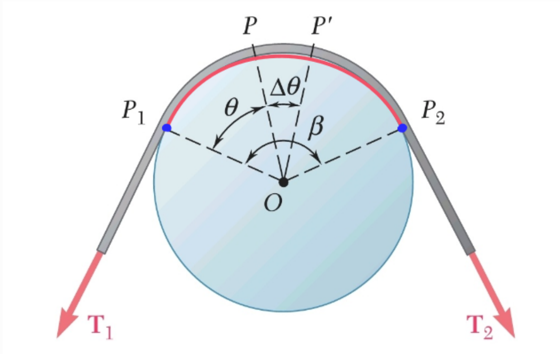
belt has negligible mass

length over which belt is in contact with drum



Tension forces at the ends of belt

Detach a small element PP' making an angle $\Delta\theta$



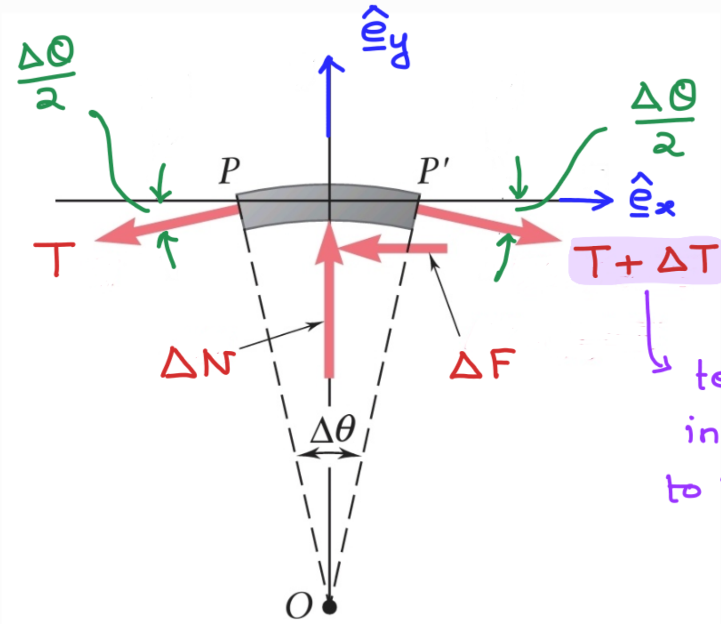
Draw an FBD of the element of belt

1) Belt mass is ignored

$\Rightarrow$ inertia effects $= 0$

2) No relative motion of between belt and pulley

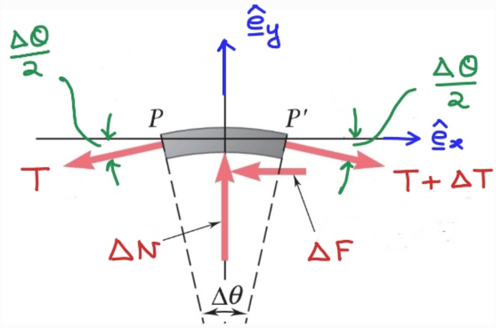
$\Rightarrow$ belt is in momentary at rest relative to pulley's surface



tension increases to the right

Therefore, the belt can be analyzed assuming static equilibrium of the infinitesimal segment of belt:

Choosing the coordinates axes as $\hat{e}_x - \hat{e}_y$, we can write equations of static equilibrium for infinitisimal segment PP':



$$\xrightarrow{+} F_{R_x} = 0: \quad (\cancel{T} + \Delta T)\cos\frac{\Delta\theta}{2} - \cancel{T\cos\frac{\Delta\theta}{2}} - \mu_s \Delta N = 0$$

$$\Rightarrow \quad \Delta N = \frac{\Delta T}{\mu_s}\cos\frac{\Delta\theta}{2} \quad \text{—} \quad (1)$$

$$+\uparrow F_{R_y} = 0: \quad \Delta N - (T+\Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0 \quad \text{—} \quad (2)$$

Substituting ΔN from eqn (2) into eqn (1)

$$\Delta T\cos\frac{\Delta\theta}{2} - \mu_s(2T + \Delta T)\sin\frac{\Delta\theta}{2} = 0$$

