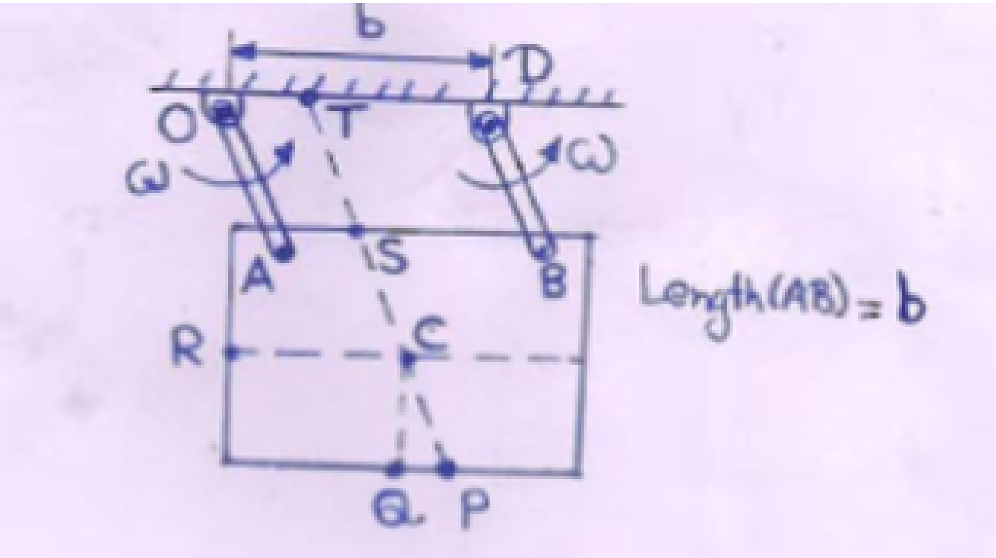
Q. 1 The expression of kinetic energy of a rigid body w.r.t. a frame F is known to be: $T = \frac{1}{2}mV_A^2 + \frac{1}{2}\left(I_{11}^A\omega_1^2 + I_{22}^A\omega_2^2 + I_{33}^A\omega_3^2\right)$. Symbols have thier usual meanings. Which of the following is true?

- ○ A can be any point fixed to the body
- ○ A may be the center of mass of the body
- ○ the expression is true only if all quantities are defined with respect to an inertial reference frame.
- ○ None of these is correct.

Q. 2 A reactnagular plate with center of mass C is pinned to two rods of length L each, which rotate in a vertical plane at a constant angular velocty ω. The line PS is parallel to the two rods at this instant. The moment of all external forces acting on the plate must be zero about



- ○ at least, Point C and Point Q,
- ○ Point C only
- ○ at least, Point C and Point P,
- ○ None of these is correct.

$0 = 0 + \omega_{plate|I} \times (\hat{e}_1)$

$\omega' \hat{k} \times \hat{e}_{1H} = \omega' \hat{k} \hat{e}_2 = 0$ A H B

$\Rightarrow \omega' = 0$

$$\underline{a}_{B|I} = \underline{a}_{A|I} + \dot{\underline{\omega}}_{plate|I} \times \underline{r}_{BA} + \underline{\omega}_{plate|I} \times (\underline{\omega}_{plate|I} \times \underline{r}_{BA})$$

$$+ 2\underline{\omega}_{plate|I} \times \underline{V}_{B|plate} + \underline{a}_{B|plate}.$$

But $\underline{a}_{B|I} = \omega^2 L \hat{e}$, $\underline{a}_{A|I} = \omega^2 L \hat{e}$

$\Rightarrow \dot{\underline{\omega}}_{plate|I} \times \underline{r}_{BA} = \dot{\omega}' \hat{k} \times H \hat{e}_1 = 0 \Rightarrow \dot{\omega}' = 0$

$\therefore$ plate is purely translating $\Rightarrow \underline{a}_{x|I} = \omega^2 L \hat{e}$ where $x \in$ plate (any point)

But

$$\underline{H}_{x|I} = \underline{\underline{I}}^x (\omega' \hat{e}_k)$$

$$\frac{d}{dt}\{\underline{H}_{x|I}\} = \frac{d}{dt}\underline{\underline{I}}^x (\omega' \hat{e}_k) + \underline{\underline{I}}^x \frac{d}{dt}(\omega' \hat{e}_k) \quad 0$$

