Solution Quiz 01

1. DO NOT submit any re-eval request unless you are absolutely sure that your response is correct, and it has been wrongly evaluated.

2. To be sure, you must compare your response with the correct answer available in this solution file before thinking of submitting any re-eval request.

3. Carelessly submitted re-eval request may result into your marks being deducted

4. No re-eval request will be accepted after the end of your tutorial class this week (Week of Feb 03)

[See Fig. 1 (left)] An accelerometer measures the acceleration vector of a point P attached to a moving vehicle with respect to (w.r.t.) the ground frame (denoted as frame 'F'), represented as $\underline{\boldsymbol{a}}_{P|F}$. The acceleration vector is expressed using three scalar components in a coordinate system fixed on the moving vehicle (frame 'm'): $\underline{\boldsymbol{a}}_{P|F}(t) = a_t(t)\hat{\mathbf{e}}_t + a_n(t)\hat{\mathbf{e}}_n + a_b(t)\hat{\mathbf{e}}_b$.
We aim to calculate the *final* velocity of point P w.r.t. to ground frame 'F', $\underline{\boldsymbol{v}}_{P|F}$, after a specific time interval $[0, t_f]$.
Under what conditions will the following relation be SATISFIED?

$$\underline{\boldsymbol{v}}_{P|F} = \left(\int_0^{t_f} a_t(t)dt\right)\hat{\mathbf{e}}_t + \left(\int_0^{t_f} a_n(t)dt\right)\hat{\mathbf{e}}_n + \left(\int_0^{t_f} a_b(t)dt\right)\hat{\mathbf{e}}_b$$

- The relation is always satisfied as long as the vehicle moves on a flat surface.
- Only when the vehicle moves in a straight line without rotating.
- The relation is always satisfied for any general motion of the vehicle.
- The relation is never satisfied for any motion of the vehicle.
- None of the above is correct.

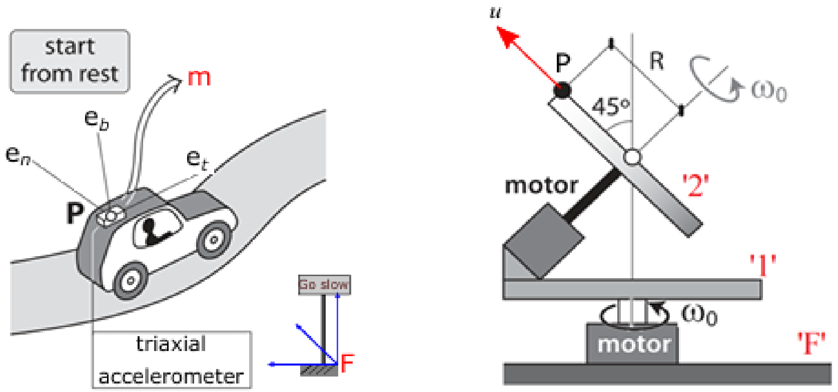


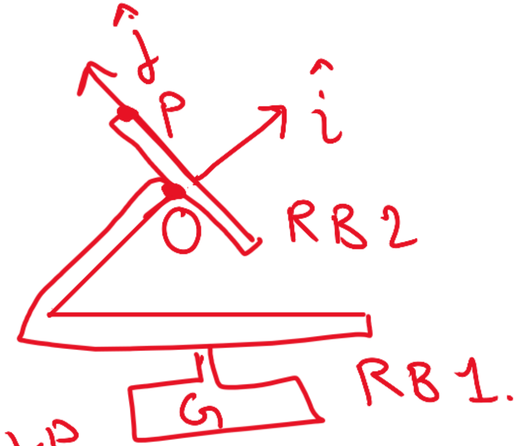
Figure 1: (Left) Moving vehicle, (Right) Rotating disk on rotating platform

[See Fig. 1 (Right)] The disk (frame '2') with radius R rotates with constant angular velocity ω_0 relative to the support (frame '1'), which in turn rotates with constant angular velocity ω_0 relative to the ground 'F'.

Point P moves radially outward along a groove on the surface of the disk at a constant speed of u w.r.t. the disk. At the current time instant, the magnitude of the acceleration of point P w.r.t. to frame '1' is:

- $\sqrt{\omega_o^4 R^2 + 4\omega_o^2 u^2}$
- $\sqrt{\omega_o^4 R^2 + \omega_o^4 R^2/2 + 2\omega_o^2 u^2}$
- $\sqrt{\omega_o^4 R^2 + \omega_o^4 R^2/4 + 2\omega_o^2 u^2}$
- 0
- None of the above is correct.