Now divide both terms by $\Delta\theta$, and impose limit $\Delta\theta \to 0$

$$\lim_{\Delta\theta\to 0} \quad \frac{\Delta T}{\Delta\theta}\,\overset{1}{\cancel{\cos\frac{\Delta\theta}{2}}} - \mu_s\left(T + \overset{0}{\cancel{\frac{\Delta T}{2}}}\right)\boxed{\overset{1}{\cancel{\frac{\sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}}}}} = 0$$

As $\Delta\theta \to 0$ $\rightsquigarrow$

ⓐ $\cos\frac{\Delta\theta}{2} \to 1$

ⓑ $\Delta T \to 0 \Rightarrow \lim_{\Delta\theta\to 0} \frac{\Delta T}{\Delta\theta} = \frac{dT}{d\theta}$

ⓒ $\frac{\sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}} \to 1$

Therefore, we get the following ordinary differential eqn:

$$\boxed{\frac{dT}{d\theta} - \mu_s T = 0}$$

$$\Rightarrow \frac{dT}{T} = \mu_s \, d\theta$$

Upon integrating the above equation from $\theta = 0$ to $\theta = \beta$ for θ and from T_1 to T_2 for belt tension

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu_s \, d\theta$$

assume μ_s does not vary with θ

$\Rightarrow \mu_s = \text{constant}$

$$\Rightarrow \ln\frac{T_2}{T_1} = \mu_s \beta$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = e^{\mu_s \beta}}$$

Belt friction formula with impending slip

The above formula is equally applicable to problems involving ropes wrapped around a post

* Note that the angle β MUST be expressed in RADIANS!

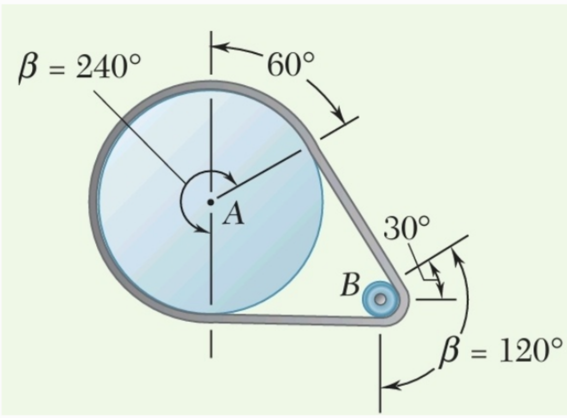
The angle β may be larger than 2π, e.g. if a rope is wrapped 'n' times around a post, $\beta = 2\pi n$

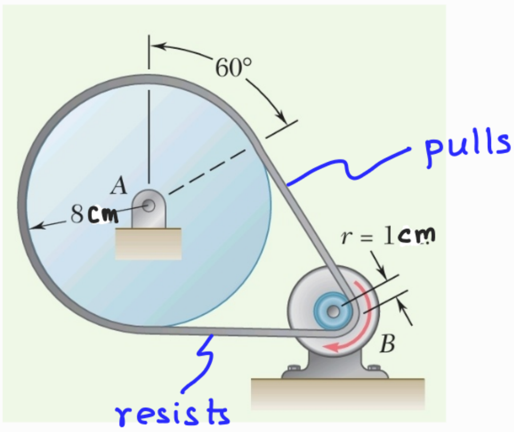
* If the belt or rope is SLIPPING, you should use coefficient of kinetic friction μ_K to find the difference in tensions

If the belt or rope is not slipping and is not about to slip, none of these formulas can be applied.

* Note T_2 is always larger than T_1. T_2 represents the tension in the part of belt that pulls

Example: A flat belt connects pulley A, which drives a machine tool, to pulley B, which is attached to the shaft of an electric motor. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. Knowing that the maximum allowable tension in the belt is 600 kg, determine the largest torque that the belt can exert on pulley A.