Since the plate has the plane as symm plane

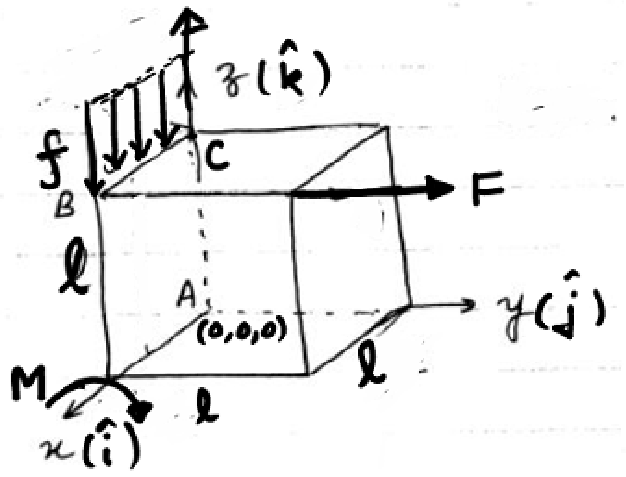
$\Rightarrow (\underline{M}_x)_3 = I^x_{33} \frac{d}{dt}(\omega') - \underline{r}_{cx} \times \underline{a}_{x|I}$

But $\frac{d}{dt}(\omega') = 0 \therefore (\underline{M}_x)_3 = 0$ if $\underline{r}_{cx} \times \underline{a}_{x|I} = 0$

for $x = c$ ✓ $\quad x = P \quad \underline{r}_{cp} \times \underline{a}_{P|I} = 0$ because $\underline{a}_{P|I} \parallel$

P ✓ $\quad$ Q: ? $r_{cQ} \times a_{Q|I} \neq 0$

Q. 3 In the shown figure f is a a distributed force (N/m) and M represents a torque (Nm). The resultant force system of the shown force system at A is comprised of a force $\underline{F}$ and a torque $\underline{C}$. Choose the correct answer.



- ○ $\underline{F} = -fl\hat{k} + F\hat{j}$, $\underline{C} = -(M + lF)\hat{i} + \frac{fl^2}{2}\hat{j} + lF\hat{k}$
- ○ $\underline{F} = -fl\hat{k} + F\hat{j}$, $\underline{C} = -(M + lF)\hat{i} + lF\hat{k}$
- ○ $\underline{F} = -fl\hat{k} + F\hat{j}$, $\underline{C} = -(M)\hat{i}$
- ○ None of these is correct.

<u>Solution Minor II</u> ①

(i) Simplest Resultant is a point force. $\underline{F}_R$

① $\underline{F}_R = -f\ell\hat{k}$ acts at centroid of Area under loading curve.

∴ acts at ① $(\ell/2, 0, \ell)$ or $(\ell/2, 0, z)$ $\forall z$ Answer

Point: E (say)

(ii) ① $\underline{F}_{RA} = -f\ell\hat{k} + F\hat{j}$

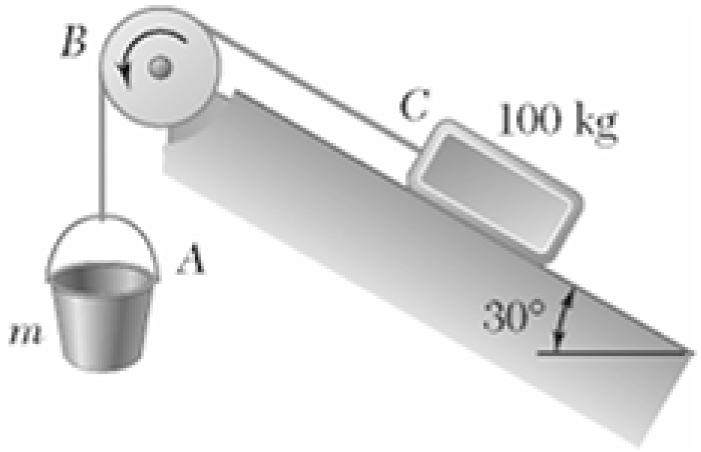
① $\underline{C}_{RA} = r_{EA} \times (-f\ell\hat{k}) + \underline{r}_{DA} \times (F\hat{j}) - M\hat{i}$