Solution



Use 2as1.F., O as1.P

$$\underline{a}_{P|1} = \underline{a}_{O|1} + \dot{\underline{\omega}}_{2|1} \times \underline{r}_{PO} + \underline{\omega}_{2|1} \times (\underline{\omega}_{2|1} \times \underline{r}_{PO}) + 2\underline{\omega}_{2|1} \times \underline{V}_{P|2} + \underline{a}_{P|2}$$

$\underline{a}_{P|2} = 0$, $\underline{V}_{P|2} = u\hat{j}$, $\underline{\omega}_{2|1} = \omega_0\hat{i}$

$\dot{\underline{\omega}}_{2|1} = 0$, $\underline{r}_{PO} = R\hat{j}$, $\underline{a}_{O|1} = 0$

$$\underline{a}_{P|1} = -\omega_0^2 R\hat{j} + 2\omega_0 u\hat{k}$$

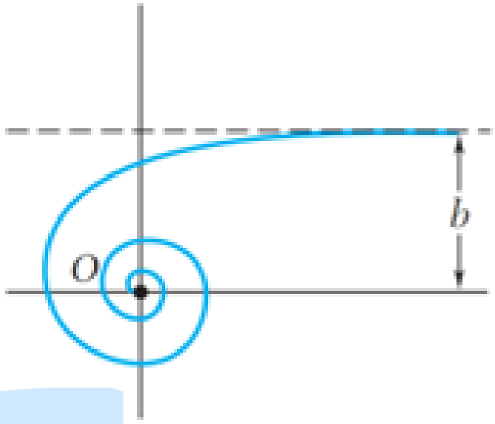
$$|\underline{a}_{P|1}| = \omega_0\sqrt{\omega_0^2 R^2 + 4u^2}$$

A particle (P) moves along the planar spiral trajectory shown in Fig. 2 (Left). The equation of this trajectory is $r\theta = b$, where (r, θ, z) are the coordinates of P in the cylindrical-polar coordinate system, and b is a constant. Given that $b = 2$m, and $\dot{\theta} = \pi$ rad/s (constant), find the radial component of the acceleration of particle P w.r.t. the ground frame at the instant when $\theta = \pi/2$ rad:

radial component of acceleration = $\ddot{r} - r\dot{\theta}^2$ where

$$r = \frac{b}{\theta}, \Rightarrow \dot{r} = -\frac{b}{\theta^2}\dot{\theta} \Rightarrow \ddot{r} = \frac{2b}{\theta^3}\dot{\theta}^2.$$

$$\ddot{r} - r\dot{\theta}^2 = \frac{2b\dot{\theta}^2}{\theta^3} - \frac{b}{\theta}\dot{\theta}^2$$



Hyperbolic spiral $r\theta = b$

Correct answers depending on your set

$-2.4\,ms^{-2}$ or $-4.8\,ms^{-2}$ or $-7.1\,ms^{-2}$

The disk shown in Fig. 2 (Right) rotates at a constant angular velocity $\omega = 50(\text{rad/s})$, in the clockwise direction, relative to the ground frame (G). The center of the disk, point A, is fixed to the ground. The sleeve D is restricted to move only in the vertical direction. The rod BD, which is $d = 300$ mm long, connects point B on the disk to the sleeve D. The angular acceleration of rod BD relative to the ground frame at the instant when $\theta = 0°$ is:

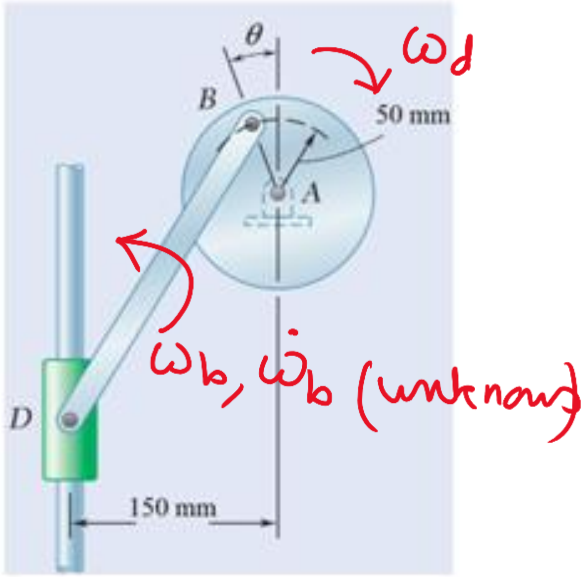
$\omega_d = 50 \text{ rad s}^{-1}$

Velocity analysis of point B

from side of A: $\underline{V}_{B|G} = \omega_d 50 \hat{i} \rightarrow (i)$

from side of D: $\underline{V}_{B|G} = \underline{V}_{D|G} + \omega_b \hat{k} \times (\underline{r}_{BD})$

$= V_0 \hat{j} + \omega_b \hat{k} \times \left[150 \hat{i} + \sqrt{d^2 - 150^2}\, \hat{j}\right]$ (ii)



(Right) Rotating disk and a connected rod

Match (i) & (ii)

$\omega_d 50 \hat{i} = V_0 \hat{j} + \omega_b 150 \hat{j} - \omega_b \sqrt{d^2 - 150^2}\, \hat{i}$.