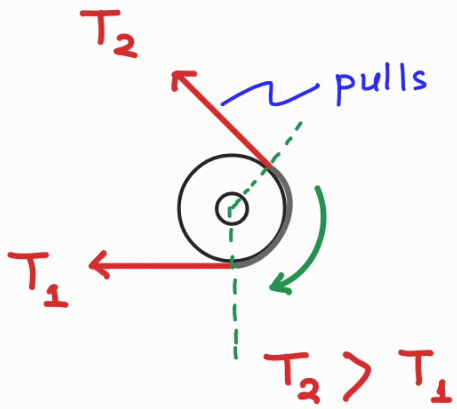
a) Identify the pulley where the slippage would first occur and then find the belt tensions

The resistance to slippage depends upon the angle of contact β between pulley and belt, as well as on μ_s.

Since μ_s is same for both pulleys, slippage would occur first on pulley B, as β is smaller for pulley B

b) Analysis using belt friction formula

Pulley B



$$T_2\big|_{max} = 600 \text{ kg} \quad \text{(given)}$$

$$\mu_s = 0.25$$

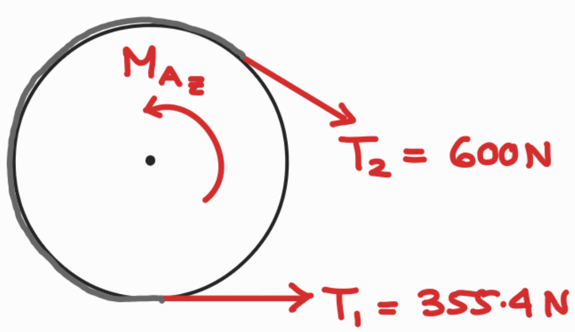
$$\beta = 120°$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \Rightarrow \frac{600}{T} = e^{0.25\left(\frac{2\pi}{3}\right)} = 1.688$$

$$\Rightarrow T_1 = 355.4 \text{ N}$$

Pulley A

FBD



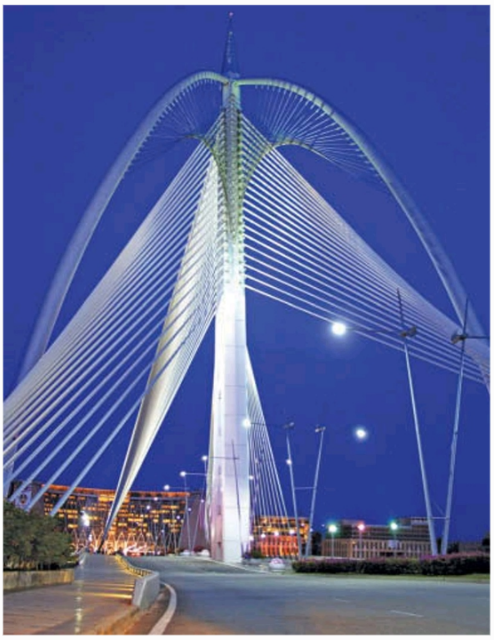
The couple M_{Az} applied by the machine tool on the pulley must be resisted by an equal and opposite torque the belt

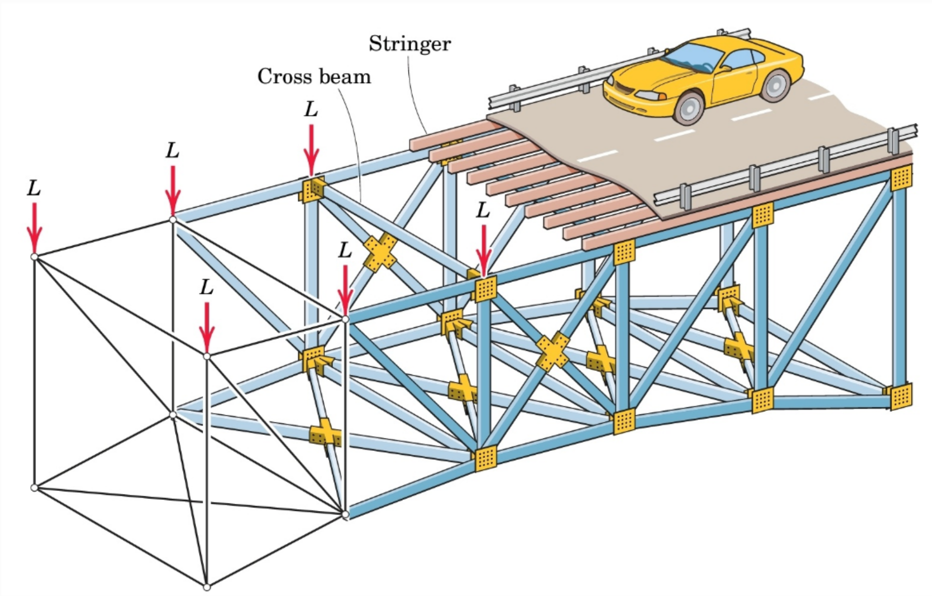
$$\stackrel{+}{\curvearrowleft} \sum M_{A_z} = 0 \Rightarrow M_A - (600\,N)(0.08\,m) + (355.4\,N)(0.08\,m)$$