$= \left(\frac{\ell}{2}\hat{i} + \ell\hat{k}\right) \times (-f\ell\hat{k}) + (\ell\hat{i} + \ell\hat{j} + \ell\hat{k}) \times (F\hat{j}) - M\hat{i}$

$= +\frac{f\ell^2}{2}\hat{j} + \ell F\hat{k} - \ell F\hat{i} - M\hat{i}$

② $\underline{C}_{RA} = -(M + \ell F)\hat{i} + \frac{f\ell^2}{2}\hat{j} + \ell F\hat{k}$ Answer

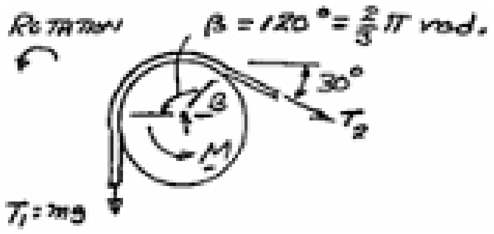
Q. 4 (answer to be checked, BJ 8.118) Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise (at a constant angular velocity driven by an external mechanism) and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will reamin at rest ($g = 9.81ms^{-2}$, $\pi = 3.1415$).



- ○ 5.9 kg
- ○ 15.8 kg
- ○ 11.7 kg
- ○ None of these is correct.

SOLUTION

<u>Free body: Drum</u>



$$\frac{T_2}{mg} = e^{\mu \frac{2}{3}\pi}$$

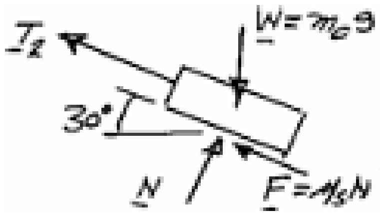
$$T_2 = mge^{2\mu\pi/3} \tag{1}$$

(*a*) <u>Smallest *m* for block *C* to remain at rest</u>

Cable slips on drum.

Eq. (1) with $\mu_k = 0.25$; $T_2 = mge^{2(0.25)\pi/3} = 1.6881mg$

<u>Block *C*:</u> At rest, motion impending ↘



$+\nearrow \Sigma F = 0$: $N - m_C g \cos 30°$

$N = m_C g \cos 30°$

$F = \mu_s N = 0.35 m_C g \cos 30°$

$m_C = 100 \text{ kg}$

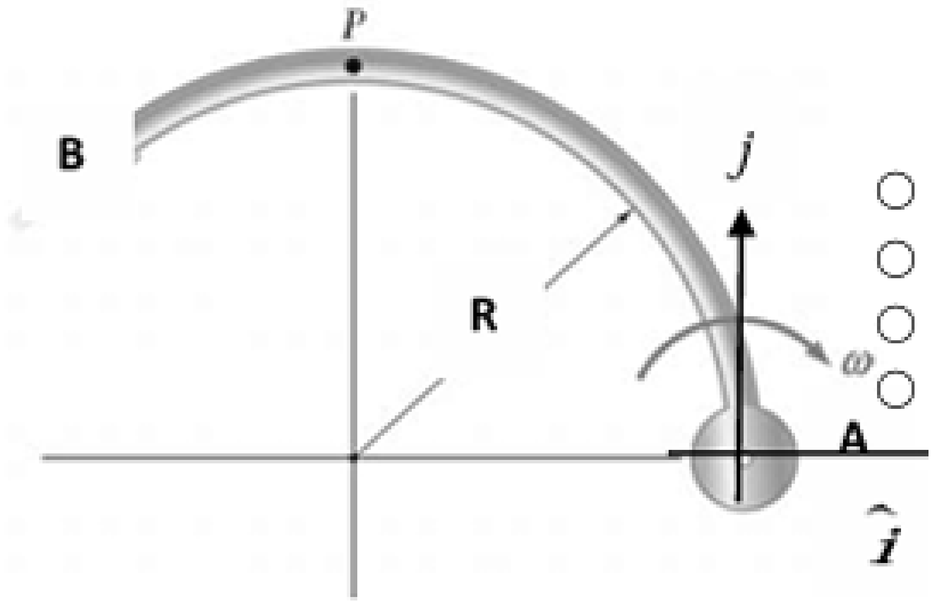
$+\searrow \Sigma F = 0$: $T_2 + F - m_C g \sin 30° = C$