$\Rightarrow \omega_d (50) = -\omega_b \sqrt{d^2 - 150^2}$

$$\omega_b = \frac{-50 \omega_d}{\sqrt{d^2 - 150^2}}.$$

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The disk shown in Fig. 2 (Right) rotates at a constant angular velocity $\omega = 50(\text{rad/s})$, in the clockwise direction, relative to the ground frame (G). The center of the disk, point A, is fixed to the ground. The sleeve D is restricted to move only in the vertical direction. The rod BD, which is $d = 300$ mm long, connects point B on the disk to the sleeve D. The angular acceleration of rod BD relative to the ground frame at the instant when $\theta = 0°$ is:

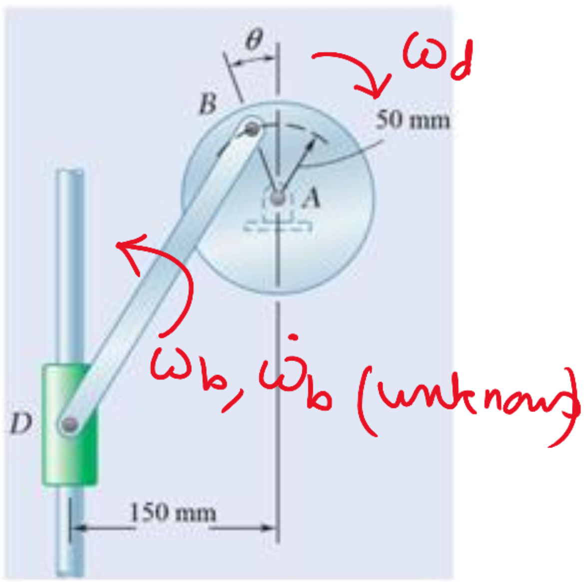
acceleration analysis of point B

from the side of A. $\underline{a}_{B|G} = -\omega_d^2\, 50\hat{j}$ (iii)

from the side of D

$\underline{a}_{B|G} = \underline{a}_{D|G} + \dot{\underline{\omega}}_b \hat{k} \times (\underline{r}_{BD}) + \omega_b \hat{k} \times (\omega_b \hat{k} \times \underline{r}_{BD})$ (iv)

$= a_0 \hat{j} + \cdots$



Right) Rotating disk and a connected rod

Match (iii) & (iv)

$-\omega_d^2\, 50\hat{j} = a_0\hat{j} + 150\dot{\omega}_b\hat{j} - \dot{\omega}_b\sqrt{d^2-150^2}\,\hat{i} - 150\,\omega_b^2\,\hat{i} - \omega_b^2\sqrt{d^2-150^2}\,\hat{j}$

$\Rightarrow \dot{\omega}_b = -150\,\omega_b^2/(d^2-150^2)^{1/2} = -150\times 2500\,\omega_d^2/(d^2-150^2)^{3/2}$

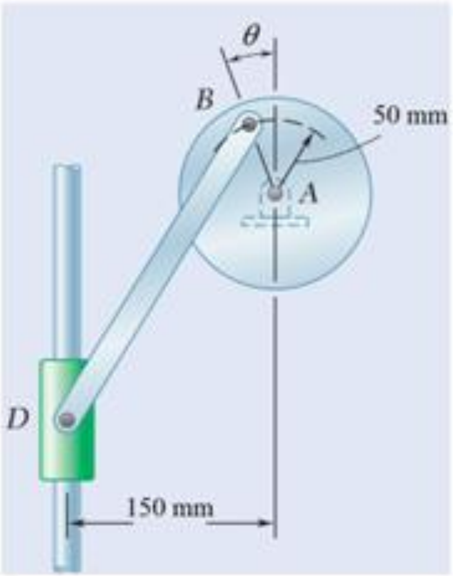
The disk shown in Fig. 2 (Right) rotates at a constant angular velocity $\omega = 50(\text{rad/s})$, in the clockwise direction, relative to the ground frame (G). The center of the disk, point A, is fixed to the ground. The sleeve D is restricted to move only in the vertical direction. The rod BD, which is $d = 300$ mm long, connects point B on the disk to the sleeve D. The angular acceleration c
ground frame at the instant when $\theta = 0°$ is:

depending on your set, correct answer

53.5 rads^{-2}

or, 18.4 rads^{-2}

or, 8.6 rads^{-2}



Right) Rotating disk and a connected rod

Which of the following options satisfies the velocity transfer relationship correctly:

$$\underline{\boldsymbol{v}}_{P|①} = \underline{\boldsymbol{v}}_{P|②} + \underline{\boldsymbol{v}}_{③|④} + \underline{\boldsymbol{\omega}}_{⑤|⑥} \times \underline{\boldsymbol{r}}_{⑦}$$

Verify your answer yourself

$$\underline{V}_{P|F} = \underline{V}_{A|F} + \underline{\omega}_{m|F} \times \underline{r}_{PA} + \underline{V}_{P|m}.$$