$$\Rightarrow M_A = 19.57\ Nm$$

Analysis of Engineering Structures

An engineering structure is any connected system of members (or bodies) built to support or transfer forces and to safely withstand the loads applied to it.





For the next 2-3 lectures, we will analyze frames, trusses and beams, which are commonly used to build engineering structures.

To determine the forces internal to an engineering structure, we dismember the structure and analyze separate FBDs of individual members or combinations of members.

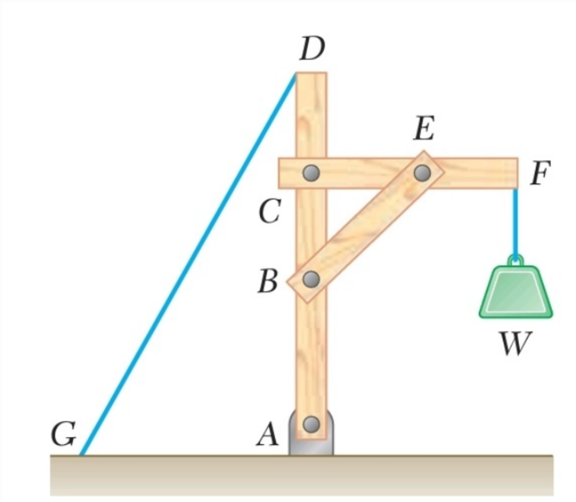
Frames → What is it?

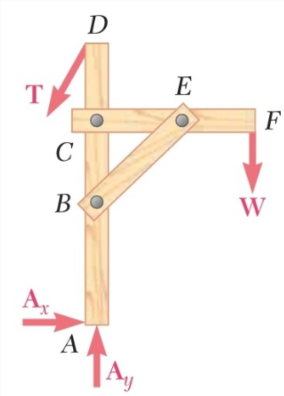
An load-bearing structure composed of several members (or RBs) connected using pin joints such that ATLEAST one member is NOT a two-force member.

In other words, frames must have at least one multi-force member, i.e. member acted upon by three or more forces

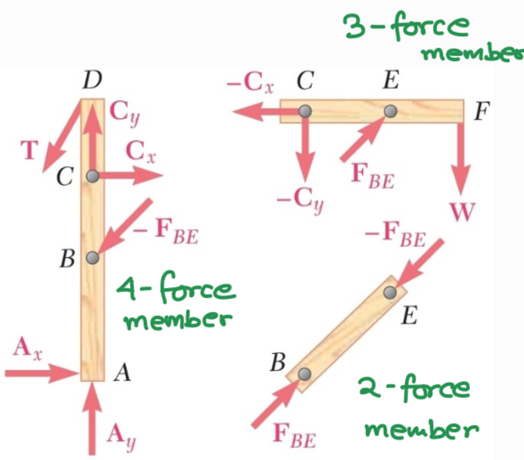
* The forces are generally not directed along the members on which they act

E.g. crane





FBD of crane



FBD of individual members

- All pin joints
- Support A is also pinned
- GD is a cable

In contrast to frames, a truss is an assembly of members where all individual members act as two-force members