$$1.6881mg + 0.35m_C g \cos 30° - m_C g \sin 30° = 0$$

$$1.6881m = 0.19689m_C$$

$m = 0.11663m_C = 0.11663(100 \text{ kg})$; $m = 11.66 \text{ kg}$ ◀

Q. 5 AB is a rigid circular tube (radius R, see figure). AB rotates with constant angular velocity 9.4 rad/s clockwise in the plane of the paper relative to the ground frame. Point A is fixed to ground. Particle P is free to move along the pipe (of small cross section). At the shown instant, the particle is moving with velocity -8$\hat{i}$ m/s with respect to the pipe. Given that the acceleration of P with respect to ground is 44.4$\hat{i}$ - 21.7$\hat{j}$ ms^{-2} at the shown instant, choose the correct answer



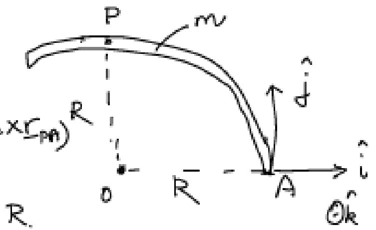
- ○ $R^2 - 1.94R + 0.72 = 0$
- ○ $R^2 + 1.94R - 0.72 = 0$
- ○ $R^2 + 1.94R + 0.72 = 0$
- ○ None of these is correct.

Question 4

We have $\omega_{m|F} = -9.4\hat{k}$ rad s^{-1}

$v_{P|m} = -8\hat{i}$

$\dot{\omega}_{m|F} = 0$



(2) $a_{P|F} = a_{A|F} + \dot{\omega}_{m|F}\times r_{PA} + \omega_{m|F}\times(\omega_{m|F}\times r_{PA}) + 2\omega_{m|F}\times v_{P|m} + a_{P|m}$

($a_{A|F} = 0$, $\dot{\omega}_{m|F} = 0$)

$a_{P|m}$ needs to written in terms of R.

$$a_{P|m} = a_{P|F} - \omega_{m|F}\times(\omega_{m|F}\times r_{PA}) - 2\omega_{m|F}\times v_{P|m} \quad (i)$$

(1) $2\omega_{m|F}\times v_{P|m} = 2(-9.4\hat{k})\times(-8\hat{i}) = 150.4\hat{j} \quad (ii)$

(1) $\omega_{m|F}\times(\omega_{m|F}\times r_{PA}) = (\omega_{m|F}\cdot r_{PA})\,\omega_{m|F} - |\omega_{m|F}|^2 r_{PA}$

Note since $v_{P|m}$ is purely along $\hat{i} \Rightarrow OP = OA = R$

(2) $= -(9.42)^2(-R\hat{i} + R\hat{j})$

$= 88.74R\hat{i} - 88.74R\hat{j} \quad (iii)$

$a_{P|F} = 44.4\hat{i} - 21.7\hat{j} \longrightarrow (iv)$ (given)

Put iv, iii, ii, in (i)

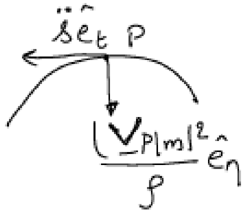
$\Rightarrow a_{P|m} = 44.4\hat{i} - 21.7\hat{j} - 88.74R\hat{i} + 88.74R\hat{j} - 150.4\hat{j}$

(2) $$\boxed{a_{P|m} = (44.4 - 88.74R)\hat{i} + (-172.1 + 88.74R)\hat{j}}$$

Answer

(4) For an observer tied to m, P is moving on a circular trajectory with radius R. Therefore

$$+\frac{|v_{P|m}|^2}{R}\hat{e}_n = (a_{P|m})_{normal}$$



or $-(a_{P|m})_{\hat{j}\text{ component}} = \dfrac{|v_{P|m}|^2}{R}$

or $-(-172.1 + 88.74R) = \dfrac{8^2}{R}$

or $-\dfrac{64}{R} = -172.1 + 88.74R$

$0 = 88.74R^2 - 172.1R + 64$

(2) $0 = R^2 - 1.94R + .72$

$0 = R^2 - 1.94R + .72$

$R = \dfrac{1.94 \pm \sqrt{1.94^2 - 4\times .72}}{2} = \dfrac{1.94 \pm \sqrt{.8836}}{2}$

(2) $$\boxed{R = \frac{1.94 \pm .94}{2} = 1.44 \text{ or } 0.5\,m}$$ Answer.